Theorem 9 *Define* $y_n : \mathcal{J} \to D$ *by the equality* $y_n((-\infty, t)) = h_n(\Delta_t^n)$. *Then* y_n *is an observable.*

Proof. It follows by properties 1 - 3 of the previous Definition.

Definition 10 *Let* $x_1, ..., x_n : \mathcal{J} \to D$ *be independent observables. Then the observable* $y_n : \mathcal{J} \to D$ *defined in previous Theorem is called the sum of observables* $x_1, ..., x_n, y_n = \sum_{i=1}^n x_i$, *i.e.*

$$(\sum_{i=1}^n x_i)((-\infty, t)) = h_n(\Delta_t^n), t \in R.$$

Remark. There has been proved in [5] that in so-called Kôpka D-posets there exists the mapping $h_n : \mathcal{M}_n \to D$ satisfying the properties stated in previous Definition.

4. Central limit theorem

We are able now to formulate and prove the main result of the paper. We shall use the following notation. If $y : \mathcal{J} \to D$ is an observable and α, β are real numbers, $\alpha \neq 0$, then $\alpha y + \beta : \mathcal{J} \to D$ is defined by the formula

$$(\alpha y + \beta)((-\infty, t)) = y((-\infty, \frac{1}{\alpha} - \beta)).$$

Theorem 11 *Let* $(x_n)_{n=1}^\infty$ *be an independent sequence of square integrable observables,* $E(x_n) = a$, $\sigma^2(x_n) = \sigma^2$, $(n = 1, 2, ...)$. *Then for any* $t \in R$

$$\lim_{n\to\infty} m\left((\frac{\sqrt{n}}{\sigma} \sum_{i=1}^n x_i - a)((-\infty, t)) \right) = \Phi(t) =$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^t e^{-\frac{x^2}{2}} dx.$$

Proof. Denote $P_n = \lambda_{F_1} \times ... \times \lambda_{F_n} : \mathcal{B}(R^n) \to [0, 1]$. Then (P_n) presents a consistent system of probability measures:

$$P_n(A \times R) = P_{n-1}(A), A \in \mathcal{B}(R), n \in N$$

By the Kolmogorov consistence theorem there exists $P : \sigma(\mathcal{C}) \to [0, 1]$ where

$$\mathcal{C} = \{A \subset R^N; A = \pi_n^{-1}(B), B \in \mathcal{B}(R_n), n \in N\}$$

such that

$$P(\pi_n^{-1}(B)) = P_n(B) = \lambda_{F_1} \times ... \times \lambda_{F_n}(B)$$

for any $B \in \mathcal{B}(R^n), n \in N$. Define $\xi_n : R^N \to R$ by the formula

$$\xi_n((u_i)_{i=1}^\infty) = u_n.$$

Therefore

$$(\sum_{i=1}^n x_i)((-\infty, t)) = h_n(\Delta_t^n) = \lambda_{F_1} \times ... \times \lambda_{F_n}(\Delta_t^n) =$$

$$= P_n(\Delta_t^n) = \{\omega; \xi_1(\omega) + ... + \xi_n(\omega) < t\}.$$

We obtained

$$m\left(\frac{\sqrt{n}}{\sigma} (\sum_{i=1}^n x_i - a)((-\infty, t)) \right) =$$

$$= m\left((\sum_{i=1}^n x_i)(-\infty, \frac{\sigma}{\sqrt{n}}(a + t)) \right) =$$

$$= P\left(\{\omega; \frac{\sqrt{n}}{\sigma} (\sum_{i=1}^n \xi_i(\omega) - a) < t\} \right).$$

Now by the classical limit theorem we obtain

$$\lim_{n\to\infty} m\left((\frac{\sqrt{n}}{\sigma} \sum_{i=1}^n x_i - a)((-\infty, t)) \right) =$$

$$\lim_{n\to\infty} m\left((\frac{\sqrt{n}}{\sigma} \sum_{i=1}^n \xi_i - a)((-\infty, t)) \right) = \Phi(t).$$

References

[1] D. J. Foulis, M. K. Bennet, The difference poset of monotone functions, *Foundations of Physics*, 24: 1325–1346, Springer-Verlag, 1994.

[2] J. Chajda, R. Halaš, J. Kühr, Every effect algebra can be made into a total algebra, *Algebra universalis*, 61:133–150, Springer-Verlag, 2009.

[3] F. Kôpka, F. Chovanec, D-posets, *Math. Slovaca*, 44: 21–34. 1994.

[4] M. Kuková, B. Riečan, Strong law of large numbers on the Kôpka D-posets. In *proceedings of the* 11^{th} *International Conference on Applications of Statistics and Probability in Civil Engineering* (ICASP11), August 1–4, Zürich, Switzerland, (to appear)

[5] B. Riečan, L. Lašová, On the probability theory on the Kôpka D-posets, *Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics*, 1: 167–176, Systems Research Institute, Polish Academy of Sciences, 2010.

[6] B. Riečan, D. Mundici, Probability on MV-algebras. *Handbook of Measure Theory*, Elsevier, Amsterdam, 869 – 909, 2002.

[7] B. Riečan, T. Nebrunn, *Integral, Measure, and Ordering*, Kluwer, Dordrecht, 1997.