

The scheduling with rejection on uniform multiprocessors

Liu Shoupeng^{1, a*}, Wei fei^{2, b}

¹Bin Zhou Medical College, Yantai, Shandong, China, 264003

²Bin Zhou Medical College, Yantai, Shandong, China, 264003

^aliushoupeng@eyou.com, ^bpeterfei3000@163.com

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Abstract: Objective To propose an approximate for the scheduling with rejection on uniform machines. **Methods** Design an on-line algorithm. **Results** We prove the competitive ratio of the algorithm. **Conclusion** The competitive ratio of the algorithm does not beyond $1 + \beta$.

Introduction

In classical deterministic scheduling problems, it is assumed that all jobs have to be processed. However, in many practical cases, mostly in highly loaded make-to-order production systems, accepting all jobs may cause a delay in the completion of orders which in turn may lead to high inventory and tardiness costs. Thus, in such systems, the firm may wish to reject the processing of some jobs by either outsourcing them or rejecting them altogether. The field of scheduling with rejection provides schemes for coordinated sales and production decisions by grouping them into a single model. Since scheduling problems with rejection are very interesting both from a practical and a theoretical point of view, they have received a great deal of attention from researchers over the last decade.

In this paper, we consider an on-line scheduling problem with rejection. The job can be processed, or be rejected, but to pay a certain penalties. It may be stated as follows: the model have n -independent, non-preemptive jobs, $J = \{ J_1, J_2, \dots, J_n \}$, are available for processing at time zero on a set of m machines $M = \{ M_1, M_2, \dots, M_m \}$ arranged according to a specific and predefined machine environment. In this paper, we consider the uniform machines. Each processor in a uniform multiprocessor machine is characterized by a speed $s_i (i = 1, 2, \dots, m)$, might as well set

$s_1 \geq s_2 \geq \dots \geq s_m = 1$. The input for a problem of scheduling with rejection includes the following

parameters: p_j , which is the processing time of job $J_j (j = 1, \dots, n)$ and r_j which is the rejection cost of job J_j . The processing time of job J_j that are processed on machine $M_i (i =$

$1, \dots, m)$ is p_j / s_i . For set $S \subseteq J$, $R(S)$ is the total rejected cost in S , namely $R(S) = \sum_{j \in S} r_j$.

Given a solution produced by the on-line algorithm SRUM (scheduling with rejection on uniform multiprocessors), A refers to the set of accepted jobs, and set R refers to the set of rejected jobs.

p_{\max} denotes the largest processing time of all accepted jobs. For their analogs in the optimal solutions we use $\bar{R}, \bar{A}, \bar{p}_{\max}$, respectively. Z^A denotes the total cost of the Algorithm SRUM and Z^{opt} is the cost achieved by the optimal solution. In the following proof, we make the $M(S) = \sum_{j \in S} \frac{P_j}{\sum S_i}$. The set $B = \{j \mid r_j \leq \frac{P_j}{\sum S_i}\}$ contains jobs with penalty less than or equal to their load.

The off-line scheduling with rejection penalty is NP-hard. It does not exist a polynomial time optimal algorithm. Therefore, we propose an approximate Algorithm SRUM. It rejects all job from set $B = \{j \mid r_j \leq \frac{P_j}{\sum S_i}\}$, and accepts part of the jobs in the rest of the jobs. Next, we propose an on-line algorithm **SRUM** and analysis the competitive ratio of it .

Algorithm SRUM:

Step 1: If a job J_i from B is arrive, reject it;

Step 2: Let P be the total penalty of all rejected jobs from $J - B$. If the job $J_j = (t_j, p_j) \notin B$ is arrive, reject it if $P + r_j \leq \alpha p_j$, otherwise, accept it and schedule it on a least loaded machine by LS algorithm.

