Performance Analysis of detecting Distributed Target Under Correlated K-Distributed Clutter Background

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Abstract. Sliding window accumulation detector and order statistics-sliding window accumulation detector for missile-borne high-resolution radar target detection are presented in this paper. Their performance under space-time correlated K-distributed clutter background has been investigated. Under null hypothesis, the approximate probability density function expressions for the test statistics are derived, the relationship between false-alarm probability, CFAR detection threshold and clutter parameters are established, based on the theory of generalized K-distribution fitting, moment matching, valid sample size estimation of relevant data and fractional order statistics computing. Both the theoretical analyses and simulation experiments demonstrate the effectiveness of the proposed methods.

Introduction

The big meaning of applying K-distribution to target detecting in broadband high resolution radar, owing to the nice fitting of K-distribution and clutter\([1,2,3]\). In the past, target detector and performance analysis have been investigated in point target and Distributed target under Gaussian clutter background \([1]\). The target detector and performance analysis under Correlated-K Distribution(C-KD) clutter have hand down to us.

Due to the requirement of real-time performance and computing resource restriction of on-board imaging radar, maximum likelihood estimation of C-KD clutter parameters and extended target scattering center is hard to realize, the optimal detection method cannot apply to missile radar. Therefore, range sliding window accumulation detector is one of the important methods in the guidance radar, which is suboptimal and low operation cost \([4,5]\). But it is difficult to analysis the detection performance and set up reasonable detection threshold in related non-Gaussian clutter, which the probability distribution of accumulation detection statistic is not easy to derive.

Sliding-window-accumulation detector and order-statistics sliding-window-accumulation detector are presented in the paper. Based on the theoretical principle of generalized K-distribution fitting, moment matching, effective sample size estimation of relevant data and fractional order statistics computing, the approximate probability density function (PDF) for the test statistics under null hypothesis are derived. The relationship between false-alarm probability and CFAR detection threshold are established. Experimental results are in good agreement with the theoretical models, which demonstrate the usefulness of the proposed approach.

1 Model of Target Detection in C-KD clutter

Set \(m\) as the range resolution cell number, then one dimensional high resolution range profile(HRRP) of cell \(m\) can be expressed as: \(\{x(m,n) | m = 0,1,...,M-1; n = 0,1,...,N-1\}\).

According to C-KD distribution clutter model, space-time related clutter can be expressed as:

\[
\varepsilon(m,n) = \sqrt{\mu_{m,n}}[w_k(m,n) + jw_r(m,n)](m = 0,1,...,M-1; n = 0,1,...,N-1) \tag{1}
\]
Where \( w_{g}(m,n) \) and \( w_{i}(m,n) \) obey normal distribution, which are independent of each other. \( 
abla_{mn} \) is the gamma function that shape parameter \( v \) and scale factor \( b \), follows a Gamma PDF as:

\[
f_{mn}(\tau) = \frac{b^{v}}{\Gamma(v)} \tau^{v-1} \exp(-b\tau) \quad (\tau \geq 0)
\]

On the assumption that \( m \) and \( n \) are different, random variables of the Gamma distribution are independent of each other. Then, space-time correlation of Clutter can be given by the correlation of \( w_{g}(m,n) \) or \( w_{i}(m,n) \). The amplitude distribution can be defined by G-KD distribution [6]:

\[
f_{(x_{m},n)}(\xi) = \frac{\sqrt{2b}}{\Gamma(v)2^{v-1}}(\sqrt{2b}\xi)^{v-1}K_{v-1}(\sqrt{2b}\xi) \quad (\xi \geq 0)
\]

Here \( K_{v-1}(\cdot) \) is a modified Bessel function of the second kind of order \( \nu - 1 \).

