Example 2 *We consider the following fuzzy integro-differential equation:*

$$u^{'}(x) = (r-1, 1-r) + \int_0^x u(t)dt, \quad u(0) = (0,0), \quad u^{'}(0) = (r-1, 1-r) \tag{11}$$

By applying Theorem (4) *and using properties of FDTM, the following relation is obtained for* $0 \le r \le 1$*:*

$$\underline{U}(k+1, r) = \left[(r-1)\,\delta(k) + \frac{\underline{U}(k-1, r)}{k} \right] \frac{k!}{(k+1)!}$$

and

$$\overline{U}(k+1, r) = \left[(1-r)\,\delta(k) + \frac{\overline{U}(k-1, r)}{k} \right] \frac{k!}{(k+1)!}$$

By using Theorem 3.1 and problem condition, we have

$$\underline{U}(0, r) = 0, \quad \underline{U}(1, r) = (r-1), \qquad \overline{U}(0, r) = 0, \quad \overline{U}(1, r) = (1-r).$$

Utilizing the above relation, we obtain:

$$\underline{U}(2,r) = 0, \quad \underline{U}(3,r) = \tfrac{r-1}{3!}, \quad \underline{U}(4,r) = 0, \quad \underline{U}(5,r) = \tfrac{r-1}{5!}, \quad \underline{U}(6,r) = 0, \quad \underline{U}(7,r) = \tfrac{r-1}{7!}, \quad \underline{U}(8,r) = 0, \quad \underline{U}(9,r) = \tfrac{r-1}{9!}, \quad \underline{U}(10,r) = 0,$$

and

$$\overline{U}(2,r) = 0, \quad \overline{U}(3,r) = \tfrac{1-r}{3!}, \quad \overline{U}(4,r) = 0, \quad \overline{U}(5,r) = \tfrac{1-r}{5!}, \quad \overline{U}(6,r) = 0, \quad \overline{U}(7,r) = \tfrac{1-r}{7!}, \quad \overline{U}(8,r) = 0, \quad \overline{U}(9,r) = \tfrac{1-r}{9!}, \quad \overline{U}(10,r) = 0,$$

Then, the following series solution is evaluated:

$$\begin{aligned} u(x) &= (r-1, 1-r)\left(x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \dots \right) \\ &= (r-1, 1-r)\sinh x \end{aligned}$$

Which is the exact solution of the fuzzy integro-differential equation

5. Conclusion

In this paper, **FDTM** is proposed for solving fuzzy Volterra integro-differential equation with separable kernels. Differential transform method is different from the traditional high order Taylor series method, which requires symbolic computation of necessary derivatives of the data function and is computationally expensive for higher order. We introduced new theorems for FDTM to solve the fuzzy integro-differential equations. We first gave their proofs and then applied to fuzzy integro-differential equations. Also, we used the concept of H-derivatives for fuzzy mappings. Some examples were examined using **FDTM** and the results have shown remarkable performance.

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