On the **Algorithm SRUM**, Let $\alpha = \frac{\sqrt{5-1}}{2}$, $\varphi = \frac{\sqrt{5+1}}{2}$, and c satisfy

$$c \geq 1 + \left(\frac{1}{s_m} - \frac{1}{\sum_{i=1}^m S_i} \right) \frac{1}{\alpha}, \quad c \geq 2 + \alpha \frac{2}{\sum_{i=1}^m S_i}$$

Theorem: The Algorithm SRUM from m machines is $1 + \beta$ competitive ,

here $\beta = \min \left\{ \sqrt{\frac{(m-1)S_1}{\sum_{i=1}^m S_i} + \frac{(m-2)S_1}{\sum_{i=1}^m S_i}}, \varphi S_1 \right\}$

Proof: First for certain optimal solution, we define $S = R \cap \bar{A} - B$ as job set, it satisfy in the optimal solution be accepted but in step 2 rejected, The completion of the machine is the first k machine M_k , Maximum hours of accepting jobs is p_{\max} , the processing time of the last job is p_l , By the algorithm LS, we obtain

$$C_k + \frac{p_l}{S_k} \leq C_i + \frac{p_l}{S_i}, (i=1,2,\dots,m) \quad (1)$$

$$C_k + \frac{p_l}{S_k} \leq C_{\min} + \frac{p_l}{S_m} \leq C_{\min} + p_{\max}, \quad (2)$$

By the formula (1) and $p_l \leq p_{\max}$, we can obtain

$$C_k + \frac{p_l}{S_k} \leq M(A) + \frac{(m-1)p_l}{\sum_{i=1}^m S_i} \leq M(A) + \frac{(m-1)p_{\max}}{\sum_{i=1}^m S_i}$$

Then

$$\begin{aligned} Z^{on} &= C_k + \frac{p_l}{C_k} + P(R) \leq M(A) + \frac{(m-1)p_{\max}}{\sum_{i=1}^m S_i} + P(R) \\ &= (M(A) + P(R) - S) + M(S) + \frac{(m-1)p_{\max}}{\sum_{i=1}^m S_i} + P(S) - M(S) \end{aligned}$$

We first consider the left half part

$$(M(A) + P(R) - S) + M(S)$$

$$M(A \cap \bar{A}) + M(\bar{A} \cap R) + P(-R \cap \bar{S}) + M(\bar{R} \cap \bar{S}) + R$$

$$\leq M(\bar{A}) + P(\bar{R}) \leq \alpha Z^T$$

Since all jobs in S are accepted by the optimal solutions, So the largest processing time is not more than \bar{p}_{\max} . By the algorithm, we can obtain $P(S) \leq \alpha \bar{p}_{\max}$. For any rejected job J by step 2, we

know $r_j \leq \alpha p_j$, Summation on both sides, we can obtain $P(S) \leq \alpha M(S) \sum_{i=1}^m S_i$. So

$$P(S) - M(S) \leq (1 - \frac{1}{\alpha \sum_{i=1}^m S_i}) P(S) \leq (1 - \frac{1}{\alpha \sum_{i=1}^m S_i}) \alpha \bar{p}_{\max} = (\alpha - \frac{1}{\sum_{i=1}^m S_i}) \bar{p}_{\max}$$

$$1) \text{ If } p_{\max} \leq \bar{p}_{\max}, \text{ then } P(S) - M(S) + \frac{(m-1)p_{\max}}{\sum_{i=1}^m S_i} \leq (\alpha - \frac{m-2}{\sum_{i=1}^m S_i}) \bar{p}_{\max}$$

2) If $p_{\max} \geq \bar{p}_{\max}$, Let P be the total penalty incurred by the jobs in Step 2 of the algorithm

until it schedules the first job with processing time is p_{\max} .

