The Research of Observer for Uncertain Nonlinear Neutral Delay Systems

R.H.Zhang
Hebei Vocational & Technical College of Building Materials
China

***Abstract*-In this paper, the observer design method is proposed for nonlinear delay neutral system, and makes the system convergence to the equilibrium point in the exponential speed. Base on the Lyapunov theorem, adaptive controller method and linear matrix inequalities (LMIs) method, an observer can be get with stability criteria to solve the exponentially stability of neutral delay system. The observer in form of adaptive control, and the stability criteria can be get in form of LMI, finally, an example is simulated to confirm the effectiveness of the proposed approach.**

Keywords-observer; neutral delay system; linear matrix inequality (LMI)

I. INTRODUCTION

Neural system with timedelay often exists in many practical systems[1-3]. The observer design for system has been paid much attention over these years, and become research hot point in future. Adaptive control has been special skill for solve the parameter identification and system controller design process, and some other problem.

The stability is the first important thing in controller design and some other spent, then the stability problem is became the most important thing for systemdesign[4-6]. Lyapunov stability theorem is became the most important theory because of the stability criteria design process in control theorem[7-8].

In this paper, the adaptive observer is design to guarantee the close system stability and the Lyapunov stability theorem is used to prove the stability of the system, and an integral type of Lyapunov functional chooses. The controller can be getting in form of LMI, which is convenient for later calculation by Matlab. The simulation is proposed to prove heperformance of the observer in the paper.

II. SYSTEM DESCRIPTION

Consider the nonlinear neutral-type system with time delay and parameter uncertainties described as following:

$$\begin{aligned} \dot{x}(t) - A_d\dot{x}(t-h) &= (A+\Delta A(t))x(t) + A_h x(t-h) \\ &\quad + Bu(t) + Ef(t) \\ y &= Cx \\ x(\tau) &= \phi(\tau), \tau \in [-h, 0] \end{aligned} \tag{1}$$

where $x(t) \in R^n$, $u(t) \in R^m$, $y(t) \in R^p$ are states, inputs and outputs respectively, and $f(t) \in R^r$ are unknown function; A, A_d, A_h, B, E, C are known constant real matrices of appropriate dimensions.

Assumption1: $\|\dot{f}(t)\| \le f_1$, and $0 \le f_1 < \infty$.

Assumption2: $\Delta A(t) = EF(t)H$,and $F^T(t)F(t) \le I$, E , H are known matrices.

Lemma1. For any constant matrix $P \in R^{n\times n}, P > 0$ and differentiable vector function x(t), with appropriate dimensions, theinequality holds as follows

$$\left[\int_{t-d}^{t} \dot{x}(\tau)d\tau\right]^T P\left[\int_{t-d}^{t} \dot{x}(\tau)d\tau\right] \le d\int_{t-d}^{t} \dot{x}^T(\tau)P\dot{x}(\tau)d\tau$$

Lemma2. For any real vectors x, y $\in R^n$ and any positive-definite matrix $G > 0$, we have

$$XY + Y^T X^T \le XGX^T + Y^T G^{-1} Y$$

Lemma3. Let E , H , H_h be real matrices of appropriate dimensions with F satisfying $F^T F \le I$. Then, for any scalar Q,

$$EFH + H^T F^T E^T \le EQE^T + H^T Q^{-1} H$$

III. ADAPTIVE OBSERVER DESIGN

The proposed observer is

$$\begin{aligned} \dot{\hat{x}}(t) - A_d\dot{\hat{x}}(t-h) &= A\hat{x}(t) + A_h\hat{x}(t-h) + Bu(t) \\ &\quad + E\hat{f}(t, x(t)) + L(y - C\hat{x}(t)) \end{aligned} \tag{2}$$

L is gain of the design observer , which will be chosen. $\hat{f}$ is an adaptive estimate of $f(t)$,and its adaptive law is given by

$$\begin{aligned} \dot{\hat{f}}(t, x(t)) = -MF(C\dot{e}(t) + Ce(t)) = -MFC[\dot{y}(t) - C\dot{\hat{x}}(t) + y(t) \\ - C\hat{x}(t)] \end{aligned} \tag{3}$$

