for any $n > n_0$ and $g_n \nearrow f$. Analogously, one can prove that $h_n \searrow f$. Now, from the monotonicity of the fuzzy integrals, we may write

$$\int_X^{\otimes} g_n \; d\mu \leq \int_X^{\otimes} f_n \; d\mu \leq \int_X^{\otimes} h_n \; d\mu$$

for any $n = 1, 2, \ldots$. Putting $c = \int_X^{\otimes} f \; d\mu$, then, according to the presumption of the theorem, there exists n_0 such that

$$\mu(X \upharpoonleft \{m \mid f_{n_0} > c\}) < \top$$

and $g_{n_0}(m) = \bigwedge_{k \geq n_0} f_k(m) \leq f_{n_0}(m)$ for any $m \in M$. Then

$$\{m \mid g_{n_0} > c\} \subseteq \{m \mid f_{n_0} > c\}$$

and from the monotonicity of μ we obtain

$$\mu(X \upharpoonleft \{m \mid g_{n_0} > c\}) < \top.$$

Let $a \in L$, $a < \top$. From Theorems 4.5 and 4.6, there exists n_0, m_0 such that for any $n \geq \max(n_0, m_0)$ we have

$$a < \int_X^{\otimes} f \; d\mu \rightarrow \int_X^{\otimes} g_n \; d\mu$$

and also

$$a < \int_X^{\otimes} h_n \; d\mu \rightarrow \int_X^{\otimes} f \; d\mu.$$

From the monotonicity of the residuum in its arguments and $g_n \leq f_n \leq h_n$, we obtain

$$a < \int_X^{\otimes} f \; d\mu \rightarrow \int_X^{\otimes} f_n \; d\mu$$

and

$$a < \int_X^{\otimes} f_n \; d\mu \rightarrow \int_X^{\otimes} f \; d\mu$$

which implies

$$a < (\int_X^{\otimes} f \; d\mu \rightarrow \int_X^{\otimes} f_n \; d\mu) \wedge (\int_X^{\otimes} f_n \; d\mu \rightarrow \int_X^{\otimes} f \; d\mu) = (\int_X^{\otimes} f_n \; d\mu \leftrightarrow \int_X^{\otimes} f \; d\mu)$$

for any $n > \max(n_0, m_0)$. Hence, we obtain $\int_X^{\otimes} f_n \; d\mu \rightarrow \int_X^{\otimes} f \; d\mu$. □

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