By the condition Step 2 of the algorithm , $\alpha p_j \leq P+r_j$.On the other hand $P+r_j \leq P(S)+P(\bar{R})$,as all rejected jobs in Step 2 are in $S \cup \bar{R}$, and also the job j with processing time is in \bar{R} , since $p_{\max} \geq \bar{p}_{\max}$.Thus

$$\begin{aligned} P(S)-M(S)+\frac{(m-1)p_{\max}}{\sum_{i=1}^m S_i} &\leq \left(\alpha-\frac{1}{\sum_{i=1}^m S_i}\right)\bar{p}_{\max}+\frac{m-1}{\sum_{i=1}^m S_i} \cdot \frac{1}{\alpha}(P+r_j) \\ &\leq \left(\alpha-\frac{1}{\sum_{i=1}^m S_i}\right)\bar{p}_{\max}+\frac{m-1}{\sum_{i=1}^m S_i} \cdot \frac{1}{\alpha}(P(S)+P(\bar{R})) \\ &\leq \left(\alpha+\frac{m-2}{\sum_{i=1}^m S_i}\right)\bar{p}_{\max}+\frac{m-1}{\alpha \sum_{i=1}^m S_i} P(\bar{R}) \end{aligned}$$

If $\alpha = \sqrt{\frac{m-1}{S_1 \sum_{i=1}^m S_i}}$, $\left(\alpha+\frac{m-2}{\sum_{i=1}^m S_i}\right)\bar{p}_{\max}+\frac{m-1}{\alpha \sum_{i=1}^m S_i} P(\bar{R})$ achieve Minimum value , So

$$P(S)-M(S)+\frac{(m-1)p_{\max}}{\sum_{i=1}^m S_i} \leq \beta_1 Z^{opt} , \text{ here } \beta_1 = \sqrt{(m-1)s_1 / \sum_{i=1}^m s_i + (m-2)s_1 / \sum_{i=1}^m s_i} .$$

By Formula (2),we can obtain

$$\begin{aligned} Z^{om} &= C_k + \frac{p_l}{S_k} + P(R) \leq C_{\min} + p_{\max} + P(R) \\ &\leq M(A) + P(R) + \left(\frac{1}{S_m} - \frac{1}{\sum_{i=1}^m S_i}\right) p_{\max} \\ &= (M(A) + P(R-S) + M(S)) + P(S) - M(S) + \left(\frac{1}{S_m} - \frac{1}{\sum_{i=1}^m S_i}\right) p_{\max} \end{aligned}$$

From the above proof , we can obtain

$$M(A) + P(R-S) + M(S) \leq M(\bar{A}) + P(\bar{R}) \leq z^{opt}$$

$$P(S) - M(S) + \left(\frac{1}{S_m} - \frac{1}{\sum_{i=1}^m S_i}\right)T \leq \left(\alpha + \frac{1}{S_m} - \frac{2}{\sum_{i=1}^m S_i}\right)\bar{p}_{\max} + \left(\frac{1}{S_m} - \frac{1}{\sum_{i=1}^m S_i}\right)\frac{1}{\alpha}P(\bar{R})$$

We know $\alpha = \varphi - 1, c = \varphi + 1$ and satisfy the following inequality

$$c \geq 1 + \left(\frac{1}{S_m} - \frac{1}{\sum_{i=1}^m S_i}\right)\frac{1}{\alpha}, \quad c \geq 2 + \alpha \frac{2}{\sum_{i=1}^m S_i}$$

So we have

$$Z^{on} \leq Z^{opt} + (c-1)(P(\bar{R}) + \bar{T}) = Z^{opt} + \varphi(P(\bar{R}) + \bar{T}) \leq (1 + \varphi S_1)Z^{OPT}$$

In summary, we have $\frac{Z^{on}}{Z^{OPT}} \leq 1 + \beta$, here $\beta = \min\left\{\sqrt{\frac{(m-1)S_1}{\sum_{i=1}^m S_i} + \frac{(m-2)S_1}{\sum_{i=1}^m S_i}}, \varphi S_1\right\}$.

The Conclusion

In this paper, we propose an on line algorithm for the scheduling with rejection penalty and analysis the competitive ratio of it. It extend the results of Yair bartal to uniform machines. But some open problems remain. This requires that we continue to work hard.

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