











- the degree of monotonicity of  $f$  is smaller or equal than the degree of monotonicity of the rule base based on  $f$ ;
- or informally speaking, the monotonicity of  $f$  implies the monotonicity of  $\mathcal{R}_f$ ;
- but this not the other way round.

### 3. Conclusion

In this contribution, we have “fuzzified” the monotonicity property of a FRB into the graded one, i.e.,  $a$ -monotonicity where  $a$  represents the particular degree in which a given FRB is monotone. Furthermore, it has been extended to an arbitrary function from fuzzy sets to fuzzy sets. Contrary to the previous work [1] where we worked within a logical framework without a direct interpretation using the degrees of truth, we emphasize here an immediate evaluation of grades. This approach may become useful for readers that are not familiar with a formal logical notational style. And moreover, we have presented examples supporting our way of generalization. In this way, we point out that degrees play their role also in this particular problem relating to FRBs and it is not only an artificial generalization. Finally, the last section showed a connection with functions (with a special domain and range) and foreshadowed the promising area for future work.

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