Spatial Error Modeling and Simulation on Sheet Metal Assembly in Single Station

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Abstract—Based on analyzing error model of Sheet Metal Assembly, namely SMA, at home and abroad, this paper establishes the propagation model of spatial error of SMA in single station, and founds the rigid error model by differential motion theory and coordinate conversion theory. On condition of satisfying the closed form and closed force, the paper analyzes the assembly deformation errors in positioning and clamping process, connecting process and spring-back process, and sets up the spatial error model of SMA in single station, and then simulates assembly errors by using MATLAB, finally compares them with the results from CATIA-TAA, and there is a good agreement between these results. The model is of significance in the work of assembly error prediction, fixture optimization design, fixture error diagnosis and tolerance synthesis design.

Keywords—SMA(Sheet Metal Assembly); Spatial Error; PCCR (Positioning; Clampin; Connecting and Releasing); Closed Form and Closed Force

I. INTRODUCTION

Sheet metal subassembly is widely used in automobile, aviation, shipbuilding and instrumentation industries, and the precision of sheet metal subassembly influences its performance, such as the sealing and aerodynamic performance for automobile, the sealing performance and fatigue strength for ship and aircraft [1]-[5]. Nearly two-thirds engineering changes in the automotive and aviation fields are caused by the failure of subassembly [1][2]. Sheet metal part errors, tool errors, clamping deformation errors, connecting deformation errors and spring-back deformation errors are inevitable in the assembly process, and they propagate, accumulate and reduce along the assembly process, and appear to be highly nonlinear and coupling each other, and more importantly the shape and force are also closed in the assembly process [2][13][14]. Traditional superposition theory for rigid body assembly is no longer applicable for sheet metal assembly, therefore more and more scholars pay close attention to the error modeling about sheet metal assembly[4]-[14].

Sheet metal assembly is divided into single station assembly of no relocation and multi-station assembly which is of relocation. Sheet metal assembly in single station should follow PCCR assembly process, including positioning, clamping, connecting and releasing spring-back.[11][13] Hu and Liu established the assembly error model of flexible part in single station in the method of influence coefficient, namely MIC.[7] Camelio and Ceglarek established the assembly error model in single/multi-station by MIC and state space method, and the model was applied to the fixture error diagnosis and error sensitivity analysis [8][9]. Dahlstrom and Liao analyzed and simulated nonlinear contact problem of sheet metal assembly in nonlinear finite element method, namely FEM [10][11]. Stewart analyzed the assembly errors in PCCR assembly process by the piecewise linear elastic method, namely PLEM.[12] Huang established the assembly error model about the open assembly process by linear elastic theory, and deduced in-plane errors and out of plane errors are independent on condition of small deformation, and the fixture and tool errors affecting the assembly deformation can be ignored in open assembly process[14]. Liu and Hu analyzed three kinds of basic connecting ways which had an effect on the assembly error, and deduced that the overlap-joint way can absorb in-plane error and the butt-joint way can magnify out of plane error [17]. Franciosa modeled and simulated sheet metal assembly error by using SVA-FEA [13]. Huang, Long and Cai simulated the errors on sheet metal assembly in single station by finite element method. The foreign toolkits for error analysis and Simulation, such as 3DCS-FEA, VisSVA-FEA and CATIA-TAA, were encapsulated and difficult to reuse. [13]-[17] Professor Chen guan-long at SJTU constructed the design system in the assembly process based on case reasoning, called BBA, which can generate and optimize the assembly sequence and the joint way [18].

This thesis analyzes the error sources of the assembly process, establishes the rigid error model on sheet metal assembly by differential motion and coordinate transformation, and builds the deformation error model on sheet metal assembly in accordance with the PCCR assembly process on the basis of the analysis of clamping deformation, assembly connecting deformation and spring-back deformation, and finally simulates the errors on sheet metal assembly by MATLAB, and the simulation results are compared with those of CATIA-TAA module.

