Then we have

Theorem [Theorem 4.2 in [10]] *Łukasiewicz* logic is complete w.r.t. complete MV–algebras.*

Similarly, to obtain completeness of Semi–divisible Monoidal logic, it is necessary and sufficient that we add another infinitary inference rule

$$\text{From } [\neg\neg\alpha \Rightarrow \neg\alpha \underline{\&} \cdots \underline{\&} \neg\alpha]\ \forall n \text{ infer } \neg\alpha, \quad (41)$$

where the new logical connective $\underline{\&}$ is defined recursively via $\neg\alpha \underline{\&} \neg\beta = \neg\neg[\neg\alpha \& \neg\beta]$.

Then we have

Theorem *Semi–divisible Monoidal logic is complete w.r.t. complete semi–divisible residuated lattices.*

However, we have not been able to prove or disprove that strongness is preserved under MacNeille completion. If this conjecture is true, then we have

Theorem *Strong semi–divisible Monoidal logic is complete w.r.t. complete strong semi–divisible residuated lattices.*

Theorem *A predicate formula α is derivable in Strong Semi–divisible Monadic logic if, and only if its double negation $\neg\neg\alpha$ is derivable in Łukasiewicz* logic.*

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