Calculation of the Effective Permeability Tensor Used in the Naturally Fractured Reservoirs Simulation

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Abstract—This paper presents a numerical model to calculate effective permeability tensor and to simulate the fluid flow in Naturally Fractured Reservoirs. This simulation includes three parts. A discrete fracture network is realized to capture the features of the target Naturally Fractured Reservoirs. The discrete fracture network is discretized into a limited number of grids whose effective permeabilities are calculated. Matching the production rate histories confirmed the correctness of the effective permeability tensor calculated in this way.

Keywords- natural fractured reservoir; effective permeability tensor; numerical simulation

I. INTRODUCTION

The development of Naturally Fractured Reservoirs provides opportunities for progress in reservoir numerical simulation techniques. Due to the complexity of reservoirs, Modeling of fluid flow in naturally fractured reservoirs is an important issue in the areas of petroleum engineering and hydrology.

There is still no determined method for Naturally Fractured Reservoirs simulation. Dual porosity and discrete fracture models are the main approaches to simulate fluid flow in naturally fractured reservoirs. [1, 2].

Currently, this thesis is aimed to calculate the effective permeability tensor and to simulate the fluid flow in Naturally Fractured Reservoirs. There are for Naturally Fractured Reservoirs simulation: the Equivalent Continuum Model (ECM), the Discrete Fracture Network Model (DFNM) and the Hybrid Model, all of which can be divided into deterministic model and stochastic model according to the inputs. The ECM is appropriate to field-scale reservoir simulation while the DFNM is well known for its ability to describe heterogeneity of local-scale regions. The equivalent medium model presented in this work is a hybrid model who takes advantages of both the ECM and DFNM.

II. MATHEMATICAL FORMULATION

In this formulation, effective permeability is described as a full tensor that relates the average pressure gradient ∇P to the average fluid velocity V as:

$$V = -K\nabla P \quad (1)$$

Where K is the local permeability tensor describing the directional effect of fracture or set of fractures on fluid flow. The effective permeability tensor needs to be symmetric and positive definite in order to have a physical meaning, i.e. $k_{xy} = k_{yx}$ and $(k_{xx}k_{yy} \geq k_{xy}^2, k_{xx} > 0, k_{yy} > 0)$. [3].

In the above definitions, k_{xx} and k_{yy} are diagonal terms and k_{xy} and k_{yx} are off-diagonal terms of permeability tensor K . Assuming that fluid flow in matrix and fracture obeys Darcy's law and cubic law, [4], respectively, the governing equations of fluid flow in a two-dimensional reservoir are the following: [5].

Matrix:

$$\frac{\partial}{\partial x} \left(k_{xx} \frac{\partial p}{\partial x} + k_{xy} \frac{\partial p}{\partial y} \right) + \frac{\partial}{\partial y} \left(k_{yx} \frac{\partial p}{\partial x} + k_{yy} \frac{\partial p}{\partial y} \right) + Q_m = 0 \quad (2)$$

Fracture:

$$\frac{\partial}{\partial L} \left(k_f \frac{\partial p}{\partial L} \right) + Q_f + q_{ff} = 0 \quad (3)$$

Where $k_f = 7.842 \times 10^{12} h^2$, h is fracture aperture, L is one-dimensional coordinate, and subscripts m and f represent matrix and fracture, respectively. Terms Q_m and Q_f represent flow interaction between the fracture and matrix. q_{ff} is flow at the point of intersection by intersecting fractures. Matrix permeability is assumed to be constant in horizontal and vertical directions as in a homogenous system.

For notation simplicity, we consider the above equations in general form of a linear Poisson's equation in domain A as follows:

$$\nabla \cdot (k \nabla p(x)) + Q(x) = 0 \text{ in } A, \quad (4)$$

Where k denotes the permeability, $x = (x_1, x_2)$ is an arbitrary point in interior of A and $Q(x)$ characterizes the source term associated with interior domain A . In case, A is a medium or long fracture, the term $Q(x)$ is defined as $(Q_f + q_{ff})$. The boundary integral equation for the above equation pertaining to an arbitrary point $\xi = (\xi_1, \xi_2)$ is written as:

$$c(\xi)p(\xi) = \int_S G(x, \xi) v(x) ds(x) - \int_S F(x, \xi) p(x) ds(x) + \int_A Q(x) G(x, \xi) dA(x) \quad (5)$$

Where $v = k \nabla p \cdot \bar{n}$. $\bar{n}$ is the exterior normal vector of S at ξ , and S is the boundary of domain A . G is the fundamental

solution, $F = k \frac{\partial G}{\partial n}$, which is its corresponding flux, and $Q(x) = \sigma(x - \xi)$, which is the Dirac delta function. The coefficient $c(\xi)$, which is a function of the internal angle of the boundary S at point ξ , equal to 1 if $\xi \in A$, 0 if $\xi \notin A \cup S$ and $\frac{\alpha}{2\pi}$ if $\xi \in S$, where α denotes the interior angle of S at ξ (α equals π at a smooth point on S). [6].