If within target, the target may occupy multiple range cells in HRRP. Then, to non-sparse target (i.e., the distribution of energy in each resolution cell is continuous). The trial is arranged to test the following hypotheses:

\[
H_{0} : x(m,n) = x(m,n)(0 \leq m \leq M - 1; 0 \leq n \leq N - 1) ; \text{Clutter only.}
\]

\[
H_{1} : x(m,n) = x(m,n) ; \text{Clutter with target occupied range at least } m_{D_{\text{max}}}.\]

The following detection scheme is put forward: at first, in the coarse detection, take the advantages of amplitude or power information of HRRP before onboard 2-D imaging, without using target signal phase change in different period; second, Doppler beam sharpening or synthetic aperture processing in possible strong scattering target area; finally, accurate detect. Achieve high detection probability under low false alarm, and effectively reduce the computational.

Set \( (m,n) \) as white Gaussian random process, supposing the target scattering cross section distribution is sparse, but not very serious. Depending on the classical linear model of maximum likelihood detection theory on the constraint conditions, considering the practical application, the clutter is heterogeneity, so the area between inspection area and reference area cannot be too far away.

Structure the optimal test statistics of hypothesis testing problem, and the arbitrary radial unit \( m_{i} \), can be handled as Figure 1:

![Figure 1 Testing block diagram of \( m_{i} \)](image)

Normalized the range power, then the test statics can be expressed as follow:

\[
T(X,m_{i}) = \frac{M' / 2}{m_{D_{\text{max}}}} \times \frac{P_{\text{test}}}{P_{\text{reference}}}
\]

\[
P_{\text{test}} = \sum_{n=0}^{N-1} \sum_{m=m_{i}}^{m_{i}+m_{D_{\text{max}}}-1} |x(m,n)|^{2}, P_{\text{reference}} = \sum_{n=0}^{N-1} \sum_{m=m_{i}-M'/2}^{m_{i}+M'/2} |x(m,n)|^{2} + \sum_{m=m_{i}+m_{D_{\text{max}}}+M'/2}^{m_{i}+m_{D_{\text{max}}}+M'/2} |x(m,n)|^{2}
\]

In(4), the range on each reference area is \( M' / 2 \), the range between the test and reference area is \( M'' / 2 \) (\( M'' > m_{D_{\text{max}}}-m_{D_{\text{max}}} \)). \( P_{\text{test}} \) is all power of the test area, \( P_{\text{reference}} \) is the sum of the reference power. Search each \( m_{i} \) orderly by sliding window, if \( T_{c}(X,m_{i}) \) larger than the CFAR detection threshold, then detect the distributed target (starting at range \( m_{i} \), the largest length is \( m_{D_{\text{max}}} \), the
minimum length is $m_{D_m}$.

In strong interference (false target jamming) or multi-objectives situation, in order to filter the strong interference in test area on both side, avoid the decrease of the detection probability by high power in reference area, using order statistic in the reference area, the test statistics can be modified as:

$$T_{l}(X,m_n) = \frac{1}{Nm_{D_m}} \sum_{n=0}^{N-1} \sum_{m=m_n}^{m_{max}-1} |x(m,n)|^2$$

So $\Omega = [m_1 - M'' / 2 - M' / 2, m_1 - M'' / 2] \cup [m_1 + m_{D_m} + M'' / 2, m_1 + m_{D_m} + M'' / 2 + M' / 2]$ is the region of reference area, set the sort of $L$ for the power of all ranges as $S_{l}(m_i)$.

2 Performance Analysis of Sliding Window Detector in C-KD Clutter

It is an important problem need to be solved that the detection threshold design and performance analysis, detecting extended target based on sliding window accumulation in C-KD clutter.

Strictly speaking, it is Complicated know the PDF for the sum of multiple C–KDs. The relationship between false-alarm probability, CFAR detection threshold and clutter parameters (i.e., shape and scale parameters, Spatial-Temporal correlation coefficient,) need to be built; when choose the right CFAR threshold according to different clutter parameters for onboard detector. To solve all questions, it is necessary to know the approximate PDF expressions.

Depending on the practice of SAR imaging, G-KD with proper parameters has a nice fitting to the sum of multiple C-KDs, which the deviation of PDF can be ignored [7]. Hence, adopt G-KD fitting to predict the accurate and reasonable of detection performance. Unfortunately, the quantitative relationship between the parameters of G-KD and C–KD has yet to establish. The moment matching method, that put forward to solve the problem in the next section.

