The Research of Fault Diagnosis with Time-Delay Switched Systems

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Abstract--The time delay switched system fault diagnosis control is proposed based on LMI skill. The observer is designed based on Lyapunov stability theorem with switched system, and the LMI form of uniformly bounded stability result for fault estimation error system is given. Finally, the simulation is carried by MATLAB/SIMULINK software, to verify the feasibility of the proposed algorithm.

Keywords- switched systems; fault diagnosis; LMI

I. INTRODUCTION

The main reason for fault diagnosislaid which increasing demand for high reliability and safety in many actual engineering control systems [1-3]. A lot of fruitful results can be found recently, most design schemes of fault diagnosis architectures for dynamic system with linear system, nonlinear system and switched system [4-6]. For the switched systems, which exist in the practical systems widely, and the switched phenomenon exists in industrial control nature or in the life of people widely. Unavoidable modeling uncertainties, which arise due to modeling errors and time delay, then the adaptive control method, can be found in this subject [7-8]. So the fault diagnosis research of switched systems with time-delay has attracted the attention of many scholars. Motivated by these considerations, the fault diagnosis with delay switched system is considered, the observer design for fault is designed and the error system is proved by Lyapunov stability. The result in form of linear matrix inequality, and Lyapunov is applied to the strict proof. At last, the simulation of MATLAB in a numerical example proved the validity of the conclusion.

II. SYSTEM DESCRIPTION

Time-delay switched systems can be described as

$$\begin{aligned} \dot{x}(t) &= A_{\sigma(t)}x(t) + A_{d\sigma(t)}x(t-d) \\ &\quad + B_{\sigma(t)}u(t) + E_{\sigma(t)}f(t) \\ y(t) &= C_{\sigma(t)}x(t) \\ x(\theta) &= \phi(\theta), \forall\theta \in [-d,0] \end{aligned} \tag{1}$$

where $x(t) \in \Re^n$ express the state vectors of system, $u(t) \in \Re^m$ expresses the system input, $y(t) \in \Re^p$ is the system output, $f(t) \in \Re^r$ is the actuator fault of the system, $A_{\sigma(t)}, A_{d\sigma(t)}, B_{\sigma(t)}, C_{\sigma(t)}, E_{\sigma(t)}$ arethe system matrixes with appropriate dimension, for switching signal, $\sigma(t):[0,+\infty) \to \psi = \{1,\cdots,N\}$, where $N \ge 1$, especially, when $N=1$, system become ordinary time-delay systems, when $N>1$,it is supposedthat the i subsystem work, that is $\sigma(t)=i$,

so the formula was established

$$A_{\sigma(t)} = A_i, A_{d\sigma(t)} = A_i, B_{\sigma(t)} = B_i, C_{\sigma(t)} = C_i, E_{\sigma(t)} = E_i$$

where $(A_i \quad C_i)$ is observable, the matrix E_i is a line full rank matrix, that is $rank(E_i) = r$,actuator fault satisfies the norm bounded condition $\|\dot{f}(t)\| \le f_1$, $\|f(t)\| \le f_2$, $\phi(\theta)$ is the initial function.

Lemma 1(Schur Complement lemma), for a givensymmetric matrix

$$\boldsymbol{Q} = \begin{bmatrix} \boldsymbol{Q}_{11} & \boldsymbol{Q}_{12} \\ \boldsymbol{Q}_{12}^T & \boldsymbol{Q}_{22} \end{bmatrix} < 0$$

Where $\boldsymbol{Q}_{ii}$ is a symmetric matrix of $r_i \times r_i$, r_i is a positive integer, $i = 1,2$, the following conditions are

(1) $\boldsymbol{Q} < 0$;

(2) $\boldsymbol{Q}_{11} < 0$ and $\boldsymbol{Q}_{22} - \boldsymbol{Q}_{12}^{\mathrm{T}}\boldsymbol{Q}_{11}^{-1}\boldsymbol{Q}_{12} < 0$;

(3) $\boldsymbol{Q}_{22} < 0$ and $\boldsymbol{Q}_{11} - \boldsymbol{Q}_{12}\boldsymbol{Q}_{22}^{-1}\boldsymbol{Q}_{12}^{\mathrm{T}} < 0$。

Lemma 2 For definite matrix composed of any positive constant $\boldsymbol{Z} \in \Re^{n\times n}$, $\boldsymbol{Z} = \boldsymbol{Z}^{\mathrm{T}} > 0$,the scalars are $b > a \ge 0$,the vector function is $\boldsymbol{w}:[a,b] \to \Re^n$,the integral inequality was established:

$$(b-a)\int_a^b \boldsymbol{w}^{\mathrm{T}}(s)\boldsymbol{Z}\boldsymbol{w}(s)\mathrm{d}s \ge \left(\int_a^b \boldsymbol{w}(s)\mathrm{d}s\right)^{\mathrm{T}} \boldsymbol{Z}\left(\int_a^b \boldsymbol{w}(s)\mathrm{d}s\right)$$