For short fractures and the part of matrix which is not affected by fluid flow in medium and long fractures, the source term $\int_A QGdA$ is neglected and Eq. (5) is rewritten as :

$$c(\xi)P(\xi) = \int_S G(x, \xi)v(x)ds(x) - \int_S F(x, \xi)pds(x) \quad (6)$$

The interface boundary conditions for short fractures are expressed as:

$$P_{mi} = P_{fi}, \text{ and } v_{mi} = -v_{fi}$$

Boundary conditions along the medium and long fracture boundaries are written as:

$$P_{fi} = P_{av} \\ v_{mi}^+ - v_{mi}^- = Q_i$$

Where $v_{mi}^+ = \vec{v}_m^+ \cdot \vec{n}$, $v_{mi}^- = \vec{v}_m^- \cdot \vec{n}$, P_{av} is the average pressure inside the fracture and p_{fi} represents the pressure along the fracture boundaries. $\vec{v}_m^+$ and $\vec{v}_m^-$ are velocities on the opposite nodes on the fracture faces and Q_i depends on the source strength of the fracture faces and represents the flow interaction between the matrix m and fracture I . Matrix pressure on the common matrix-fracture interface and on the exterior boundaries of the area around the fracture are unknown and will be calculated by applying the periodic boundary condition during the solution process.

The periodic boundary condition is considered for nodes along the block boundaries, which requires to treat all fracture edges as being inside the grid block. [7]. Assume that Γ_1 and Γ_2 are two opposite faces of the grid-block in the x_2 direction and Γ_3 and Γ_4 are two opposite faces of the grid-block in the x_1 direction. Pressure at an arbitrary point $x=(x_1, x_2)$ in the grid-block can be expressed as :

$$p(x) = p_0 + J(x - x_0) \quad (7)$$

Where x_0 is the center of the region under consideration, p_0 is the pressure at x_0 , and $j=(j_1, j_2)$ is the local pressure gradient. The periodic boundary conditions over the unit cell are written as:

$$\begin{aligned} p(x_1, 0) &= p(x_1, 1) - j_2 \text{ on } \Gamma_1 \text{ and } \Gamma_3 \\ v(x_1, 0)\vec{n}_1 &= -v(x_1, 1)\vec{n}_2 \text{ on } \Gamma_1 \text{ and } \Gamma_3 \\ p(0, x_2) &= p(1, x_2) - j_1 \text{ on } \Gamma_2 \text{ and } \Gamma_4 \\ v(0, x_2)\vec{n}_2 &= -v(1, x_2)\vec{n}_4 \text{ on } \Gamma_2 \text{ and } \Gamma_4 \end{aligned}$$

A constant pressure difference in the x_1 direction and a zero pressure difference in the x_2 direction ($j_1 \neq 0$ and $j_2 = 0$) are applied to calculate the first two terms of permeability tensor k_{xx} and k_{yx} . The remaining two components of permeability tensor, k_{xy} and k_{yy} can be calculated in the same way by carrying the direction of pressure gradient to ($j_1 = 0$ and $j_2 \neq 0$)

III. APPLICATION

There is a Naturally Fractured Reservoir called JA field located in China. We have built the structural modeling after

collation of the data. The Petrophysical modeling was built with the method of Sequential Gaussian simulation. In this method, we got the curve of the history matching in Fig. 1.

FIGURE 1. HISTORY MATCHING CURVE WITH THE METHOD OF SEQUENTIAL GAUSSIAN SIMULATION.

In this curve, we can see the red line means the Field Oil Production Rate History, the blue one is the Field Oil Production Rate, the green one means the Field Water Production Rate, the purple line means the Field Water Production Rate History. We can get the information that the history matching effect is not ideal. The oil production at 2005 and the water production at the 2010 is lower than the date of history in the method of the Sequential Gaussian simulation.

Another property modeling was modified with the method of effective permeability tensor. We got the history matching curve in Fig.2.

FIGURE II. HISTORY MATCHING CURVE WITH THE METHOD OF THE EFFECTIVE PERMEABILITY TENSOR.

Obviously, this history matching curve is better than the last one, It means the method of the effective permeability tensor is one of the best ways for the reservoirs simulation.

IV. CONCLUSIONS

A model has been developed to calculate effective permeability tensor in naturally fractured reservoirs. Fluid flow through matrix, fracture and flow interaction between matrix and fracture are considered. From the results of this method following conclusions can be drawn:

- 1) The model presented in this study is able to calculate permeability tensor of naturally fractured reservoirs by considering the fact that the porous matrix, fractures of different sizes, (isolated), and interconnected fractures all contribute to the permeability of the reservoir.
- 2) Application of the effective permeability tensor method on the property modeling, the history matching curve has become better than the Sequential Gaussian Simulation. Its correctness had been confirmed.

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