Error modeling and Sensitivity Analysis of 5-DOF Parallel Tripod Head

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Abstract. Aiming at 5-dof parallel tripod head, the integrated parameter modeling of parallel mechanism with less degree of freedom is researched in this paper. Firstly, the kinematics parameters analysis of each component of 5-dof parallel mechanism is carried out and kinematics model of 5-dof mechanism is established, mathematical modeling of 5-dof mechanism error then is systematically proposed with integrated modeling method within the identical framework, which constructs the corresponding relationship between end-effector’s pose errors and geometric error and clearly discloses the influence of geometric error source on end effector’s uncontrollable errors. Finally, by the given the structure parameters, distribution rule of the uncontrollable errors coming from different height of workspace can be figured out with Monte Carlo simulation method, sensitivity analysis can then be carried out to diminish these errors. Error sensitivity represented by the Jacobin matrix shows that $e_\omega$ is extremely sensitive to $\delta h$. Moreover, in order to obtain the required accuracy, standard deviation caused by $\delta h$ is limited within 0.02mm/m, tolerance of $\delta h$ should precisely be 0.0449mm. Consequently, uncontrollable errors caused by the UPS chains of the 5-dof parallel tripod head, which can not be compensated by the theoretical method, need more stringent restrictions in manufacturing and assembly stages.

Introduction

Vision system is a major way to obtain information from outside, under the background of computer technology and artificial intelligent technology, computer vision comes to be a new subject, it has began to develop towards strong practicability, such as active vision, active measurement and multi-sensor fusion. For the precise forecasting, tracking, detection and segmentation respecting target, the research and invention of vision tripod head have got some achievement. Parallel mechanism [1] are utilized widely in computer vision tripod head due to their advantages of few drive elements, low cost and compact structure [2]. The 5-dof visual monitoring parallel tripod head based on TriVariant mechanism is proposed in this paper, it has the advantages of simple and compact structure, strong bearing capacity, well stability, small accumulated error [3], error can be compensated and large working space of end effector. Besides, the moving platform can change and adjust the motion states by constraint driven limb structure resulted from good dexterity, driving limb fails to impose constraint and interference on moving platform. Furthermore, kinematics limb is relatively less and the drive near the base is easy to control. Nevertheless, in order to obtain clearer object in visual monitoring process, geometric precision of mechanism need higher requirements [4-5]. Therefore, the geometric precision analysis [6] is quite important in terms of 5-dof parallel tripod head. This paper puts forward a error modeling [7] methodology based on screw [8] and establishes a mapping model between geometric error and end effector’s pose error for 5-dof parallel mechanism by integrated modeling method to analyze the influence of geometric errors on end effector’s pose error. Besides, clearly separate the geometric error source of effector’s uncontrollable error [9].

Architecture description and degree of freedom analysis

Architecture description. The 5-dof parallel tripod head mentioned in section 1 is 2-UPS&UPU structure, which is shown in Figs. 1. This mechanism consists of a base, a moving platform(end effector), a UPU chain and two UPS chains, these three limbs connecting the moving platform with the base.
As presented in Fig. 2, which is rotated from axis $x_i$ by $\phi_i$ and then rotated from axis $v_i$ by $\gamma_i$, the rotation matrix can be expressed as

$$
R_i = \begin{bmatrix}
\cos \gamma_i & 0 & \sin \gamma_i \\
-\sin \phi_i \sin \gamma_i & \cos \phi_i & -\sin \phi_i \cos \gamma_i \\
-\cos \phi_i \sin \gamma_i & \sin \phi_i & \cos \phi_i \cos \gamma_i
\end{bmatrix}
$$

(1)

Where $u_i$, $v_i$, $w_i$ is unit vector. The orientation of each limb can be denoted by

$$
\gamma_i = \arcsin(w_i), \phi_i = \arctan(-\frac{w_i}{w_{0}})
$$

(2)

In proposed kinematic model, $O_i$ is chosen as the reference point that can provide necessary input information to servo control and provide essential mathematical model to subsequent analysis.

The closed-loop constraint equations under the fixed coordinate frame $P_i-x_iy_iz_i$ can be obtained by

$$
r_i = l_i s_i
$$

(3)

$$
r_i = p_i + l_i s_i - o_i = p_i + l_i s_i - R_i o_i, i = 1, 2
$$

(4)

where $l_i$, $s_i$ represents length and unit vector of limb $i$ respectively, and $o_i$ indicate the position vector of point $o_i$ under the moving coordinate frame $O_i-x_iy_iw_i$. The $l_i$ and $s_i$ can be derived based on Eqs. (3) and (4):

$$
l_i = \|r_i\|
$$

(5)

$$
s_i = \frac{r_i}{l_i}
$$

(6)

According to Eqs. (2), orientation $\gamma_i$ and $\phi_i$ of $UPU$ chain can be calculate. Rotation matrix $R_i$ can be obtain from Eqs. (1), we can get the length of $UPS$ chain $l_i$ through taking $R_i$ into Eqs. (4).
then we have
\[ l_i = \|r_i + \alpha_i - p_i\| \quad i = 1, 2 \] (7)
\[ w_i = \frac{r_i + \alpha_i - p_i}{l_i} \quad i = 1, 2 \] (8)

Orientation \( \varphi_i \), \( \theta_i \) and rotation matrix \( R_i(i=1,2) \) for UPScan can be represented based on Eqs (2).