III. TIME-DELAY SYSTEM FAULT OBSERVER DESIGN

The observer can be design as following:

$$\begin{aligned} \dot{\hat{x}}(t) &= A_{\sigma(t)}\hat{x}(t) + A_{d\sigma(t)}\hat{x}(t-d) + B_{\sigma(t)}u(t) + E_{\sigma(t)}\hat{f}(t) \\ &\quad + L\left(y - C_{\sigma(t)}\hat{x}(t)\right) \\ \hat{x}(\theta) &= \phi(\theta), \forall\theta \in [-d,0] \end{aligned} \tag{2}$$

Definite the error state as

$$e_x = x - \hat{x}, e_f = f - \hat{f}$$

Then the error system can be get as

$$\dot{e}_x(t) = \left(A_{\sigma(t)} - LC\right)e_x(t) + A_{d\sigma(t)}e_x(t-d) + E_{\sigma(t)}e_f(t)$$

If there are positive definite symmetric matrices P_i, Q_i and an arbitrary matrix Y_i satisfy the follow matrix inequality $\Xi < 0$, then the fault estimation error system is uniformly bounded stable, where

$$Y_i = P_i L_i, E_i^T P_i = F_i C_i$$

$$\dot{\hat{f}} = -\Gamma^{-1}\left(FCe_x + FC\dot{e}_x + \sigma\hat{f}\right)$$

The residence time of switching signal meets:

$$T_a > T_a^* = \frac{\ln(\mu)}{\alpha}$$

Then the system is asymptotically exponentially stable and there is $\mu \geq 1, P_i \leq \mu P_j, Q_i \leq \mu Q_j, Z_i \leq \mu Z_j$, where:

$$\Xi = \begin{bmatrix} \begin{matrix} A_i^T P_i + P_i A_i - C_i^T L_i^T P_i \\ -P_i L_i C_i + Q_i \\ -e^{-\alpha d}\frac{1}{d^2}Z_i \end{matrix} & \begin{matrix} P_i A_{di} + E_i^T P_i A_{di} \\ e^{-\alpha d}\frac{1}{d^2}Z_i \end{matrix} & P_i E_i + E_i^T P_i\left(A_i - L_i C_i\right) & \left(A_i - L_i C_i\right)Z_i \\ * & -e^{-\alpha d}Q_i - e^{-\alpha d}\frac{1}{d^2}Z_i & 0 & A_{di}Z_i \\ * & * & -2E_i^T P_i E_i & E_i Z_i \\ * & * & * & -Z_i \end{bmatrix}$$

Proof: Lyapunov function is selected as following

$$V_i(t) = V_{1i}(t) + V_{2i}(t) + V_{3i}(t) + V_{4i}(t)$$

where

$$V_{1i}(t) = e_x^T(t)P_i e_x(t)$$

$$V_{2i}(t) = e^{-\alpha t}\int_{t-d}^{t} e^{\alpha s}e_x^T(s)Q_i e_x(s)ds$$

$$V_{3i}(t) = d^{-1}e^{-\alpha t}\int_{-d}^{0}\int_{t+\sigma}^{t} e^{\alpha s}\dot{e}_x^T(s)Z_i\dot{e}_x(s)dsd\sigma$$

$$V_{4i}(t) = e_f^T(t)\Gamma e_f(t)$$

Along (1), take the derivative of Lyapunov functional:

$$\dot{V}_{1i}(t) = e_x^T(t)\left(\left(A_i - L_i C_i\right)^T P_i + P_i\left(A_i - L_i C_i\right)\right)e_x(t) + 2e_x^T(t)P_i A_{di}e_x(t-d) + 2e_x^T(t)P_i E_i e_f(t)$$

$$\dot{V}_{2i}(t) \leq -\alpha V_{2i} + e_x^T(t)Q_i e_x(t) - e^{-\alpha d}e_x^T(t-d)Q_i e_x(t-d)$$

$$\dot{V}_{3i}(t) \leq -\alpha V_{3i} + \dot{e}_x^T(t)Z_i\dot{e}_x(t) - d^{-1}e^{-\alpha d}\int_{t-d}^{t}\dot{e}_x^T(s)Z_i\dot{e}_x(s)ds$$

Because of the system so that there is:

$$\dot{e}_x^T(t)Z_i\dot{e}_x(t) = \xi^T(t)\begin{bmatrix} \left(A_i - L_i C_i\right)^T \\ A_{di}^T \\ E_i^T \end{bmatrix} Z_i\left[A_i - L_i C_i \quad A_{di} \quad E_i\right]\xi(t)$$