Set $r_{nw}(\alpha, \beta)$ as the space-time autocorrelation function of $w_{R_k}(m,n)$ or $w_l(m,n)$ speckle part of K distribution clutter, which satisfy the following properties: $r_{nw}(\alpha, \beta) = E[w_{R_k}(m + \alpha, n + \beta)w_{R_k}(m,n)]$.

Further, if the space-time correlation is separable, the correlation function can be simplified to:

$$r_{nw}(\alpha, \beta) = e^{-a/\alpha}e^{-b/\beta}$$

Where $a$ and $b$ represent the normalized correlation distance (related range/width of range resolution cell) and normalized related time (related time/HRRP period), respectively.

According to the type (2), the moment feature of the gamma distribution:

$$E[\eta_{m,n}] = \int_0^\infty \tau f_{\eta_{m,n}}(\tau)d\tau = \frac{\Gamma(v+1)}{b\Gamma(v)} \int_0^\infty \frac{b^{v+1}}{\Gamma(v+1)} \tau^v \exp(-b\tau)d\tau = \frac{v}{b},$$

$$E[\eta_{m,n}^2] = \frac{v^2}{b^2} + \frac{v}{b^2}, \text{var}[\eta_{m,n}] = E[\eta_{m,n}^2] - (E[\eta_{m,n}])^2 = \frac{v}{b^2}$$

Combined normal distribution moment features (zero mean and unit variance), the moment feature of space-time C-KD clutter(1) is deduced:

$$E[|\varepsilon(m,n)|^2] = E[\eta_{m,n}]E[w_{R_k}^2(m,n) + w_l^2(m,n)] = \frac{v}{b}[1 + 1] = \frac{2v}{b}$$

$$E[|\varepsilon(m,n)|^4] = E[\eta_{m,n}^2] \times E[w_{R_k}^4(m,n) + 2w_{R_k}^2(m,n)w_l^2(m,n) + w_l^4(m,n)] = \frac{8v^2}{b^2} + \frac{8v}{b^2}$$

$$\text{var}[|\varepsilon(m,n)|^2] = E[|\varepsilon(m,n)|^4] - \{E[|\varepsilon(m,n)|^2]\}^2 = \frac{4v^2}{b^2} + \frac{8v}{b^2}$$

(7)
Normalize (4) (the normalized molecule and denominator expressed as $S_T$ and $S_C$), multiply the molecule and the denominator by $b / 2v$ (Reciprocal value the average power of C-KD), respectively.

Under the null hypothesis (no goals) condition $S_T$ can be expressed by G-KD, which the shape parameters are $\bar{\beta}_1 = \beta_{T1}$ and $\bar{\beta}_2 = \beta_{T2} = Nm_{D_{mn}} (N$ is cumulative frequency of each resolution cell, same to Multi-look SAR imagery), scale parameter is $b_s = b_T$, the PDF is given by[1,7,9]:

$$f(\beta_1, \beta_2, b_s; \xi) = \frac{2}{\Gamma(\beta_1)\Gamma(\beta_2)} b_s^{\bar{\beta}_1 + \bar{\beta}_2} \xi^{(\bar{\beta}_1 + \bar{\beta}_2) - 1} K_{\bar{\beta}_1 - \bar{\beta}_2} (2b_s \sqrt{\xi})$$

(8)

According to (4)(moment matching method for space-time C-KD,(7)) and PDF of G-KD(8), the expectation and variance expression of $S_T$ is:

$$E[S_T] = \frac{\beta_{T1}\beta_{T2}}{b_T^2}, \quad \text{var}[S_T] = \left(\frac{1}{\beta_{T1}} + \frac{1}{\beta_{T2}} + \frac{1}{\beta_{T1}\beta_{T2}}\right)\left(\frac{1}{\beta_{T1}} + \frac{1}{\beta_{T2}} + \frac{1}{\beta_{T1}\beta_{T2}}\right)$$

