The Design of the Multi-user MIMO Nonlinear Precoding System

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I. INTRODUCTION

In order to reduce the interference between the parallel data flow, we can use precoding matching transmission channel both in the transmitter data flow (precoding) and the processing of received signal (equality). Therefore, commonly widely used linear precoding scheme has low complexity based on forced to zero (ZF) and minimum mean square error (MMSE) criterion. Despite the very low complexity, linear precoding scheme also has capacity loss problems. And both in the transmitter or receiver, nonlinear precoding processing can provide alternative methods which can improve the performance of linear precoding. The implementation method of high complexity in practice includes linear precoding combined with decision feedback equalization (DFE) [1], vector perturbation [2], Tom Lin Senyuan island (TH) precoding [3] and ideal dirty paper coding [4] these four methods. Vector perturbation has capacity loss problems. And both in the transmitter or receiver, nonlinear precoding scheme can achieve performance better than the linear force based on perfect CSI and quantitative CSI zero (ZF) precoding.

II. THE TRANSMISSION SYSTEM WITH QUANTITATIVE CSI

In the actual cases, there are not perfect CSI presenting in the transmission system. For example, in the FDD system transmitting end through each receiver downward limit of feedback B get CSI. According to the literature [6] for quantitative research of CSI, channel in each vector can be quantitative, its corresponding index feedback to the transmitter at the receiving end through a wrong channel without delay. A transmitter and the receiver can read the code $W = \{w_1, \cdots, w_n\} (w_i \in C^{i \times n})$, direction vector can be written as after the quantitative of the receiving end first k selecting.

$$\hat{h}_k = \arg \max_{w_j \in W} \left\{ \| \hat{h}_k w_j \|^2 \right\}$$

(1)

Here $\| \hat{h}_k \|$ is the channel direction vector of user k. Here we use the RVQ code. The code in the measurement vector n is independent of shaft and the $n_T$ inside spherical space is fragmented. Although RVQ this is not the most suitable for limited space system code, but it is suitable for the most close to the optimal quantitative analysis and its performance. For the user k, we can get

$$h_k = h_k \cos \theta_k + h_k \sin \theta_k$$

(2)

Here $\cos^2 \theta_k = \| \hat{h}_k \hat{H}_k^H \|^2_2$, $\hat{h}_k = C^{i \times n}$ is a baseline vector distribution of same axial units in the orthogonal complement of subspace $\hat{h}_k$ and $\sin \theta_k$ independent of each other. H can be written as:
\[ \mathbf{H} = \Gamma (\Phi \mathbf{H} + \Omega \mathbf{H}) \] (3)

Here
\[
\begin{align*}
\Gamma &= \text{diag}(\rho_1, \cdots, \rho_n), \quad \rho_i = \| \mathbf{h}_i \| \\
\Phi &= \text{diag}(\cos \theta_1, \cdots, \cos \theta_n) \\
\Omega &= \text{diag}(\sin \theta_1, \cdots, \sin \theta_n), \quad \mathbf{\hat{H}} = [\mathbf{\hat{h}}_1^T, \cdots, \mathbf{\hat{h}}_n^T]^T, \quad \mathbf{\tilde{H}} = [\mathbf{\tilde{h}}_1^T, \cdots, \mathbf{\tilde{h}}_n^T]^T
\end{align*}
\]

As a simple analysis, in this article we reference for the quantitative element approximation method in [7]. Here quantitative unit is envisioned as a spherical theissen polygon area, whose the surface area is \( \frac{1}{n} \) of the total \( n_T \) dimensions of the spherical surface. For a given code \( \mathbf{W} \), the actual unit vector quantization \( \mathbf{w}_i \), \( \mathbf{r}_i = \frac{\mathbf{h}_i - \mathbf{f}_i}{\| \mathbf{h}_i - \mathbf{f}_i \|} \), \( \forall i \neq j \) can be written as:

\[ \mathbf{\tilde{r}}_i \approx \frac{\mathbf{h}_i}{\| \mathbf{h}_i \|} \geq 1 - \delta \], \( \delta = 2 \pi \angle_i \)