FIGURE I. THE SPATIAL ERROR PROPAGATION KEY POINT ON SHEET METAL PART A AND B
Spatial Error Propagation Model on SMA in Single Station
In order to carry out the research smoothly without loss of generality, the thesis assumes the sheet metal part/subassembly is of in-plane rigidity and out-of-plane flexibility, while out-of-plane deformation belongs to small elastic deformation, which does not affect stiffness matrix of sheet metal part, that is, the stiffness matrix is a constant matrix. The assembly tools, including fixture and connecting tool, are regarded as rigid parts, and these assumptions are fit for 70-80% sheet metal assembly [1] [2] [13]. Based on the analysis of influence factors and propagation process, the spatial error propagation model on SMA in single station is established in accordance with the PCCR assembly process, as shown in figure 1. The influence between the in-plane error and the out-of-plane error in the process is represented by double arrows in the figure. The force and moment are always closed in PCCR assembly process because the out of plane form is closed. Because of in-plane rigidity, the rigid error sources include positioning tool error, clamping fixture error and the connecting tool error, on the other hand, the out-of-plane deformation errors exist in the clamping process, connecting process and spring-back process.

II. SPATIAL ERROR ANALYZING AND MODELING IN SINGLE STATION
A. Rigid Spatial Error Modeling on SMA
In the assembly process, the in-plane errors and out-of-plane errors influence each other, and the assembly errors in each process have affected each other. The in-plane errors are mainly determined by rigid errors, while out-of-plane errors are determined by rigid errors and deformation errors, so the rigid errors of the key point * on the assembly part J should be denoted as spatial errors, according to the frontal error expression, that is:

\[
\begin{align*}
\mathbf{\delta}_j &= [\mathbf{\delta}_{x_j}, \mathbf{\delta}_{y_j}, \mathbf{\delta}_{z_j}, \mathbf{\delta}_{x'y_j}, \mathbf{\delta}_{x'z_j}, \mathbf{\delta}_{y'z_j}]^T, \\
\mathbf{\delta}_{x_j} &= (M_{0,0}, Q_{0,0}, \nu_{0,0}, A_{0,0})_j \end{align*}
\]

In the formula, \( A_{0,0} \) can be denoted as:

\[
\begin{align*}
[\mathbf{\delta}_{x_1}, \mathbf{\delta}_{y_1}, \mathbf{\delta}_{z_1}, \mathbf{\delta}_{x'y_1}, \mathbf{\delta}_{x'z_1}, \mathbf{\delta}_{y'z_1}]^T &= (M_{0,0}, Q_{0,0}, \nu_{0,0}, A_{0,0})_j
\end{align*}
\]

B. Deformation Error Analyzing and Modeling on SMA in Single Station
1) Deformation Error Analyzing and Modeling in Clamping Process
In all part J, external forces on positioning and clamping point 1 should satisfy the closed forces, as shown in Figure 2, that is:

\[
\sum F_j = 0, \sum \mathbf{M}_j = 0, \sum \mathbf{M}_j = 0
\]

As is shown in Figure 3, the rigid errors of point i on part J and the fixture errors jointly determine the clamping force and the clamping deformation error, and also generate the clamping deformation error on any connecting point j and any measurement point k, expressed as the formula (3) and (4) respectively.

\[
\begin{align*}
\mathbf{F}_j &= \begin{bmatrix}
K_{iJ} & K_{iJ} & K_{iJ} \\
K_{jJ} & K_{jJ} & K_{jJ} \\
K_{kJ} & K_{kJ} & K_{kJ}
\end{bmatrix}
\begin{bmatrix}
\mathbf{\delta}_i \\
\mathbf{\delta}_j \\
\mathbf{\delta}_k
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\mathbf{F}_j &= \begin{bmatrix}
K_{iJ} & K_{iJ} & K_{iJ} \\
K_{jJ} & K_{jJ} & K_{jJ} \\
K_{kJ} & K_{kJ} & K_{kJ}
\end{bmatrix}
\begin{bmatrix}
\mathbf{\delta}_i \\
\mathbf{\delta}_j \\
\mathbf{\delta}_k
\end{bmatrix}
\end{align*}
\]

2) Deformation Error Analyzing and Modeling in Connecting Process
After any sheet metal part J is connected, the part J should satisfy the closed force and the close shape on the connecting point J, and the closed forces are shown as Figure 2. The internal and external forces on part J should keep balance, that...
In the formula, $F_{ij}$, $M_i$ and $M_j$ stand for the internal force and moments of connecting point j on part J. $F_{ij}$, $M_i$ and $M_j$ stand for the external force and moments of the positioning and clamping fixture errors and $\delta_i$, $\delta_j$ and $\delta_k$ stand for the external force and moments of the measuring point k on part J; and $F_{ij}$, $M_i$ and $M_j$ are usually zero.

