Since by (13) and (17) $I_{FGM(0)} = I_\Pi$, hence probabilistic implication (18) for $\theta = 0$ is a fuzzy implication. Now let us check whether we can obtain a probabilistic implication based on the Farlie-Gumbel-Morgenstern copula with parameter $\theta \neq 0$.

Thus, by Th. 6 we have to examine whether condition (12) holds for any $\theta \in [0, 1]$. Taking any $u_1, u_2, v \in [0, 1]$ such that $u_1 \leq u_2$ we can see that (12) is satisfied when

$$[u_1 v + \theta u_1 v(1 - u_1)(1 - v)]u_2 \geq$$
$$\geq [u_2 v + \theta u_2 v(1 - u_2)(1 - v)]u_1$$

which is equivalent to

$$\theta(1 - u_1) \geq \theta(1 - u_2)$$

and holds if and only if $\theta \geq 0$. Therefore, we may conclude that

$$I_{FGM(\theta)} \in \mathcal{FI} \Leftrightarrow \theta \geq 0,$$

i.e., the probabilistic implication (18) based on the Farlie-Gumbel-Morgenstern copula is a fuzzy implication not for all possible values of parameter θ but only for $\theta \geq 0$. ■

6. Conclusions

A new family of implication operators, called probabilistic implications were introduced in the paper. It was discussed when probabilistic implications are fuzzy implications. Some examples illustrating these new tools were also given. As they are called, probabilistic implications give a promising link from probability to theory of fuzzy implications that might be useful in approximate reasoning. However, many problems and questions are still open. One of the most important is to characterize a family of the copulas leading to probabilistic fuzzy implications. It would be desirable to find mathematical formulae which are not only efficient but also providing a clear and natural interpretation. Next problem is to examine properties of the probabilistic implications based on different families of copulas.

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