Dynamic Analysis of Cable-towed System during Ship in 180° U-turn MANEUVER

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Abstract—this paper analyses nonlinear dynamics of cable-towed system. A numerical model of marine cables with bending stiffness is presented based on three-dimensional lump-parameter approach and validated by Orcaflex. The dynamic response of a cable-towed system during ship in 180° U-turn maneuver is studied using the presented model. Additionally, corresponding parameters are intensively examined via simulation. Deep discussions upon the results can provide an insight into the tactical aspects of towed systems' applications.

Keywords—lumped parameter approach; towed cable system; u-turn maneuver

I. INTRODUCTION

The marine cable-towed systems are widely adopted in science, defense and industry. Due to the complexity of the undersea environment and the system, accurate prediction for the location and precise evaluation for the maneuverability and hydrodynamic characteristics of a specific cable-towed system are of significance before the system has been constructed [1]. A dynamic, mathematic model of cable is to find the configuration and tension on the cable as a function of time and space. Majority of the investigations on cable dynamics have concentrated on the model development or algorithm improvement. Present mathematic models are based on the finite difference method which has been popularly adopted in numerical simulations of underwater cable structures and can be attributed to A blow & Schechter [2].

The main purpose of this paper is to give an expansion to the previous analysis on transient dynamic response of cable-towed system during U-turn maneuver. A simple case is under research which will be carried out with different directions of current, tow-length to loop radius ratio, and current speed to tow speed ratio. Quantitative and qualitative conclusions drawn from numerical computations can be applied in the tactical aspects of cable-towed systems.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>Mass per unit length of cable</td>
</tr>
<tr>
<td>$EA$</td>
<td>Axial stiffness</td>
</tr>
<tr>
<td>$EI$</td>
<td>Bending stiffness</td>
</tr>
<tr>
<td>$s$</td>
<td>Unstretched length</td>
</tr>
<tr>
<td>$S$</td>
<td>Stretched length</td>
</tr>
<tr>
<td>$L$</td>
<td>Total length of cable</td>
</tr>
<tr>
<td>$Q$</td>
<td>External moment per unit length</td>
</tr>
<tr>
<td>$r$</td>
<td>Position vector</td>
</tr>
<tr>
<td>$T_e$</td>
<td>Effective tension vector</td>
</tr>
<tr>
<td>$V$</td>
<td>Shear force vector</td>
</tr>
<tr>
<td>$F$</td>
<td>Hydrodynamic force vector</td>
</tr>
<tr>
<td>$w$</td>
<td>Wet weight per unit length</td>
</tr>
<tr>
<td>$M$</td>
<td>Internal moment vector</td>
</tr>
<tr>
<td>$M_A$</td>
<td>Nodal mass matrix</td>
</tr>
</tbody>
</table>

II. FORMULATION OF LUMPED PARAMETER MODEL

In lumped parameter approach, a cable is divided into $N$ cable segments (elements) which are modelled by straight mass less model segments with a node at each end [3]. The model segments only model the axial and torsional properties of the line. The other properties (mass, weight, buoyancy etc.) are all concentrated on the nodes which are connected by means of springs.

To derive the governing equations of motion for the cable, we assume that: (1) the cross-section of cable is annular and homogeneous; (2) the effects of shear deformation, rotational inertia and structure damping can be neglected; (3) the hydrodynamic forces applied on the cable can be described using Morison’s formulations.

By taking the force and moment equilibrium of this model segment, the governing equations of motions for the marine cable are written as:

$$\frac{\partial T_e}{\partial S} + \frac{\partial V}{\partial S} + F + w = m \frac{\partial^2 r}{\partial t^2}$$

$$\frac{\partial M}{\partial S} + \frac{\partial r}{\partial S} \times V + Q = 0$$

Since towed cables are neither designed nor used to sustain any distributed external moments, we set $Q = 0$.

Next, we replace the space derivative terms in the above governing equations with the below finite-difference approximations:

$$\frac{\partial T_e}{\partial S_i} = \frac{(T_{e,i} - T_{e,i-1})}{\Delta S_i}$$

$$\frac{\partial V}{\partial S_i} = \frac{(V_i - V_{i-1})}{\Delta S_i}$$

$$\frac{\partial M}{\partial S_i} = \frac{(M_i - M_{i-1})}{\Delta S_i}$$
\[
\frac{\partial r_i}{\partial S_i} = \frac{\left( r_i - r_{i+1} \right)}{\Delta S_i}
\]

(6)

Here, the subscript \( i = [0,1,\ldots,N - 1] \) denotes the node number with \( i = 0 \) representing the free end (towed body end) node. After the previous discretization, the motion governing equations of the \( i \)th node can be written as the following partial nonlinear system of equations:

\[
\begin{align*}
\frac{dv_i}{dt} &= v_i \\
M_i \frac{dv_i}{dt} &= T_{i-1} - T_{i+1} + V_i - V_{i+1} + w_i (\Delta S_i + \Delta S_{i+1})/2 + F_i
\end{align*}
\]

(7)

The system of equations (7) can be solved by 4th order Runge-Kutta method with given initial conditions.