Thus:

$$\beta_{T1} = \frac{Nm_{D_{mn}} (Nm_{D_{mn}} + 1)v_1}{2Nm_{D_{mn}} + v_1 \left(\sum_{m,s \in [0, N-1]} r_n^2(|m-u|,|n-v|) + m_{D_{mn}} \sum_{n \in \in [0, N-1]} r_n^2(0,|n-v|)\right)}$$

(9)

$$b_T = \sqrt{\beta_{T1}\beta_{T2}}$$

(10)

In the same way, $S_C$ can be expressed by G-KD, which the shape parameters as $\bar{\beta}_1 = \beta_{C1}$ and $\bar{\beta}_2 = \beta_{C2} = NM'$, scale parameter as $b_s = b_C$, the PDF same as(8). According to the principle of moment matching:

$$\beta_{C1} = \frac{NM'(NM'+1)v_1}{2NM' + v_1 \left(\sum_{m,u \in [0, N-1]} r_n^2(|m-u|,|n-v|) + M' \sum_{n \in [0, N-1]} r_n^2(0,|n-v|)\right)}$$

(11)

$$b_C = \sqrt{\beta_{C1}\beta_{C2}}$$

(12)

The cumulative distribution function (CDF) of G-KD, with parameters $\beta_1$, $\beta_2$, $b_s$, can be derived by using [9] and(8):

$$F(\beta_1, \beta_2, b_s; \xi) = \pi \csc[\pi(\beta_2 - \beta_1)] \times \left\{\frac{(b_s^2 \xi)^{\beta_1} F_2(\beta_1; 1 - \beta_2, 1 + \beta_2; b_s^2 \xi)}{\Gamma(\beta_2)\Gamma(1 - \beta_2)\Gamma(1 + \beta_2)} - \frac{(b_s^2 \xi)^{\beta_2} F_2(\beta_2; 1 + \beta_2 - \beta_1, 1 + \beta_2; b_s^2 \xi)}{\Gamma(\beta_1)\Gamma(1 + \beta_2 - \beta_1)\Gamma(1 + \beta_2)}\right\} (\xi \geq 0)$$

(13)

Where $\rho F_q(\cdot)$ is the generalized hyper geometric function.

Due to the certain interval between inspection and reference area, ignoring the correlation, according to the quotient of two random variables probability density, the relationship between false alarm probability $P_{F_1}$ (the probability of $S_T > r_{Di} S_C$) and detection threshold $r_{Di}$ is given by the
following express:
\[
\int_0^\infty [1 - F(\beta_{T_1}, \beta_{T_2}, b_T; \xi_{r_D})] f(\beta_{C_1}, \beta_{C_2}, b_C; \xi) d\xi = P_{F_1}
\] (14)

Calculate (14) offline by the numerical calculation method, to the given \( P_{F_1} \), store the relationship between correlation coefficient and clutter parameters in the radar storage.

3 Performance Analysis of Order Statics-Sliding Window Detector in C-KD Clutter

It is an important problem need to be solved that the detection threshold design and performance analysis, detecting extended target based on sliding window accumulation in C-KD clutter.

\[\ln(8)(9)(10), \text{ set } m_{d_{\text{min}}} = 1, S_L(m_i) \text{ can be expressed by G-KD , which the shape parameters as }\]
\[\beta_1 = \beta_{x_1} \text{ and } \beta_2 = \beta_{x_2} = N, \text{ scale parameter as } b_\xi = b_{\xi}, \text{ that is:}\]
\[\beta_{x_1} = \frac{2N + v}{N(N + 1)\nu} \sum_{k=0, k \neq 1} r_w^2(0, k - \ell)\]
\[b_{\xi} = \sqrt{\beta_{x_1}/\beta_{x_2}} \quad \text{(15)}\]