In quantitative CDI launch, the transmitting terminal get feedback precoding matrix \( \mathbf{F} \) and feedback matrix \( \mathbf{B} \) through its channel matrix QR decomposition using the same method of QR decomposition of matrix \( \mathbf{\hat{H}} \) which uses the same decomposition methods with the same QR decomposition matrix \( \mathbf{H} \). And here this matrix \( \mathbf{\hat{r}} \) and \( \mathbf{\tilde{Q}} \) are respectively with the same structure like \( \mathbf{R} \) and \( \mathbf{Q} \). we can get \( \mathbf{F} = \mathbf{\tilde{Q}}^H/ \mathbf{\hat{r}} \) and \( \mathbf{B} = (\text{diag} (\mathbf{\hat{r}}))^{-1} \mathbf{\tilde{Q}} - I \). In addition, the scaling matrix at the receiving end becomes:

\[ \mathbf{G} = \sqrt{\frac{k}{p}} (\Gamma \Phi \text{diag} (\mathbf{\hat{r}}))^{-1} \] (4)

At the receiving end we use the same method under the condition of the optimal CSI to test and detect signals, and the detected signal vector \( \mathbf{\hat{y}} \) is associated with the channel and the RVQ feedback of quantify CSI, we will get the statistical distribution of interference parts in formula

\[ \frac{P_k}{\rho_k^2} \mathbf{\hat{h}}_k \mathbf{\hat{Q}}^H \mathbf{\hat{h}}_k^T \sin^2 \theta_k \]

As known to all, \( \rho_k^2 \) has a \( x_{n_T}^2 \) distribution and \( \sin^2 \theta_k \) distribution has been given in [10]. Because we can use \( \mathbf{\hat{h}}_k \) \( k = 1, \cdots, K \) to determine \( \mathbf{\hat{h}}_k \) \( k = 1, \cdots, K \) is associated with \( \mathbf{\tilde{Q}} \). But the distribution of \( \mathbf{\hat{h}}_k \) is not known yet, and the accuracy cannot be ignored. The theory next can reveal precision of the interference of distribution.

For \( 1 < K < n_T \), and the random variable \( \mathbf{\epsilon}_k = \| \mathbf{\hat{h}}_k \mathbf{\hat{Q}}^H \|_2^2, k = 1, \cdots, K \) follow the same beta distribution until \( (K-1), (n_T-K) \), then \( \mathbf{\epsilon}_k \) can be noted as \( \text{Beta}(K-1, n_T-K) \). In addition, the probability density function \( \beta(p, q, f) \) of \( \mathbf{\epsilon}_k \) can be given

\[ f_{\epsilon_k}(x) = \frac{1}{\beta(K-1, n_T-k)} x^{K-2}(1-x)^{n_T-k-1} \] (7)

Here \( \beta(a, b) = \int_0^1 t^{a-1} (1-t)^{b-1} dt \) is a beta function. When \( k = 1 \), there is no interference.

When \( K = n_T \), \( \mathbf{\epsilon}_k = \| \mathbf{\hat{h}}_k \mathbf{\hat{Q}}^H \|_2^2 \) is the constant of 1.

The logistic expectations of interfere \( \mathbf{\epsilon}_k \) are given as follows

\[ E_{\epsilon_k}[-\log_{2}(\| \mathbf{\hat{h}}_k \mathbf{\hat{Q}}^H \|_2^2)] = \log_2 e \times \sum_{n=k+1}^{n_T} \sum_{j=0}^{n_T-n} \frac{(n_T-2)}{m!n!(n_T-m-2-j)!} \]

(8)
B. Above the Average rate of Total loss And Total Rate

User \( k \) under optimal CSI and quantitative CSI instantaneous gain rate can be respectively written as:

\[
R_{p,k} = \log_2(1 + \xi_k) \\
R_{Q,k} = \log_2(1 + \gamma_k)
\]

Based on this, under the condition of each user feedback section B, quantitative CSI feedback user average total loss rate of \( k \) can reach maximum

\[
\Delta R = E_{H_{<R}} \left( R_{p,k} - R_{Q,k} \right) \leq \Delta R = \log_2(1 + cP2^{-\frac{b}{n+t-1}})
\]

\[
+ \frac{\log_2(e)^{n-t} - \sum_{i=1}^{n-t+1} \beta(n, \frac{i}{n+t-1})}{n+t-1}
\]

Here \( C = \frac{(K-1)n_t}{k(n_t - 1)} \) and the size of the code is \( n = 2^b \).