III. SIMULATION RESULTS AND ANALYSIS

In the 180° U-turn maneuver we studied, the tow ship makes a half-circle turn with a certain radius \( R \) from a straight-tow course and then returns to a 400s' straight-tow course in the reverse direction. It should be noted that the ship tows in a constant speed \( V \) in the whole simulation and the system starts in a steady state with the ship towing the system in a straight-tow equilibrium configuration. For the towed system, a tow cable is linked by a body with a zero volume, indicating that the effects of drag and added mass of the towed body is neglected. The state of the body is taken as a boundary condition of the lower end of the cable, while the upper boundary condition is depended by the moving tow ship.

The characteristics of the towed system we used are given as follows:

**Cable characteristics:**

- Wet weight per unit length \( w = 5 \) N/m
- Mass per unit length \( m = 0.751 \) kg/m
- Diameter \( d = 0.041 \) m
- Total length \( L = 100 \) m
- Normal drag coefficient \( C_d = 2.0 \)
- Tangential drag coefficient \( C_l = 0.015 \)
- Axial stiffness \( EA = 4.4 \times 10^7 \) N
- Bending stiffness \( EI = 237.9 \) N m²

**Towed body characteristics:**

- Wet weight \( W = 15 \) N
- Mass \( M = 2.5 \) kg

**Environmental properties:**

- Density of sea water \( \rho = 1024 \) kg/m³
- Speed of Current \( C = 0, 0.2, 0.4, 0.6 \) kn
- Tow speed \( V = 2, 4, 6, 8 \) kn

In order to verify the correctness and accuracy of the proposed model, a case with verification is simulated. At time \( t=0 \), the ship begins a 180° U-turn maneuver with a turn radius \( R=55 \) m and moves back to an 400-second opposite straight-tow finally. The simulation sets a constant tow speed \( V=2 \) kn and is carried in still water. Obviously indicated in fig.1, the results of the location of the towed body agrees well with the results got by Orcaflex, a worldwide popular marine engineering software, which shows the validity of the proposed method. As the ship turns, the body comes into an unsteady state with its depth changes continually because of the variation of the lift force. The body's depth will experience its maximum upon the finish of the U-turn.

(a) Horizontal trajectories of the towed body.

(b) Depth-time relationship of the towed body.

FIGURE I. COMPARISON BETWEEN THE PROPOSED MODEL AND ORCAFLEX.

A. Turning Radius Analysis

In order to examine the effect of turn radius on the dynamics of cable-towed system, four cases have been completed with four different turn radii for a fixed tow cable length \( L=100\) m and tow speed \( V=2\) kn in still water conditions.

FIGURE II. HORIZONTAL TRAJECTORIES OF SHIP AND TOWED BODY DURING U-TURNING MANEUVERS FOR SHIP-TURNING RADIUS R=35, 45, 55 AND 66 M.

FIGURE III. TIME HISTORY OF TOWED BODY DEPTH DURING MANEUVERS FOR SHIP-TURNING RADIUS R=35, 45, 55 AND 66 M.

FIGURE IV. TIME HISTORY OF TOW POINT TENSION DURING MANEUVERS FOR SHIP-TURNING RADIUS R=35, 45, 55 AND 66 M.
Fig. 2. shows the horizontal trajectories of ship and towed body during U-turning maneuvers for different radii in still water. Note that the symbols 'o' in figures indicate the boundary between U-turning and final straight-tow. It is seen that the turn center of the towed body coincides with that of the tow ship during the half circle turn. As the ship-turning radius increases, the turn center of the body displaces outward from the turn center of the ship, the trajectories of the body get closer and closer to the trajectories of the ship. Fig. 3. shows the time history of the towed body depth during maneuvers with four different turn radii in still water condition. As a result, the lift force applied on the towed body reduces and the towed body dives deeper until it reaches its maximum depth slightly behind the end of U-turning and then rises back to its balance position in the reverse straight-tow. We can clearly see that the lower the ship-turning radius is, the more time the towed body gets back to the original steady-state will take, and the greater maximum of depth the towed body will experience. The time series of tow point tension for $R=35, 45, 55$ and $65$ m are shown in Fig. 4. The value of tow point tension experiences an oscillation during the maneuver. The tension at tow point decreases upon the U-turn maneuver starts at the outset. Afterwards, the tension increases to a maximum which is quite bigger than the balance tension due to the increase of lift force after the ship exits the turn and returns to the straight-tow. Finally, the tension decreases to the value at steady-state again. The amplitude of the oscillation of the tow point tension is a function of turning radius. In each case, the maximum of tow point tension decreases with the increasing of ship-turning radius. And the amplitude of the oscillation that the tension value experiences reduces as the turn radius becomes bigger.

B. Tow Speed Analysis

In this section, a comparison is given to study the effect of tow speed on the dynamics of towed system during U-turn. The other parameter remains unchanged, and so are the following simulations.