In the assembly process, the part A and part B are connected on condition of the closed shape, as is shown in Figure 4. The displacement along the direction l and the angle error around n should satisfy the following formula.

In the formula, $\delta_{ij}(m,n)$ and $\theta_{ij}(m,n)$ stand for the position function of the connecting point j on part J before connecting and after connecting respectively, $\delta_{ij}$ and $\theta_{ij}$ are the corresponding angle function before connecting and after connecting respectively.

Similarly available the angle error around axis m: At the connecting point j and measurement point k respectively, $\delta_m$ and $\theta_m$ are the rigid errors of the connecting tools, the spring-back deformation error is recorded as $\delta_{spr}$, that is eqn (9).

Moreover, the deformation angle errors generated by $M_i$ and $M_j$ are similar to the way above.

4) Deformation Error Modeling About SMA in Single Stations

Based on the analysis of previous section, integrating the positioning and clamping deformation, the connecting deformation and the spring-back deformation errors, that is, the eqn(4), (8), (9) and (10), the deformation error along the direction l about SMA is the following:

$$\bar{\delta}_l = -(K_{ij})^{-1}(\delta_{ij} + \delta_{ij}) - K_{ij}(\delta_{ij} + (K_{ij})^{-1}(\delta_{ij} + \delta_{ij}) + (K_{ij})^{-1}(\delta_{ij} - K_{ij}(\delta_{ij} + (K_{ij})^{-1}(\delta_{ij} + \delta_{ij})) + (K_{ij})^{-1}(\delta_{ij} - K_{ij}(\delta_{ij} + (K_{ij})^{-1}(\delta_{ij} + \delta_{ij}))$$

(11)

where, $\delta_{ij}$ is the deformation error along the direction l on any measurement point k on part j. $\delta_{ij}$, $\delta_{ij}$ and $\delta_{ij}$ are the rigid errors along direction l on positioning and clamping point i, connecting point j and measurement point k respectively, $\delta_{ij}$ and $\delta_{ij}$ stand for the positioning and clamping fixture errors and the connecting tool errors along the direction l.

Similarly, the Angle error $\theta_{ij}$ is:

$$\bar{\theta}_l = -(K_{ij})^{-1}(\theta_{ij} + \theta_{ij}) - K_{ij}(\theta_{ij} + (K_{ij})^{-1}(\theta_{ij} + \theta_{ij}) + (K_{ij})^{-1}(\theta_{ij} - K_{ij}(\theta_{ij} + (K_{ij})^{-1}(\theta_{ij} + \theta_{ij})) + (K_{ij})^{-1}(\theta_{ij} - K_{ij}(\theta_{ij} + (K_{ij})^{-1}(\theta_{ij} + \theta_{ij}))$$

(12)

In the formula, each parameter is of the same meaning of the formula (10).

Thus the deformation error of point k on subassembly J is.

$$\bar{\delta}_l = \delta_{ij}, \bar{\delta}_m, \bar{\theta}_m, \bar{\theta}_m$$

(13)

C. Spatial Error Model of Sheet Metal Assembly in Single Station

According to previous analysis, the sheet metal assembly in-plane error is the rigid error, and the out-of-plane error is composed of the rigid error and the deformation error, according to eqn(1), (11), (12) and (13), the assembly error on any measurement point k on the sheet metal subassembly is.

$$\delta_l = (\bar{\delta}_l, \bar{\delta}_m, \bar{\theta}_l, \bar{\theta}_m)^T = \delta_{spr} + \lambda \bar{\delta}_l$$

(14)

where $\lambda$ is the selection coefficient of the errors. And
stands for the rigid error, and \( \delta_k \) stands for the deformation error, \( \cdot \) is the operator of the matrix multiplication.

III. SPATIAL ERROR SIMULATION AND ANALYSIS ON SMA

The simulation object about Sheet metal assembly in single station is the plat sheet metal A and Z type sheet metal B, and the material is aluminum. The specific parameter is shown in Figure 5, PLB1-8 stand for out of plane positioning block respectively, PLP1-4 stand for in-plane positioning pin, and PLP1 and PLP3 are the four-direction positioning pin, while PLP2 and PLP4 are the two-direction positioning pin, the connecting points are JP1-5 for the flat sheet A and the Z type sheet B, and the material is aluminum. The specific parameter is account the closed shape of the parts in modeling process, and the error changes on each measurement point are much gentler in the connecting process. (4) The measurement error changes of the positioning part A, that is, PMP1-PMP5, are much smaller than the changes of the part B which is no positioning part after the fixtures and the connecting tools are released.