MF is an matrices, which will be chosen later. We define $e(t) = x(t) - \hat{x}(t)$ from (1) and (2),we have

$$\begin{aligned} \dot{e}(t) - A_d\dot{e}(t-h) &= (A - LC)e(t) + A_h e(t-h) + \Delta A(t)x(t) \\ &\quad + E[f(t) - \hat{f}(t, x(t))] \end{aligned} \tag{4}$$

Theorem .Given scalars β >0, suppose that Assumptions are satisfied, then, the observer given by (2),(3) ensure that the state estimation error is asymptotically stable. If there exist symmetric positive definite matrices $P_i > 0 (i = 1,2,3,4)$, and $\Xi < 0$.

Proof. Choose the Lyapunov function candidate

$$\begin{aligned} V_1 &= e^T(t)P_1e(t) \\ V_2 &= \int_{t-h}^{t} e^{2\beta(\tau-t)}e^T(\tau)P_2e(\tau)d\tau \\ V_3 &= h\int_{-h}^{0}\int_{t+\theta}^{t} e^{2\beta(\tau-t)}\dot{e}^T(\tau)P_3\dot{e}(\tau)d\tau d\theta \\ V_4 &= [f(t)-\hat{f}(t,x(t))]^T P_4[f(t)-\hat{f}(t,x(t))] \end{aligned} \quad (5)$$

Then , the derivative of V_i is described by

$$\begin{aligned} \dot{V}_1 &= 2e^T(t)P_1\dot{e}(t) \\ \dot{V}_2 &= 2e^T(t)P_2e(t) - e^{-2\beta h}e^T(t-h)P_2e(t-h) \\ &\quad -2\beta\int_{t-h}^{t} e^{2\beta(\tau-t)}e^T(t)P_2e(t)d\tau \\ \dot{V}_3 &= h^2e^T(t)P_3e(t) - he^{-2\beta h}\int_{t-h}^{t}\dot{e}^T(\tau)P_3\dot{e}(\tau)d\tau \\ &\quad -2\beta h\int_{-h}^{0}\int_{t+\theta}^{t} e^{2\beta(\tau-t)}\dot{e}^T(\tau)P_3\dot{e}(\tau)d\tau d\theta \\ \dot{V}_4 &= 2[f(t)-\hat{f}(t,x(t))]^T P_4[\dot{f}(t)-\dot{\hat{f}}(t,x(t))] \end{aligned} \quad (6)$$

So we get

$$\begin{aligned} 2\beta V_1 &= 2\beta e^T(t)P_1e(t) \\ 2\beta V_2 &= 2\beta\int_{t-h}^{t} e^{2\beta(\tau-t)}e^T(\tau)P_2e(\tau)d\tau \\ 2\beta V_3 &= 2\beta h\int_{-h}^{0}\int_{t+\theta}^{t} e^{2\beta(\tau-t)}\dot{e}^T(\tau)P_3\dot{e}(\tau)d\tau d\theta \\ 2\beta V_4 &= 2\beta[f(t)-\hat{f}(t,x(t))]^T P_4[f(t)-\hat{f}(t,x(t))] \end{aligned} \quad (7)$$

Then we have

$$\begin{aligned} \dot{V}_1 + 2\beta V_1 &= 2e^T(t)P_1\dot{e}(t) + 2\beta e^T(t)P_1e(t) \\ \dot{V}_2 + 2\beta V_2 &= 2e^T(t)P_2e(t) - e^{-2\beta h}e^T(t-h)P_2e(t-h) \\ \dot{V}_3 + 2\beta V_3 &= h^2e^T(t)P_3e(t) - he^{-2\beta h}\int_{t-h}^{t}\dot{e}^T(\tau)P_3\dot{e}(\tau)d\tau \\ \dot{V}_4 + 2\beta V_4 &= 2[f(t)-\hat{f}(t,x(t))]^T P_4[\dot{f}(t)-\dot{\hat{f}}(t,x(t))] \\ &\quad +2\beta[f(t)-\hat{f}(t,x(t))]^T P_4[f(t)-\hat{f}(t,x(t))] \end{aligned} \quad (8)$$