Along (1), take the derivative of Lyapunov function $V_{4i}(t)$:

$$\dot{V}_{4i}(t) = -2e_f^T(t)F_i C_i e_x - 2e_f^T(t)F_i C_i\left(A_i - L_i C_i\right)e_x(t) - 2e_f^T(t)F_i C_i A_{di}e_x(t-d) - 2e_f^T(t)F_i C_i E_i e_f(t) - 2\sigma e_f^T(t)\left(e_f(t) + f^*\right)$$

Because of $FC = E^T P$ and for the system the following inequality exists:

$$-2\sigma e_f^T(t)\left(e_f(t) + f^*\right) \leq -2\sigma e_f^T(t)e_f(t) + 2\sigma e_f^T(t)f^* \leq -\sigma e_f^T(t)e_f(t) + \sigma f^{*T}f^*$$

Select $\alpha = \sigma\lambda_{\min}\left(\Gamma^{-1}\right)$,soitcan be obtained:

$$\dot{V}_{4i}(t) \leq -\alpha V_{4i} - 2e_f^T(t)E_i^T P_i e_x - 2e_f^T(t)E_i^T P_i\left(A_i - L_i C_i\right)e_x(t) - 2e_f^T(t)E_i^T P_i A_{di}e_x(t-d) - 2e_f^T(t)E_i^T P_i E_i e_f(t) + \sigma f_2^2$$

As a result:

$$\dot{V}_i(t) \leq -\alpha V_i(t) + \xi^T\Xi\xi + \delta$$

If $\Xi < 0$, where $\delta = \alpha f_2^2$, so the following will be got in $[t_0, t]$

$$V_i(t) \leq -e^{-\alpha t}V_i(t_0) + \frac{\delta}{\alpha}$$

So that, there is

$$V_i(t) \leq -e^{-\alpha(t-t_0)}\mu^{\beta}V_i(t_0) + \frac{\delta}{\alpha} \leq -e^{-\left(\alpha - \frac{\ln\mu}{T_a}\right)(t-t_0)}\mu^{\beta}V_i(t_0) + \frac{\delta}{\alpha}$$

Fault estimation error system is uniformly bounded stable.The theorem is proven.

IV. SIMULATION EXPERIMENT

In order to verify the feasibility and effectiveness of the theoretical results, the simulation is proposed by MATLAB/SIMULINK software. To solved by LMI toolbox of Matlab, but in the theorem $E^T P = FC$ is required. To solve $E^T P - FC = 0$, The following inequality is required to add into the process of simulation experiment.

$$\min \eta$$
$$s.t. \begin{bmatrix} \eta I & E^T P - FC \\ * & \eta I \end{bmatrix} > 0$$

Through a small enough positive number η, $E^T P - FC = 0$ can be realized.

Subsystem 1:

$$A_1 = \begin{bmatrix} -1 & 0 \\ 0 & -0.8 \end{bmatrix}, A_{d1} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, E_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, C_1 = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

Subsystem 2:

$$A_2 = \begin{bmatrix} -2 & 1 \\ 1 & -3 \end{bmatrix}, A_{d2} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}, B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, E_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, C_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

Through the algorithm of the above theorem, the LMI toolbox of the Matlab can carry out the simulation, we can get:

$$L_1 = \begin{bmatrix} 13.6138 \\ 1.6245 \end{bmatrix}, L_2 = \begin{bmatrix} 1.6245 \\ 8.7539 \end{bmatrix}$$

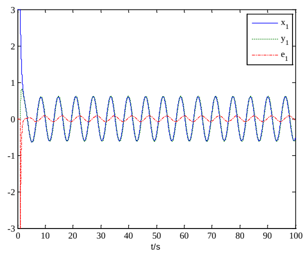

FIGURE I. STATE AND ERROR OF THE SYSTEM

As can be seen from the simulation graph Ⅰ, the state of the observer y_1 can track the system state x_1 soon, the error of the e_1.

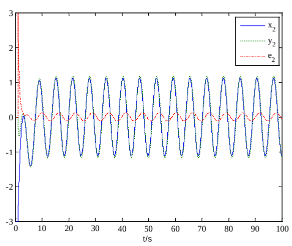

FIGURE II. STATE AND ERROR OF THE SYSTEM

As can be seen from the simulation graph Ⅱ, the state of the observer y_2 can track the system state x_2 soon, the error of the e_2.

V. CONCLUSION

In this paper, the observer ofswitched system fault diagnosis is presented for a class of lag switched systems. The uncertain term is estimation by adaptive control method and the error system is proved to stable based on Lyapunov theory to verify the effectiveness.

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