Then

- $i \neq j \Rightarrow M_i \cap M_j = \emptyset$;
- $\bigcup_{i=1}^{n} M_i = \mathbb{R}$;
- $E = \bigvee_{i=1}^{n} A(M_i, \alpha_i)$;
- $E^{\alpha_i} = \bigcup_{j=1}^{i} M_j$
  where E^{α_i} as the α_i-cut of fuzzy set E .

Taking into account the property of addition of fuzzy numbers:

$(\bigoplus_{i=1}^{n} z(a_i, \alpha_i))(t) =$

$$= \begin{cases} 1, t \leq 0, \\ \alpha_1,\ 0 < t \leq a_1, \\ \ldots \\ \alpha_{i+1},\ a_1 + \ldots + a_i < t \leq a_1 + \ldots + a_{i+1}, \\ \ldots \\ 0,\ t > a_1 + \ldots + a_n, \end{cases}$$

we obtain

$$\int_E f\, d\mu = \bigoplus_{i=1}^{n} \int_{E(\alpha_i, M_i)} f\, d\mu = \bigoplus_{i=1}^{n} z(\int_{M_i} f d\nu, \alpha_i) =$$

$$= \begin{cases} 1,\ t \leq \int_{M_1} f d\nu, \\ \ldots \\ \alpha_i,\ \sum_{j=1}^{i} \int_{M_j} f d\nu < t \leq \sum_{j=1}^{i+1} \int_{M_j} f d\nu, \\ \ldots \\ 0,\ t > \sum_{j=1}^{n} \int_{M_j} f d\nu, \end{cases} =$$

$$= \begin{cases} 1,\ t \leq \int_{E^{\alpha_1}} f d\nu, \\ \ldots \\ \alpha_i,\ \int_{E^{\alpha_i}} f d\nu < t \leq \int_{E^{\alpha_{i+1}}} f d\nu, \\ \ldots \\ \alpha_n,\ \int_{E^{\alpha_{n-1}}} f d\nu < t \leq \int_{E^{\alpha_n}} f d\nu, \\ 0,\ otherwise. \end{cases}$$

5.3. Integration over NMF E

As was already mentioned every NMF E can be presented as the limit of a non-decreasing sequence of SNMF. To describe this sequence we use the same logic as in the previous subsection. Let us take a sequence $(E_n)_{n \in \mathbb{N}}$ such as:

- for all $n \in \mathbb{N} : E_n(\mathbb{R}) = \{\alpha_1^n, \ldots, \alpha_{k_n}^n\}$;
- for all $n \in \mathbb{N}$ $\alpha_i^n > \alpha_{i+1}^n$, $i = 1, \ldots, k_n - 1$;
- $M_1^n = \{x \mid E(x) = \alpha_1^n\}$,
  $M_i^n = \{x \mid \alpha_i^n \leq E(x) < \alpha_{i-1}^n\}$, $i = 2, \ldots, k_n$;
- $E_n = \bigvee_{i=1}^{k_n} E(\alpha_i^n, M_i^n)$, $n \in \mathbb{N}$;
- $E = \bigvee_n E_n$.

Denoting $I = \int_E f\, d\mu$ and $I_n = \int_{E_n} f\, d\mu$ we get

$$I = Sup\{\int_{E_n} f\, d\mu \mid n \in \mathbb{N}\} = Sup\{I_n \mid n \in \mathbb{N}\}.$$

From the last equality we can get an approximate value of I by fixing n. Obviously, the integral accuracy in this case will be dependent on n .

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