By using Lemma 1.we have

$$\begin{aligned} &-he^{-2\beta h}\int_{t-h}^{t}\dot{e}^T(\tau)P_3\dot{e}(\tau)d\tau \\ &\le -e^{-2\beta h}\left[\int_{t-h}^{t}\dot{e}^T(\tau)d\tau\right]^T P_3\left[\int_{t-h}^{t}\dot{e}^T(\tau)d\tau\right] \\ &= -e^{-2\beta h}\left[e(t)-e(t-h)\right]^T P_3\left[e(t)-e(t-h)\right] \end{aligned} \quad (9)$$

$$\begin{aligned} &2[f(t)-\hat{f}(t,x(t))]^T P_4[\dot{f}(t)-\dot{\hat{f}}(t,x(t))] \\ &= 2[f(t)-\hat{f}(t,x(t))]^T P_4[\dot{f}(t)-MF(C\dot{e}(t)+Ce(t))] \end{aligned} \quad (10)$$

By using Lemma 2.and $\left\|\dot{f}(t)\right\| \le f_1$, there exist $N > 0$, then we have

$$\begin{aligned} &2[f(t)-\hat{f}(t,x(t))]^T P_4[\dot{f}(t)-\dot{\hat{f}}(t,x(t))] \\ &\le [f(t)-\hat{f}(t,x(t))]^T N[f(t)-\hat{f}(t,x(t))] + \lambda_{\max}(P_4N^{-1}P_4)f_1^2 \\ &-2[f(t)-\hat{f}(t,x(t))]^T P_4MF(C\dot{e}(t)+Ce(t)) \end{aligned}$$

(11)

We define

$$e_f(t) = f(t) - \hat{f}(t,x(t)) \quad (12)$$

From (8) - (12)we have

$$\begin{aligned} \dot{V} + 2\beta V &= 2e^T(t)P_1\dot{e}(t) + 2\beta e^T(t)P_1e(t) \\ &+2e^T(t)P_2e(t) - e^{-2\beta h}e^T(t-h)P_2e(t-h) \\ &+h^2e^T(t)P_3e(t) - e^{-2\beta h}\left[e(t)-e(t-h)\right]^T P_3\left[e(t)-e(t-h)\right] \\ &+[f(t)-\hat{f}(t,x(t))]^T N[f(t)-\hat{f}(t,x(t))] + \lambda_{\max}(P_4N^{-1}P_4)f_1^2 \\ &-2[f(t)-\hat{f}(t,x(t))]^T P_4MF(C\dot{e}(t)+Ce(t)) \\ &+2\beta[f(t)-\hat{f}(t,x(t))]^T P_4[f(t)-\hat{f}(t,x(t))] \end{aligned}$$

(13)

From (4) we have

$$\begin{aligned} \dot{e}(t) - A_d\dot{e}(t-h) &= (A-LC)e(t) + A_he(t-h) + \Delta A(t)x(t) \\ &+ E[f(t)-\hat{f}(t,x(t))] \end{aligned}$$

(14)

We define

$$\xi = [\dot{e}^T(t) \quad \dot{e}^T(t-h) \quad e^T(t) \quad e^T(t-h) \quad e_f{}^T(t)] \quad (15)$$

By using (13)- (15) we get

$$2\beta V + \dot{V} \le \xi^T \Xi \xi$$

So we have

$$\dot{V} \le -2\beta V$$

So the observer given by (2), (3) ensure that the state estimation error is asymptotically stable.

