4.2. Generating new 'original' results

Now we are going to demonstrate how our theory could be (ab)used to generate new 'original' results matching presented patterns. We can formulate results that (probably) nobody formulated so far. First, we could generalize known results to other classes of algebras:

Corollary 31 (Type 1.) *Let* $\boldsymbol{A}$ *be an* IMTL-*algebra. The following statements are equivalent.*

1. *Every* IMTL-*filter on* $\boldsymbol{A}$ *is an implicative* IMTL-*filter.*
2. $\{\overline{1}\}$ *is an implicative* IMTL-*filter on* $\boldsymbol{A}$.
3. $\boldsymbol{A}$ *is Boolean algebra.*

Corollary 32 (Type 2.) *Boolean and implicative filters of any* IMTL-*algebra coincide.*

Corollary 33 (Type 3.) *Let* $\boldsymbol{A}$ *be an* IMTL-*algebra. If* F *is a implicative filter, then every filter* G *containing* F *is also a implicative.*

Corollary 34 (Type 4.) *A filter* F *of an* IMTL-*algebra* $\boldsymbol{A}$ *is implicative iff every filter on the quotient algebra* $\boldsymbol{A}/F$ *is implicative.*

Second, we could also define new classes of filters (with as good motivation as those defined in the literature have) and prove all four types of results.

Definition 35 *Let* $\boldsymbol{A}$ *be a* MTL-*algebra. A filter* F *is called a contractional filter on* $\boldsymbol{A}$ *if for all* $x \in A$ *we have* $x \to y \in F$ *whenever* $x \to (x \to y) \in F$.

As we know that MTL + contrational property (expressed as consecution) is Gödel logic, we immediately obtain:

Corollary 36 (Type 1.) *In any* MTL-*algebra* $\boldsymbol{A}$ *the following conditions are equivalent:*

1. *Every filter on* $\boldsymbol{A}$ *is a contractional filter.*
2. $\{\overline{1}\}$ *is a contractional filter.*
3. $\boldsymbol{A}$ *is a Gödel algebra.*

Corollary 37 (Type 2.) *Contractional and implicative filters of any* MTL-*algebra coincide.*

Corollary 38 (Type 3.) *Let* $\boldsymbol{A}$ *be an* MTL-*algebra,* $F, G \subseteq A$ *filters. If* $F \subseteq G$ *and* F *is a contractional filter, then* G *is also a contractional filter.*

Corollary 39 (Type 4.) *Let* $\boldsymbol{A}$ *a* MTL-*algebra and* F *an* MTL-*filter. Then* F *is an contractional filter on* $\boldsymbol{A}$ *if and only if every* MTL-*filter on the quotient algebra* $\boldsymbol{A}/F$ *is a contractional filter.*

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