Under the correlation condition, for a measurement series of length \( M' \), the effective sample size (ESS) is defined as the number \(^{M'_{\varepsilon}} < M'\) of independent samples, which would provide the same information as the \( M'\)-size sample. Then the order statistics problems of \( M'\) relevant sample can be approximately equivalent to \(^{M'_{\varepsilon}}\) independent sample [10][11]. Set \( L' = LM'_{\varepsilon} / M'\), according to the principle of order statistics, the PDF of \( S_L(m_i)\) (in the sort of L of \( M'\) related random variables obey G–KD) can be expressed as:
\[f_L(\beta_{x_1}, \beta_{x_2}, b_{\xi}; \xi) = \frac{\Gamma(M'_{\varepsilon} + 1)}{\Gamma(L')\Gamma(M'_{\varepsilon} - L' + 1)} F^{L'-1}(\beta_{x_1}, \beta_{x_2}, b_{\xi}; \xi)\]
\[\times [1 - F(\beta_{x_1}, \beta_{x_2}, b_{\xi}; \xi)]^{M'_{\varepsilon} - L'} f(\beta_{x_1}, \beta_{x_2}, b_{\xi}; \xi) \quad (\xi \geq 0)\]

Clearly, \(^{M'_{\varepsilon}}\) and \( L'\) are not restricted to integer, called as fractional order statistics of related data in this paper. Easy to get the CDF of the fractional order statistics \( S_L(m_i)\) as:
\[F_L(\beta_{x_1}, \beta_{x_2}, b_{\xi}; \xi) = \int_0^\xi f_L(\theta) d\theta\]
\[= \frac{\Gamma(M'_{\varepsilon} + 1)}{\Gamma(L')\Gamma(M'_{\varepsilon} - L' + 1)} \int_0^{F(\beta_{x_1}, \beta_{x_2}, b_{\xi}; \xi)} s^{L'-1} (1 - s)^{M'_{\varepsilon} - L'} ds\]

\(^{M'_{\varepsilon}}\) is called the ESS of related data. Based on the theory presented in literatures [10,11], that \(^{M'_{\varepsilon}} / M'\) can be defined as variance ratio of the independent sample and related sample, which extended into order statistics-sliding window accumulation detector under C-KD clutter background, can be defined as:
\[\text{var} \left\{ \sum_{n=0}^{N-1} |x(m,n)|^2 \right\} \left( \text{independent simples of } \Omega \right) / \text{var} \left\{ \sum_{n=0}^{N-1} |x(m,n)|^2 \right\} \left( \text{related samples of } \Omega \right)\]
According to the statistical properties of C-KD distribution, easily derived:

\[
M' = M' \times \frac{2}{v_1} + \frac{1}{N} \sum_{n,v \in [0,N-1]} r^2_v(\infty, |n-v|) + \frac{1}{N} \sum_{n,v \in [0,N-1]} r^2_v(0, |n-v|)
\]

When the interval between the test area and reference area far enough, ignore the correlation, the relationship between the false alarm probability \(P_{F2}\) (probability of \(S_T > r_{D2}S_\Sigma\)) and detection threshold \(r_{D2}\):

\[
\int_0^\infty \left[1 - F(\beta_1, \beta_2, b_1; \xi_{D2})\right] f_L(\beta_1, \beta_2, b_1; \xi) d\xi = P_{F2}
\]

Using numerical calculation method, to a given \(P_{F2}\), calculate and store the relationship between detection threshold and the clutter parameter.

4 Simulation

In this section, results for the C-KD clutter generated by Monte-Carlo simulation and G-KD parameters estimation by moment matching are presented. According to a typical application case, set simulation parameters, Table 1.

<table>
<thead>
<tr>
<th>Maximum /minimum range number</th>
<th>Reference /reserves number</th>
<th>Cumulative frequency</th>
<th>Normalized related distance /related time</th>
<th>Shape /scale parameter</th>
<th>The sort of L of the order statics</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m_{\text{max}}), (m_{\text{min}})</td>
<td>(M'), (M'')</td>
<td>(N)</td>
<td>(a), (b)</td>
<td>(v), (b)</td>
<td>5</td>
</tr>
<tr>
<td>10/5</td>
<td>12/8</td>
<td>10</td>
<td>2.5/2</td>
<td>1/2</td>
<td></td>
</tr>
</tbody>
</table>

