













### Remark 7.5

- A possible definition of when two predicates  $P$  and  $Q$  in a universe of discourse  $X \subset R$  are exact synonyms could be done by the equality,  $\mu_Q = \mu_P \circ f$ , once they are represented by fuzzy sets  $\mu_P, \mu_Q$  in  $[0, 1]^X$ , and  $f : X \rightarrow X$  is a bijective function such that:

1.  $x \leq y \rightarrow f(x) \leq f(y)$
2.  $f(Z(\mu_Q)) \subset Z(\mu_P)$
3.  $f(S(\mu_Q)) \subset S(\mu_P)$

Notice that this definition is a particular case ( $F = id$ ) of definition 7.1. Is in this sense that an exact synonym can be viewed as a migration. Then, predicates in example 7.4 can be taken as exact synonyms.

- Although definition 6.2 is only for a totally ordered universes, it is not a too limiting one since, in most practical cases (like that in 3.2, point 2), the elemental meaning comes from some equivalence in  $\mathbb{R}$ .

## 8. Conclusions

In his *Philosophical Investigations*, Wittgenstein conceived language in a way close to how people manages it. Thus, the meaning of an imprecise predicate is not given by necessary and sufficient conditions, but is built up by some similarity, that partially preserves prototypes and is done in processes that evolve over tie and finish when the predicate becomes obsolete. Notwithstanding, for more complex predicates like  $P = beautiful$  in a set of art objects, it could be not clear enough the existence of prototypes and, since in such cases it could be  $S(\mu_P) = \emptyset$ , the study of the family resemblance for these predicates remains an open problem.

It should be pointed out that the definition of family resemblance introduced in this paper is essentially done through monotonic functions between the (totally ordered) universes of discourse. It is for this reason that it seems suitable to consider its relation with some Galois Connection.

In natural language, families  $[\mu]$  of ‘relatives’ must be ‘open’, but not ‘closed’ like they were defined. These families are here static (sets), but in natural language they should have a dynamic character. With time, an element  $\sigma$  that was not in  $[\mu]$ , could be included in  $[\mu]$  and thus generating a new family, indeed, changing the relation **fr**. Actually, **fr** is not a permanent relation in natural language, it is a changing one. Only with families of resemblance taken as classical sets, human thought seems to be impossible (see [5]). It is for reasons like this, that this paper cannot be seen as anything else than some unended initial reflections on the representation of the concepts

of family resemblance and predicates linguistic migration. It yet lacks, for instance, a study of family resemblance in non-totally ordered universes.

## References

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