













## 6. Conclusion

In the last few years many researchers focused on the approximation of fuzzy numbers. Between the most important requirements that an approximation operator should satisfy are: the invariance to translations, the scale invariance, additivity or continuity. One would expect that an approximation operator would possess as many of these properties as possible. In the present contribution we found sufficient conditions for approximation operators over the space of fuzzy numbers (with respect to well-known distances) to be invariant to translations and scale invariant, respectively. An immediate consequence of these results is that most of the approximation operators proposed so far in the literature satisfy these requirements. But, according to Theorems 1 and 4, the class of approximation operators which possess these two properties is even larger. This could be a useful tool in finding new approximation operators so that most of the basic requirements are fulfilled. The next goal is to characterize the general class of the additive approximations and/or continuous approximations of fuzzy numbers.

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