Generate complex related space-time C-KD data \((x(m,n) | m = 0,1,\ldots,24; n = 0,1,\ldots,9)\), in each Monte-Carlo simulation\([12,13]\). Repeat the simulation \(10^5\) times, \(m \in [0,5] \cup [19,24]\) as the reference area, \(m \in [6,9] \cup [15,18]\) as the reserves area between the reference area and inspection area. Sampling frequency of variable \(x(m,n)\), get the PDF \(f_1(\xi_1)\) and the CDF \(G_1(\xi_1)\); same to the variable \(x(m,n)\), get \(f_2(\xi_2)\) and \(G_2(\xi_2)\); in each experiment, calculate for all reference ranges, in the sort of L of the \(M'\) value marked \(\xi_1\), get \(f_3(\xi_3)\) and \(G_3(\xi_3)\).

Get the parameters by the moment matching theory: \(\beta_1 = 8.9581\), \(\beta_2 = 50\), \(b_1 = 21.1638\); \(\beta_{11} = 23.0875\), \(\beta_{12} = 120\), \(b_{11} = 52.6355\); \(\beta_{21} = 3.6915\), \(\beta_{22} = 10\), \(b_{21} = 6.0758\); \(M' = 7.652\), \(L' = 3.1885\). Calculate the PDFs \((f(\beta_{11}, \beta_{12}, b_{11}; \xi), f(\beta_{21}, \beta_{22}, b_{21}; \xi))\) and CDFs \((F(\beta_{11}, \beta_{12}, b_{11}; \xi), F(\beta_{21}, \beta_{22}, b_{21}; \xi))\) of G-KD fitting by numerical integral method, respectively.
The PDF and CDF plots for the test area $S_T$ of C-KD simulation clutter and G-KD fitting are shown in Figure 2, respectively; the PDF and CDF plots for reference area $S_C$ of C-KD simulation clutter and G-KD fitting are shown in Figure 3, respectively; the PDF and CDF plots for the order statistics of C-KD simulation clutter and G-KD fitting are shown in Figure 4, respectively.

The results show that: a) the PDF between the G-KD fitting and C-KD simulation have some differences, but their variation trends are similar; b) the high consistency CDF between the C-KD simulation and G-KD fitting; c) the effectiveness of method put forward in this paper, that use the moment matching of G-KD fitting method and the independent data length and the fractional order statistical fitting method.

In the coarse detection, according to the theory formula of G-KD false alarm and detection threshold (16),(22), shown in Figure 5. Set false alarm as $P_F = 0.2$, then CFAR threshold is calculated: $r_{D1} = 1.4265$, $r_{D2} = 2.1002$; In Monte-Carlo experiments, frequency statistics of the value of the variable $\xi_4 = \xi_1 / \xi_2$, the probability of $\xi_4 > r_{D1}$ is $P_{F1} = 0.2073$, same to the variable $\xi_5 = \xi_1 / \xi_3$, the probability of $\xi_5 > r_{D2}$ is $P_{F2} = 0.2006$, and approximately equal to the set value, which proves that the fitting method proposed in this paper, has a nice control to the detection threshold and the false alarm probability.

![Figure 2](source_url) The PDF (a) and CDF(b) plots for $S_T$ of C-KD simulation clutter and G-KD fitting

![Figure 3](source_url) The PDF (a) and CDF(b) plots for $S_C$ of C-KD simulation clutter and G-KD fitting
Figure 4 The PDF (a) and CDF(b) plots for $S_{\nu}(m)$ of C-KD simulation clutter and G-KD fitting

Figure 5 Relationship between CFAR detection threshold and false-alarm probability of theory G-KD

Summary

This paper presents a range accumulation of extended target detection design and false-alarm probability calculation method, has not yet given formula of the detection performance in different ratio, under C-KD clutter background. The test statistics probability distribution is not only related to clutter parameters, but also with the space-time distribution of target scattering on the fluctuation characteristics, which will be more complicated in the fitting process. How to get a simple, suitable performance assessment formula for engineering application remains to be further research.

References