Fig. 6. shows that the increase of the tow speed leads to a substantial increase in the depth of the body in the whole time domain. Other than the curve for $V=2$ kn in Fig. 6., the other three curves show a differentiation that the depth of body will be lower than the steady-state depth at a certain period. This differentiation gets more prominent as the tow speed increases. It is seen that the tension at tow point grows significantly due to the raise of tow speed in Fig. 7. Furthermore, during the U-turning maneuver, the tension value of the tow point is never bigger than the steady-state tension. We can conclude that a high tow speed eliminates the oscillatory behavior of the tension around the moment that U-turning transits to straight tow maneuver shown in Fig. 3.

C. Current Speed Analysis

Four cases with 4 current speed $C=0, 0.2, 0.4$ and $0.6$ kn are simulated individually in this section. In these simulations, the current is in -x direction, which is parallel and opposite to the original straight tow direction.

Fig. 8. shows that the increase of the current speed leads to a substantial increase in the depth of the body in the whole time domain. Other than the curve for $C=0$ in Fig. 8., the other three curves show a differentiation that the depth of body will be lower than the steady-state depth at a certain period. This differentiation gets more prominent as the current speed increases. It is seen that the tension at tow point grows significantly due to the raise of current speed in Fig. 9. Furthermore, during the U-turning maneuver, the tension value of the tow point is never bigger than the steady-state tension. We can conclude that a high current speed eliminates the oscillatory behavior of the tension around the moment that U-turning transits to straight tow maneuver shown in Fig. 3.
Fig. 8 gives a plan view of the ship undergoing 180° U-turn maneuver in the presence of current. As the current speed increases, the trajectories of the towed body deviate further and further from the ship’s trajectories, which delays the new equilibrium that the towed system will reach in the end. Fig. 9 shows the time history of towed body depth during maneuver in the presence of current. Four curves correspond to four different current speed $C=0$, 0.2, 0.4, and 0.6kn. Compared to the case in still water (i.e. $C=0$), the towed system changes its steady configuration after the U-turn maneuver in the other three cases. In addition, the maximum depth of the towed body in time history increase substantially accompanying the addition of the speed of current in -x direction. Fig. 10 illustrates the variation of the tension at tow point during maneuvers for four different current speed. Though four curves have a similar trend with the maneuver executed, the value of tow point tension varies more dramatically as the increase of the current speed. As the current speed becomes bigger, the time that the tension value decrease will extend, and so is the time the value will rise.

D. Current Directions Analysis

The effect of changing the magnitude of current speed is investigated in the previous section, here we will analyze the effect of the current direction on the dynamic characteristic of cable system during U-turn maneuver. Four cases with a same magnitude $C=0.4$ kn of current in -x, +x, -y, +y direction are simulated respectively. Many differences appeared during the maneuver for the four cases and the corresponding results are given as the follow plots.

An obvious effect of current direction is to change the horizontal trajectories of the towed body (fig. 11.). Fig. 12. is the time history of towed body for current in different directions, which shows that current in x direction changes the balanced depth of towed body while the other two don’t. Among the four cases, the case with current in -y direction firstly comes to a new steady-state and the case with current in -x direction will be the last to finish unsteady motion. Additionally, the curve for the case of -y direction current has an uncommon trend in time domain: increases at first and then reduces back to steady-state. Fig. 13. shows the time history of tow point tension during maneuvers for current in the four directions. For cases with current in y direction, the tow point tension in final steady-state coincides with the original balanced tension due to y-current has no component in x direction which is parallel to the straight-tow's direction. A significant difference due to current direction is that the time the tension reaches its maximum. For the case with current in +y direction, the tow point tension reaches its maximum at the time $t=204.2$ s (i.e. the maximum occurs during the reverse straight-tow maneuver), while for other cases, the tension maximum occurs during the 180° U-turn maneuver. Moreover, the peak tension at tow point for current in +y direction is bigger than that of anyone in other three cases.

FIGURE XI. HORIZONTAL TRAJECTORIES OF SHIP AND TOWED BODY DURING U-TURNING MANEUVERS FOR CURRENT IN DIFFERENT DIRECTIONS.

FIGURE XII. TIME HISTORY OF TOWED BODY DEPTH DURING MANEUVERS FOR CURRENT IN DIFFERENT DIRECTIONS.

FIGURE XIII. TIME HISTORY OF TOW POINT TENSION DURING MANEUVERS FOR CURRENT IN DIFFERENT DIRECTIONS.

IV. Conclusions

A new hydrodynamic model for marine cable has been formulated with lumped parameter method and verified by Orcaflex. The dynamic behavior of a cable-towed system which results from a tow ship doing U-turn maneuver from a steady straight-tow state is numerical examined with the proposed model. A sequence of sensitivity (turning radius, tow speed, current speed and current direction) studies were carried out using the presented model to test the performance of cable-towed system during ship in 180° U-turn maneuver.

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References