**Error sources analysis and error modeling**

For error modeling, establishing reference coordinate frame \( \{R_{pi}\} \) with origin point \( P_3 \) in the base, \( z \)-axis is perpendicular to plane \( P \) and the direction of \( x \)-axis points middle point of \( PP_2 \). Moreover, instantaneous reference coordinate system \( \{R_{pi}\} \) can be obtained whose coordinate axis is parallel to that of \( \{R_{pi}\} \) instantaneously. In the \( i \)th UPS chain \( (i=1,2) \), system \( \{R_{pi}\} \) can be got through the \( \{R_{pi}\} \) is rotated about \( z \)-axis by \( \beta_i \). In UPU chain, reference coordinate frame \( \{R_{pi}\} \) coincides with the system \( \{R_{pi}\} \). For the sake of describing the joint geometry error of each limb \( i \) \( (i=1,2,3) \), the origin point \( P_{ji} \) of body-fixed coordinate frame \( \{R_{ji}\} \) fixed on the parts consisting of the joint, \( z_{ji} \)-axis coincides with the joint axis, \( x_{ji} \)-axis is perpendicular to \( z_{ji+1} \)-axis. Let point \( P_{ji} \) be the intersection point of \( z_{ji} \)-axis and \( x_{ji} \)-axis, then \( P_{ji} \) is intersection point of \( z_{ji} \)-axis and \( x_{ji} \)-axis.

The error model of UPS chain is derived through performing inner product simultaneously both sides of Eqs. (9) by \( S_{01,2,3} \) and then by \( S_{03,3,3} \), the general error model for UPS chain are given by
\[
J_s = \begin{bmatrix}
S_{01,11} & S_{01,12} & S_{01,13} & S_{01,21} & S_{01,22} & S_{01,23} & S_{01,31} & S_{01,32} & S_{01,33}
\end{bmatrix}
\] (13)
\[
\hat{S}_{01,1} = \begin{bmatrix}
\hat{s}_{11}
\end{bmatrix}, \quad \hat{S}_{01,2} = \begin{bmatrix}
\hat{s}_{21}
\end{bmatrix}, \quad \hat{S}_{01,3} = \begin{bmatrix}
\hat{s}_{31}
\end{bmatrix}
\] (14)
\[
\hat{S}_{01,1} = \begin{bmatrix}
(a_i - l_i s_{12}) \times s_{12}
\end{bmatrix}_1, \quad \hat{S}_{01,2} = \begin{bmatrix}
(a_i - l_i s_{12}) \times s_{12}
\end{bmatrix}_2, \quad \hat{S}_{01,3} = \begin{bmatrix}
0
\end{bmatrix}_3
\] (15)
\[
\hat{S}_{01,1} = \begin{bmatrix}
(a_i - l_i s_{12}) \times s_{12}
\end{bmatrix}_1, \quad \hat{S}_{01,2} = \begin{bmatrix}
(a_i - l_i s_{12}) \times s_{12}
\end{bmatrix}_2, \quad \hat{S}_{01,3} = \begin{bmatrix}
0
\end{bmatrix}_3
\] (16)

The error model of UPU chain is derived through performing inner product simultaneously both sides of Eqs. (11) by \( S_{01,3,3} \) and then by \( S_{03,3,3} \), the general error model of parallel mechanism can be expressed as
\[
J_a S_a = J_p \Delta \alpha_a + E_a \epsilon_a
\] (20)
\[
J_s = E_a \epsilon_a
\] (21)

in which \( \epsilon_a \) and \( \epsilon_a \) is the given independent geometric error sources. Some items of Eq. (20) and (21) can be explained as follows:
$$J_a = \begin{bmatrix} S_{a,1,1} & S_{a,1,2} & S_{a,1,3} \end{bmatrix}^T$$

