The Optimization Design of Two Stages Helical Gear Reducer Based on GA and Nastran
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Abstract. Paper taking the volume of gear transmission system of two stages helical gear reducer as mathematical model and write a GA program based on MATLAB soft to get the relevant result. Next, paper draw gear model based on UG soft and analysis the gear’s bending stress in Nastran solver which contained in UG soft. The result shows that optimized gear transmission system can not only meet job requirement, also reduced the volume greatly.

Introduction
As a kind of power transfer device, gear reducer is widely used in aircraft automotive, machine tools, marine machinery etc[1], because of its simple structure, high transmission efficiency, good stability.

Traditional reducer design method is combined with the experience and test way, which has many disadvantages, such as, results is reasonable, waste materials, tedious design work, etc. Therefore, the optimum design of reducer has the important practical significance.

Discussed problems
The reducer of one type of band conveyors pass by 4.5kw power, total drive ratio is 7.53, the speed of high-speed spindle is 480r/min. The structure as shown in figure 1, the material of two small gears is 40Cr, two big gears’ material is 45 # steel. The design requirement is that volume of the reducer as small as possible under meet the condition of performance.

![Gear reducer structure diagram](image)

Fig.1: Gear reducer structure diagram

Optimal design of gear reducer
Optimum design combined with the optimization theory in mathematics and computer technology, its aim is to seek the best design parameters under the given constraint conditions. Optimization design mainly consists of four steps[2]: requirement analysis, build mathematical modeling, choose optimization methods, write computer programming to solve the problem.

3.1 The selection of design variables.
When selecting the design variables, we should make sure the variables little and sensitive, which means we should choose the variables has greater influence on the design requirements. The main optimization goal in the paper is volume of the reducer, which could be expressed in gear teeth counts, modulus, spiral angle and tooth width coefficient. After comprehensive consideration, paper choose normal module, teeth counts and helix angle of the two small gears and the ration between high-speed gear and middle-speed gear as the variables and gear width coefficient is 1. Relevant variables could be written as:
\[ X = [x_1, x_2, x_3, x_4, x_5, x_6, x_7]^T = [m_1, z_1, m_2, z_2, i, \cos \beta_1, \cos \beta_2]^T \] (1)

### 3.2 The construction of objective function.

Volume of the gear transmission system could be expressed as four gears’ volume up. Taking the variables into the expression and handle it, the object function can written as:

\[
F = \frac{\pi}{4} \times \left\{ \frac{x_1^3}{\cos^3 x_6} \times (1 + x_5^3) + \frac{x_3^3}{\cos^3 x_7} \times \left[ 1 + \left( \frac{7.53}{x_5} \right)^3 \right] \right\} \]

(2)

### 3.3 The structure of the constraint condition [3].

To prevent the gear tooth fracture, bending strength of gear shall not exceed the allowable value, we usually take tooth root as the dangerous section for checking:

\[
\sigma_r = \frac{KF \gamma_{fa} \gamma_\beta}{b m_n \varepsilon_\alpha} \leq [\sigma_r] \]

(3)

To prevent the gear pitting corrosion, stress on unit contact line shall not exceed the allowable value, we usually take nodes in mesh as the dangerous section for checking:

\[
\sigma_n = \frac{KF_f (u+1) \varepsilon_\alpha}{b m_n} \leq [\sigma_n] \]

(4)

- \( K \) — load factor circumferential force; \( F_f \) — helical gear tooth shape coefficient; \( \gamma_{fa} \) — helical gear stress correction factor; \( \gamma_\beta \) — spiral angle influence coefficient; \( m_n \) — normal module; \( \varepsilon_\alpha \) — helical gear transverse contact ratio; \( b \) — gear width.

Generally speaking, low-speed gear will not get break as long as the high-speed gear satisfied the working condition. Now taking two small gears’ strength as the constrain condition. Refer to relevant date, we can get the formula as follows:

\[
G_1(x) = \frac{229188}{x_1^3 x_2^3} \sqrt{x_6^3 (x_2 + 1)} - 1200 \leq 0 \]

(5)

\[
G_2(x) = \frac{229188}{x_1^3 x_2^3} \sqrt{x_6^3 (x_2 + 7.53)} - 1200 \leq 0 \]

(6)

\[
G_3(x) = \frac{1042843}{x_1^2 x_2^2} x_6^2 - 560 \leq 0 \]

(7)

\[
G_4(x) = \frac{3190766}{x_3^2 x_4^2} x_2^2 - 560 \leq 0 \]

(8)

There are many methods for processing of constraint function, the most widely used is the penalty function method. The paper using the penalty function method to deal with constraint conditions, the objective function changed as follows:

\[
F = \frac{\pi}{4} \times \left\{ \frac{x_1^3}{\cos^3 x_6} \times (1 + x_5^3) + \frac{x_3^3}{\cos^3 x_7} \times \left[ 1 + \left( \frac{7.53}{x_5} \right)^3 \right] \right\} + 5 \times 10^8 \times \\
\left[ \frac{(1042843 x_6^2 - 560)^2}{x_1^2 x_2^2} + \frac{(3190766 x_7^2 - 560)^2}{x_3^2 x_4^2} + (229188 x_5^3 (x_2 + 7.53) - 1200)^2 \right]

(9)
N means punish factor coefficient, whose values often greater than 2. After many times to change the value of N, paper setting N as 8, which makes sure the values of objective function almost no volatility.

