the sensors is a human, by adjusting the criteria from which he will have to provide a given alarm.

7.2.2. *Adjusting the system itself*

It is also possible to optimize the mean mutual information by modifying the systems itself. It can be a parameter as for example the location of one sensor, or a fusion method.

8. Conclusion

We proposed a measure for generalizing the mutual information to belief functions. This can allow us to define a channel capacity for belief mass output-modeled channels, and to propose a methodology for evaluation and optimization of such systems.

We started an axiomatization to generalize the KL divergence to belief functions. The axioms are the boundary condition (in the probabilistic case), the positiveness, the symmetry w.r.t. the ordering of the elements, the (almost) minimum for the pignistic probability, and the (almost) nullity of the associated mutual information in the independent case. A list of properties is also given.

A reasoning is presented to obtain one expression for this generalized KL divergence. But we did not prove that this was the unique solution. so the axiomatic is not complete. To achieve it, some axioms may be added in order to guarantee a unique solution for the generalized KL divergence. For example, one could express as an axiom that the resulting capacity must not exceed its maximal probabilistic value $\log N$, where N is the number of possible inputs.

In the field of belief functions, the literature offers a panel of measures for uncertainty, including generalizations of Shannon entropy. But these works concern only one part of the Information Theory. We have proposed to generalize the other part by defining the first extension of Shannon channel capacity to belief functions. It offers then a generalization of the mutual information, and a definition of the KL divergence for mass functions. They are derived from the expression of discord.

This new measure allows us to evaluate information systems, and to increase their performance, either by controlling the input statistics, if this is possible in practice, or by modifying the system itself (design and date fusion algorithms). We showed and example of algorithm to compute the generalized capacity, and a possible application.

References

[1] Claude Shannon. A mathematical theory of communication. *Bell Syst. Technical J.*, 27:379–423 and 623–656, July and Oct. 1948.

[2] George J. Klir. *Uncertainty and Information. Foundations of Generalized Information Theory*. Wiley, US, 2006.

[3] Isabelle Bloch. Defining belief functions using mathematical morphology ; application to image fusion under imprecision. *Int. J. Approximate Reasoning*, 48:437–465, June 2008.

[4] Glenn Shafer. *A Mathematical Theory of Evidence*. Princeton Univ. Press, Princeton, NJ, 1976.

[5] Nikhil R. Pal, James C. Bezdek, and Rohen Hemasinha. Uncertainty measures for evidential reasoning I: A review. *Int. J. Approximate Reasoning*, 7:165–183, 1992.

[6] Simon Petit-Renaud. *Application de la théorie de l'information et des systèmes flous à l'estimation fonctionnelle en présence d'informations incertaines ou imprécises*. Ph.d. thesis, Université de Technologie de Compiègne, France, 1999.

[7] Solomon Kullback and Richard Leibler. On information and sufficiency. *The Annals of Mathematical Statistics*, 22:79–86, 1951.

[8] Richard E. Blahut. *Principles and practice of information theory*. Addison Wesley Publishing Company, 1990.

[9] Frank Critchley, Paul Marriott, and Mark Salmon. Preferred point geometry and the local differential geometry of the Kullback-Leibler divergence. *The Annals of Statistics*, 22:1587–1602, Sept. 1994.

[10] Carlo Bertoluzza, Mario Di Bacco, and Viviana Doldi. An axiomatic characterization of the measures of similarity. *Sankhya, the Indian J. of Statistics*, 66:474–486, Aug. 2004.

[11] Imre Csiszár. Axiomatic characterizations of information measures. *Entropy*, 10:261–273, 2008.

[12] Ivan Kojadinovic, Jean-Luc Marichal, and Marc Roubens. An axiomatic approach to the definition of the entropy of a discrete Choquet capacity. *Inf. Comput. Sci.*, 172(1–2):131–153, 2005.

[13] Philippe Smets and Robert Kennes. The transferable belief model. *Artificial Intelligence*, 66:191–234, 1994.

[14] Boutheima Ben Yaghlane, Philippe Smets, and Khaled Mellouli. Belief function independence: I. the marginal case. *Int. J. Approximate Reasoning*, 29:47–70, Jan. 2002.

[15] Boutheima Ben Yaghlane, Philippe Smets, and Khaled Mellouli. Belief function independence: II. the conditional case. *Int. J. Approximate Reasoning*, 31:31–75, Oct. 2002.