![FIGURE IV. SHEET METAL PART A AND B FOR ASSEMBLY SIMULATION](image)

TABLE IV. THE POSITIONING AND CLAMPING FIXTURE ERRORS OF SHEET METAL PART A AND B (MM)

<table>
<thead>
<tr>
<th>Part</th>
<th>PLB1</th>
<th>PLB2</th>
<th>PLB3</th>
<th>PLB4</th>
<th>PLB5</th>
<th>PLB6</th>
<th>PLB7</th>
<th>PLB8</th>
<th>PLB9</th>
<th>PLB10</th>
<th>PM1</th>
<th>PM2</th>
<th>PM3</th>
<th>PM4</th>
<th>PM5</th>
<th>PM6</th>
<th>PM7</th>
<th>PM8</th>
<th>PM9</th>
<th>PM10</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.2</td>
<td>0.15</td>
<td>0.2</td>
<td>0.15</td>
<td>0.12</td>
<td>0.15</td>
<td>0.15</td>
<td>0.13</td>
<td>0.15</td>
<td>0.13</td>
<td>0.15</td>
<td></td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>B</td>
<td>0.2</td>
<td>0.15</td>
<td>-0.15</td>
<td>-0.2</td>
<td>-0.05</td>
<td>0.1</td>
<td>0.15</td>
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</table>

(a) the clamping deformation error

(b) the connecting deformation error

(c) the spring-back deformation error

![FIGURE V. THE ASSEMBLY DEFORMATION ERROR](image)

TABLE III. CLAMPING, CONNECTING AND SPRING-BACK DEFORMATION ERRORS BY TAA (MM)

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>positioning clamping</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Assembly connecting</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Release spring-back</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
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</table>

TABLE II. CLAMPING, CONNECTING AND SPRING-BACK DEFORMATION ERRORS BY MODEL (MM)

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>positioning clamping</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Assembly connecting</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Release spring-back</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
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</tbody>
</table>

Analyzing the AB assembly process errors, we can obtain: (1) the error change rules on measurement point between assembly error model simulation and CATIA-TAA module simulation are almost similar, and moreover the errors are in the same range (±0.2). (2) the error in the assembly process are affected and controlled by the errors of fixture(±0.2). (3) the connecting error changes are much smaller than CATIA-TAA simulation, mainly because of taking into account the closed shape of the parts in modeling process, and the error changes on each measurement point are much gentler in the connecting process. (4) The measurement error changes of the positioning part A, that is, PMP1-PMP5, are much smaller than the changes of the part B which is no positioning part after the fixtures and the connecting tools are released.
IV. CONCLUSION

Based on analyzing the rigid positioning errors, the deformation errors, including the clamping errors, the connecting errors and the spring-back errors, are deduced by linear elastic theory in accordance with the PCCR process, and finally built the spatial error model. The main conclusions are summarized as:

(1) According to the PCCR cycle, established the assembly spatial error propagation model on sheet metal assembly in single station.

(2) Based on detailed analysis of deformation errors of the assembly process, including positioning and clamping process, connecting process and releasing spring-back process, the spatial error model of sheet metal assembly was established by the linear elastic theory and finite element method on condition of satisfying the closed force and the closed shape.

(3) Comparing the simulation results of the assembly error model with the CATIA-TAA module simulation results, there is a good agreement between these results, and therefore verified the correctness of the assembly error model.

The model is of significance in the work of assembly error prediction, fixture optimization design, fixture error diagnosis and tolerance synthesis design.

REFERENCE.

[10] Stefan Dahlstrom and Lars Lindkvist, Contact Modeling in Method of Influence Coefficient for Variation Simulation of Sheet Metal Assemblies, Proceedings of IMECE04, 2004;
[14] Wzh Huang, Methodologies for Modeling and analysis of stream of variation compliant and rigid assemblies[D], University of Wisconsin-Madison, 2004; Yufeng Long, Variation simulation for compliant sheet metal assemblies with applications[D], The University of Michigan, 2000;