The following proposition states that the set of filters of an equality algebra coincide with the set of (BCK-algebra) filters of its underlying BCK-algebra.

Proposition 3 *Let $\mathcal{E} = \langle E, \sim, \wedge, \mathbf{1}\rangle$ be an equality algebra. $F \in Fil(\mathcal{E})$ iff for all $a, b \in E$,*

(i') $\mathbf{1} \in F$,

(ii') $a, a \to b \in F \Rightarrow b \in F$

holds.

Proposition 4 *If $\mathcal{E}$ is an equality algebra and $F \in Fil(\mathcal{E})$ then $\Theta_F \in Con(\mathcal{E})$ and $\Theta_F = \Theta_{\overrightarrow{F}}$.*

Lemma 5 *For $\Theta \in Con(\mathcal{E})$ we have $(a, b) \in \Theta$ iff $(a \sim b, \mathbf{1}) \in \Theta$*

The next theorem establishes a connection between $Fil(\mathcal{E})$ and $Con(\mathcal{E})$.

Theorem 6 *Let $\mathcal{E} = \langle E, \sim, \wedge, \mathbf{1}\rangle$ be an equality algebra, $\Theta, \Psi \in Con(\mathcal{E})$, $F \in Fil(\mathcal{E})$. Then*

(a) $[\mathbf{1}]_\Theta \in Fil(\mathcal{E})$, *where* $[\mathbf{1}]_\Theta = \{a \mid (a, \mathbf{1}) \in \Theta\}$,

(b) $\Theta_{[\mathbf{1}]_\Theta} = \Theta$,

(c) $[\mathbf{1}]_{\Theta_F} = F$,

(d) **(1-regularity)** *if* $[\mathbf{1}]_\Theta = [\mathbf{1}]_\Psi$, *then* $\Theta = \Psi$.

Lemma 7 *The variety of equality algebras is congruence permutable and congruence distributive.*

Remark 8 Every variety in which (E3), (E4), and (b) (or (E7) and (E2) instead of (b)) holds is congruence permutable. The term $m(x, y, z) = ((x \sim y) \sim z) \wedge ((y \sim z) \sim x)$. testifies it.

Summing up, we have obtained that

Theorem 9 *The variety of equality algebras is a $\mathbf{1}$-regular, arithmetical variety.*

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