IV. ILLUSTRATION

Consider a neutral delay system described by

$$\dot{x}(t) - A_d\dot{x}(t-h) = (A+\Delta A(t))x(t) + A_h x(t-h) + Bu(t) + Ef(t)$$
$$y = Cx$$
$$x(\tau) = \phi(\tau), \tau \in [-h, 0]$$

where

$$A_d = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, A_h = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, E = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

And we assume that the time delay $h = 1$ and

$$P_1 = \begin{bmatrix} 1.1223 & -0.1623 \\ -0.1623 & 3.7126 \end{bmatrix},\ P_2 = \begin{bmatrix} 1.237 & 0.1345 \\ 0.1345 & 1.3245 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 2.1235 & -1.5467 \\ -1.5467 & 2.4566 \end{bmatrix}$$

The figure1-2 show the trajectory of the states with the neural system, in the figure, the state convergence to equilibrium point exponentially.

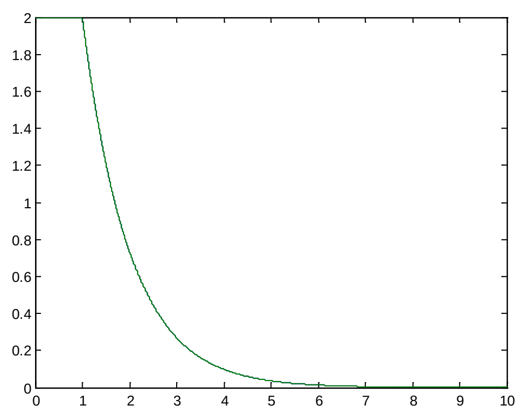

FIGURE I. STATE AND ESTIMATION OF THE STATE

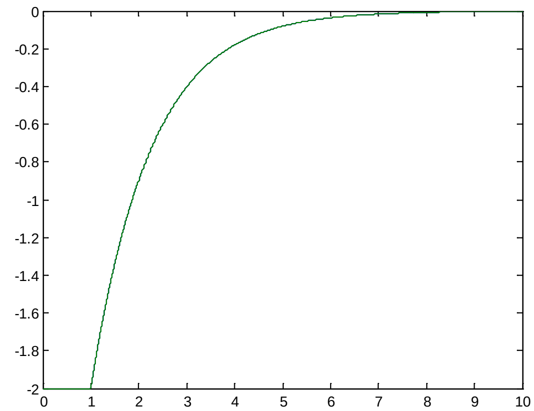

FIGURE II. STATE AND ESTIMATION OF THE STATE

V. CONCLUSIONS

The adaptive observer is designed to guarantee the system convergence to the equilibrium point in exponential speed. The controller get by LMI tool box in Matlab, then it can be get easily. The simulation show the controller result based on the theorem.

REFERENCES

[1] Chen, Y.,Stabilization of neutral time-delay systems with actuator saturation via auxiliary time-delay feedback, Automatica, 52, 242-247, 2015.

[2] Hall, M. B., Comparisons of in vitro fermentation and high moisture forage processing methods for determination of neutral detergent fiber digestibility, Animal Feed Science and Technology, 199, 127-136, 2015.

[3] Jian, J. and B. Wang, Stability analysis in Lagrange sense for a class of BAM neural networks of neutral type with multiple time-varying delays. Neurocomputing, 149, 930-939, 2015.

[4] Krishnasamy, R. and P. Balasubramaniam, A descriptor system approach to the delay-dependent exponential stability analysis for switched neutral systems with nonlinear perturbations. Nonlinear Analysis: Hybrid Systems, 15, 23-36, 2015.

[5] Liao, X., Exponential estimates and exponential stability for neutral-type neural networks with multiple delays.Neurocomputing, 149, 868-883,2015.

[6] McAdams, R.Control and calculation of the titanium sublimation pumping speed and re-ionisation in the MAST neutral beam injectors. Fusion Engineering and Design, 90, 47-54, 2015.

[7] Milošević, M.Convergence and almost sure exponential stability of implicit numerical methods for a class of highly nonlinear neutral stochastic differential equations with constant delay.Journal of Computational and Applied Mathematics, 280, 248-264, 2015.

[8] Rajagopalan, V., Critical determinants of mitochondria-associated neutral sphingomyelinase (MA-nSMase) for mitochondrial localization.Biochimica et Biophysica Acta (BBA) - General Subjects, 1850(4), 628-639,2015.