Therefore, when only quantitative CSI effect in transmitter, the second item in right formula (11) comparing with linear precoding can be considered as derivative of nonlinear precoding. In addition, the result of linear ZF beam, rate loss of the nonlinear prediction coding is also an increasing function of system signal to noise ratio \( P \). Therefore, with accurate feedback rate system under the condition of high SNR signal is limited, it can be got in the following theorem.

Under TH precoding quantitative CSI cases through the feedback section B, the average total rate range of user \( k \) are

\[
r_{n,k} \leq \log_e \left( \sum_{m=0}^{n-t+1} \sum_{l=0}^{n-t-m} \frac{(n_t - 2)!}{m!(n_t - m - 2 - l)!} \frac{1}{m+1} \frac{1}{n_t - 1} \right)
\]

Here, the code range are \( n = 2^b \).

If \( n \to \infty \), the upper bound is 0. Therefore, when \( n \) with SNR linear expansion, for any given constant, we can find a number of a normal \( N(\varepsilon) \), when \( n > N(\varepsilon) \),

\[
\Delta R \leq \log_2(1 + cP2^{-\frac{b}{n+t-1}}) + \varepsilon
\]

In order to shape a zoom the sufficient condition for feedback rate, setting the right side of (13) the item is the item in maximum allowable difference \( \log_2 b \). Through some simple processing, we can get

\[
B = (n_t - 1) \log_2 P - \log_2(b - 2^s - 1) + \log_2 c = (n_t - 1) \log_2 \frac{10}{10} P_{dB}
\]

\[
- \log_2(b - 2^s - 1) + \log_2 c
\]

Figure 1(a) showed a system average total rate curve under the case of \( n_t = 4, K = 4 \). Feedback rate is set according to the given relationship between the proportions in (14). Noted that when \( B \) is big enough, \( \varepsilon \) can be very small. In the simulation we set \( \varepsilon = 0 \) to get strong conditions than formula (14). In the optimal CSI TH precoding, limited feedback is respectively about 4 dB and 5.5 dB when \( b = 3 \) and \( b = 4 \).

IV. THE RESULTS OF SIMULATION

Setting \( n_t = K = 4 \), then the system SNR is classified as P. Figure 1(b) shows TH precoding and linear ZF precoding performance under the optimal CSI and quantitative CSI, and feedback section of each user \( B = 4, 8, 15 \). We can see that TH precoding performance is superior to linear precoding under optimal CSI and quantitative CSI circumstances. When SNR is small and slow, the average total rate obtained by TH precoding of quantitative CSI is superior to linear ZF precoding under optimal CSI.

Figure 1(c) showed that when the SNR of the system is 25 dB. Taking the average total loss rate of each user as a function obtained in the case of TH and ZF precoding the number of feedback section. We can also see from the picture that nonlinear precoding is more affected by the imperfect CSI than linear precoding. However, when the SNR is not very big or quantitative feedback processing is very thorough, nonlinear precoding performance is superior to linear precoding. In addition, we can notice that the upper limit of TH precoding is very close to the actual loss rate. With the increase of \( B \) at the same time, its convergence speed is faster than linear precoding in the literature.
V. CONCLUSION

This paper studies the multi-user MIMO systems to realize TH precoding with quantitative CSI in case in the downlink. Especially when the number of users $K$ in the system transmitting antenna number is less than or equal to $K$ system, this scheme can be extended to general system. In this paper, by derived average total rate and the average total rate expression of the maximum loss under the condition of quantitative CSI studied the average total rate of the scheme. With quantitative results showed in both the optimal CSI and CSI cases, nonlinear TH precoding get far superior performance of linear ZF precoding. In addition, with the increase of number of feedback section, derived from TH precoding ceiling linear ZF precoding had faster convergence than the real rate.

REFERENCES