$$J_e = \begin{bmatrix} \hat{S}_{e,1,1} \end{bmatrix}^T$$

$$J_{mc} = \text{diag}\{ S_{a,1,1}, S_{a,1,2}, S_{a,1,3} \}$$

$$E_{mc} = \text{diag}\{ S_{a,1,1}, S_{a,1,2}, S_{a,1,3} \}$$

$$E_i = \{ A_{i,0}, A_{i,1}, A_{i,2}, A_{i,3}, A_{i,4}, A_{i,5}\}, k = 1,2,3$$

when $k = 3$, that is to say, the analyzed object is UPU chain

$$E_{ce} = \text{diag}\{ S_{a,1,1}, S_{a,1,2}, S_{a,1,3} \}$$

In light of Eqs. (19), error screw $s_{\alpha}$ can be compensated by kinematic calibration that adjusting kinematic error of active joints, let $s_{\alpha} = 0$, we have

$$\Delta \theta_e = - J_{mc,1} E_{wa}$$

However, error screw $s_{\alpha}$ cannot be compensated. Thus, the geometric error sources $e_{ce}$ of $s_{\alpha}$ should be paid more attentions in manufacturing and assembly stage in order to reduce or eliminate the effect to pose error.

### Sensitivity analysis

**Sensitivity analysis model.** To investigate the influences of each geometric error on the pose accuracy of the moving platform, a probability model should be proposed. $J_{ce} s_{ce} = E_{ce} e_{ce}$ is modified into a unified form and then simultaneously multiplying both side of the equation by $J_{ce}^{-1}$, we can obtain

$$e = J_{ce} e_{ce}$$

Error vector of the end is denoted by $e$, geometric error sources vector is denoted by $e_{ce}$, and error transformation matrix between the two vector is denoted by $J_{ce}$. Now some sensitivity indices can evaluate the influences of each geometric error on the pose errors that can not be uncompensated of moving platform, which are

$$\mu_i = \left( \frac{\int \mu_i dV}{V} \right)$$

$$\mu_i = \sum_{j=1}^{n} J_{eq}^2$$

note that $\mu_i$ denotes the sensitivity of $e$ relative to $e_{ce}$, which represent the standard deviation of $e$ caused by the unit standard deviation of $e_{ce}$. $\mu_i$ denotes the universe value of $\mu_i$.

**Sensitivity analysis.** Based on the indices defined by Eqs.(31), the sensitivity analysis can be carried out for the mechanism shown in Fig.2. The investigated workspace, which takes $O_i$ as reference point, is a cylinder area of $\phi500mm \times 220mm$, the dimension parameters of base and moving platform are identified as

$$O_i = \begin{bmatrix} 85(-1)^{'} & 45 & 0 \end{bmatrix}, p_i = \begin{bmatrix} 425(-1)^{'} & 240 & 0 \end{bmatrix}$$

the error sources of uncontrollable orientation error on end effector can be simplified as

$$e_{oa} = \begin{bmatrix} \delta h_1 & \delta h_2 & \delta a \end{bmatrix}$$

The maximum value is $0.0352mm/m$, minimum value is $0.0329mm/m$, and average value reaches $0.0340mm/m$. The global sensitivity coefficient that $e_{oa}$ is about geometrical parameter errors $\delta h_1, \delta h_2$ and $\delta a$ are $\mu_{p_1} = 1.334rad/m$, $\mu_{p_2} = 1.183rad/m$, $\mu_{p_3} = 1.291rad/m$. As shown in Fig. 4, $\delta h_1$ has the greatest influence on $e_{oa}$. based on $3\sigma$ criterion, the tolerance of $\delta h_1$ should be calculated as

$$T(\delta h_1) = 3 \times 0.02 / 1.334 = 0.0449mm$$
Table 1 The effects of constraint limb on the position error of end effector

<table>
<thead>
<tr>
<th>Geometric error</th>
<th>Standard deviation/mm</th>
<th>Maximum orientation error of the end effector/mm/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta h_1$</td>
<td>0.01</td>
<td>0.0352</td>
</tr>
<tr>
<td>$\delta h_2$</td>
<td>0.01</td>
<td>0.0329</td>
</tr>
<tr>
<td>$\delta a$</td>
<td>0.02</td>
<td>0.0340</td>
</tr>
</tbody>
</table>

Fig. 4 Distribution of uncontrolled error

Conclusions

i) Comparing with the existing error model based on space vector chain and vector differential method, the integrated error model, which is on account of variational screw for parallel mechanism, reflects more accurately nonlinear mapping relation between pose error and geometric error.

ii) The sensitivity indice $\mu_1$ can be defined in the sense of statistics. The error brought by the two non-intersecting axes of universal joint, the position error result from the zero point of universal joint and the error in torsional angle of sleeve guide, all of which have great influences on the end effector’s errors by the sensitivity analysis.

iii) According sensitivity analysis, the error sources $\delta h_1, \delta h_2, \delta a, \delta p$ and $\delta o$ have remarkable effect on the uncontrollable error, and $\varepsilon_v$ is sensitive to the linear error result from $\delta h_1$. Thus, the tolerance of $\delta h_1$ must meet the design requirements in advance.

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