

for every $x_\lambda \in Pt(L^X)$, $u \in L^X$. Then $(\mathcal{U}_\mathcal{T}, X \times M(L), L^X)$ is an L -fuzzy soft neighborhood set. We call it generated by the L -fuzzy topology \mathcal{T} .

Conversely, given an L -fuzzy soft neighborhood set $(\mathcal{U}, X \times M(L), L^X)$ we define the mapping $\mathcal{T}_\mathcal{U} : L^X \rightarrow L$ by the equality $\mathcal{T}(u) = \bigwedge_{x_\lambda \in \widehat{qu}} \mathcal{U}_{x_\lambda}(u)$. Then $\mathcal{T}_\mathcal{U}$ is an L -fuzzy topology on X . We call the L -fuzzy topological space $(X, \mathcal{T}_\mathcal{U})$ induced by the L -fuzzy soft neighborhood set $(\mathcal{U}, X \times M(L), L^X)$.

Besides $\mathcal{U}_{\mathcal{T}_\mathcal{U}} = \mathcal{U}$ and $\mathcal{T}_{\mathcal{U}_\mathcal{T}}$.

Proposition 6.3 Let $(\mathcal{U}, X \times M(L), L^X)$ and $(\mathcal{V}, Y \times M(L), L^Y)$ be two L -fuzzy soft neighborhood sets. Then

$$(\psi, (\psi^\rightarrow)^\rightarrow) : (\mathcal{U}, X \times M(L), L^X) \rightarrow (\mathcal{V}, Y \times M(L), L^Y)$$

is an L -fuzzy soft mapping if and only if for all $x_\lambda \in Pt(L^X)$ and all $u \in L^Y$, it holds

$$\mathcal{U}_{x_\lambda}(\psi^\leftarrow(u)) \geq \mathcal{V}_{\psi(x_\lambda)}(u)$$

.

From this proposition we easily obtain the next

Theorem 6.4 Let $\psi : X \rightarrow Y$ be a mapping. Then

$$(\psi, (\psi^\rightarrow)^\rightarrow) : (\mathcal{U}, X \times M(L), L^X) \rightarrow (\mathcal{V}, Y \times M(L), L^Y)$$

is a soft mapping if and only if the mapping of the corresponding L -fuzzy topological spaces $\psi : (X, \mathcal{T}_\mathcal{U}) \rightarrow (Y, \mathcal{T}_\mathcal{V})$ is continuous.

From the above results we have the following.

Theorem 6.5 Let a lattice L be fixed. By assigning to an L -fuzzy soft neighborhood set $(\mathcal{U}, X \times M(L), L^X)$ the L -fuzzy topological space $(X, \mathcal{T}_\mathcal{U})$ and assigning to a soft mapping $(\psi, (\psi^\rightarrow)^\rightarrow) : (\mathcal{U}, X \times M(L), L^X) \rightarrow (\mathcal{V}, Y \times M(L), L^Y)$ of L -fuzzy soft neighborhood sets the (continuous) mapping $\psi : (X, \mathcal{T}_\mathcal{U}) \rightarrow (Y, \mathcal{T}_\mathcal{V})$ of the corresponding L -fuzzy topological spaces we get a functor $\Psi : \mathbf{NFSOFSET}(L) \rightarrow \mathbf{FTOP}(L)$ which establishes isomorphism between the category of L -fuzzy topological spaces $\mathbf{FTOP}(L)$ and the category of L -fuzzy soft neighborhood sets $\mathbf{NFSOFSET}(L)$.

Acknowledgements This research was done during the stay of the first named author at the University of Latvia from September 2010 till January 2011. This stay was organized in the frames of the Erasmus exchange programme between the University of Latvia and Kocaeli University. Banu Pazar Varol gratefully acknowledges financial support received from the Erasmus programme and from the Kocaeli University.

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