In order to prevent the tooth profile root cutting, we commonly take \( Z \geq 17 \).

In order to guarantee the need of transmission power, we usually take modulus \( m \geq 2 \).

The bigger the spiral Angle the greater gear bearing capacity and the more stability transmission, we general setting the spiral angle as \( 8^\circ \leq \beta \leq 20^\circ \), as well as \( 0.9659 \leq \cos \beta \leq 0.9903 \).

In order to slow down the speed, we usually take ratio as \( 2 \leq i \leq 4 \).

**Genetic algorithm based on MATLAB**

Through analysis of the objective function, we know that the objective function is nonlinear function. There is a variety of solutions to such problems, the literature[5] compare different algorithms for solving the problem of reducer center distance, through which we can get the conclusion that GA has good robustness, fast convergence rate advantages, when refer to the related problem. GA mainly contains four parts[6]: coding, fitness function, genetic operators and control parameters.

Encoding is the process of building relation between the variables and the chromosome. Common encoding methods mainly include binary code, gray code, in order to guarantee the algorithm to search conveniently, the paper use gray coding and coding length is 7.

Fitness is individual adaptive value, which used to measure of chromosomes, in this paper, optimization goal is the minimum volume and the fitness is bottom of the objective function value of chromosomes. The smaller the objective function the higher the fitness, corresponding chromosome is more easy to choose. Algorithm consist iteration until find out the optimal answer.

Genetic operators are mainly contain copy, crossover, mutation operator, the copy operator judging the size fitness and determines the chromosome is eliminated or copied to the next generation; crossover operator is used to achieve two chromosome and form a new individual; mutation operator make sure the chromosome encoding varies randomly based on settled rate and forming a new chromosome.

The control parameters of the algorithm usually refers to population size, genetic algebra, crossover probability, mutation probability, which have a great influence on the results obtained. The settle of the control parameters in general is based on the relevant experience. This paper set genetic algebra as 100, population size 50, crossover probability as 0.9, mutation probability as 0.05.

As the MATLAB language simple and easy to understand and the function is rich, paper use MATLAB software to write GA program. The program logic diagram show in Figure 2, the result of running the program is shown in Figure 3, the hereditary algebras and fitness diagram shown in Figure 4.

Table 1 shows the design parameters before and after optimized, from which we can see that the gear volume decreased by about 34%, the optimized result is satisfied. At the same time, through analysis the number of iterations and the fitness diagram, we can see that in front of the forty generation, the fitness value volatility is relatively large, this shows that the population generate a number of new chromosomes, which ensures the diversity of population and after the forty generation, fitness tends to be stable, which shows algorithm have found the approximate optimal answer and the algorithm is basically achieved global convergence.
Encoding and Initialization
Calculation fitness
Satisfied the constrain?
No
Choose the chromosome
Mutant
Crossover
Yes
Decode
End

Fig. 2: GA logic diagram

**********best particle number**********
zbest =

2.0301 17.2940 2.0137 20.5137 3.5005 0.9633 0.9558

Fig. 3: Optimized result

Fig. 4: Fitness and evolution algebra diagram

<table>
<thead>
<tr>
<th>Name</th>
<th>High speed and middle speed gears variable</th>
<th>volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
<td>Modulus/m</td>
<td>Tooth number</td>
</tr>
<tr>
<td>before</td>
<td>3/3</td>
<td>19/23</td>
</tr>
<tr>
<td>optimize</td>
<td>2.5/3</td>
<td>17/21</td>
</tr>
</tbody>
</table>

Gears static analysis based on Nastran

In order to verify the correctness of the optimization results, paper using finite element method to analysis gear bending stress. The basic idea[7] of finite element method is through discrete geometry into limited individual units, and further set certain nodes between each unit, and then
through the balance condition of each unit in node set up a corresponding finite element equations and eventually get the approximate solution of the solution domain.

As the stress analysis process are similar, paper take the medium speed big gear as an example. paper use Nastran as the finite element solver, which is put in UG software, with a strong analysis function. Nastran can undertake multiple kinds of finite element analysis and analysis of the reliability is high. Specific steps as follow: draw the corresponding helical gear in the UG soft and then change into the advanced simulation navigator to build the FEM simulation, the mesh rezoning choosed 4 nodes tetrahedron element, the number of nodes is 77014, unit number is 57789, as shown in figure 5. Material assigned for STEEL, and then select the round surface of the shaft hole as fixed constraint, finally select any one tooth surface to applied normal load, through the gear stress formula in "mechanical design", the normal load is 4472N, and then start to solve, the results shown in figure 6.

Through the results, we can see that the gear teeth root interface is dangerous, the maximum stress is 71.40MPa, and through the gear bending strength formula in "mechanical design", the bending stress is 76.06MPa, the allowable bending stress is 233MPa. All these date shows that optimized gear could meet the work requirement and the error between analysis value and theory value is little, which means the analysis value can be accepted. There are two main reasons for the difference between analysis value and theory value. One is mesh retoing will bring error, another reason is theory value keeps conservative.
Conclusions
The papers use the genetic algorithm to design two stage helical gear reducer, which decreased the volume of gear transmission system greatly. Besides, paper use Nastran to analysis the gear bending stress, the result shows that optimized gear could meet work requirement.

References