Optimization of Combined Scroll Profile Based on Multi-objective Genetic Algorithm

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Abstract. This In the design of scroll compressor, in order to improve the performance of the combined profile by involute of circle and high order curve, the mathematical model was established which takes the stroke volume and area utilization coefficient as the objective function. Then optimize it by the MATLAB genetic algorithm toolbox and have a contrast of the original parameters with the optimized parameters. The optimal result shows that the result after optimization has higher stroke volume and area utilization coefficient.

Introduction

Scroll compressors are widely used in air conditioning and refrigeration because of its high efficiency, small volume, light weight, low noise and simple structure etc. [1]. But the material utilization rate of uniform scroll part is low, in order to improve the material utilization rate of scroll plate, the variable cross section scroll profile by involute of circle and high order curve is put forward. To make it have higher utilization rate, we can optimize it. However, optimizations of combined profile of existing work [2] have limitations, because they are only based on the single objective to optimize. Therefore, this paper proposes to take the stroke volume and area utilization coefficient as the target function and then optimize the profile. Finally, make a comparison of the profiles between the original parameters and the optimized parameters.

Mathematical model

This paper chooses the profile composed by involute of circle and high order curve and then optimize it. Combined profile is shown in Fig1 and its equation is:

\[
\begin{align*}
    x &= R_{g1} \cos(\varphi) + R_{s1} \sin(\varphi) \\
    y &= R_{g1} \sin(\varphi) - R_{s1} \cos(\varphi)
\end{align*}
\]

(0 ≤ \varphi < \varphi_1)  \hspace{1cm} (1)

Where: \( R_{g1} = a, R_{s1} = a\varphi, a \) is the radius of base circle

\[
\begin{align*}
    x &= R_{g2} \cos(\varphi) + R_{s2} \sin(\varphi) \\
    y &= R_{g2} \sin(\varphi) - R_{s2} \cos(\varphi)
\end{align*}
\]

(\varphi_1 ≤ \varphi < \varphi_2)  \hspace{1cm} (2)

Fig.1 Combined profile
Where: \( R_{g2} = c_1 + 2c_2(\varphi - \frac{\pi}{2}) + 3c_3(\varphi - \frac{\pi}{2})^2 \)
\( R_{s2} = c_0 + c_1(\varphi - \frac{\pi}{2}) + c_2(\varphi - \frac{\pi}{2})^2 + c_3(\varphi - \frac{\pi}{2})^3 \)

When \( \varphi_1 = 2\pi, \varphi_2 = 4.5\pi, a = 2.25 \)  \( c_0 = 32.1464, c_1 = -11.8210, c_2 = 2.1066, c_3 = -0.0868 \).

The dynamic and static scroll meshing is shown in Fig 2.

![Fig. 2 profile meshing diagram](image)

**Objective function**

The stroke volume and area utilization coefficient are taken as the objective function.

Calculate stroke volume \(^3\)

When the dynamic scroll plate angle \( \theta = 0 \), compression cavity is closed, the volume of first compression chamber is stroke volume and its formula is:

\[
V = 2S_1 \cdot h
\]

Where:
\( S_1 \)— cross-sectional area of single compression cavity of the outer ring
\( H \)— the height of scroll plate

When \( \theta = 0 \), the length of the busbar of suction chamber is:

\[
L = \int_{\varphi_2-2\pi}^{\varphi_2} \sqrt{\left( R_{g2} \cos \varphi + R_{s2} \sin \varphi \right)^2 + \left( R_{g2} \sin \varphi - R_{s2} \cos \varphi \right)^2} \, d\varphi
\]

The cross-sectional area of single compression cavity of the outer ring is:

\[
S_1 = L \cdot R_{or} + \left[ R_g \left( \varphi_2 - 2\pi \right) - R_g \left( \varphi_2 \right) \right] \cdot R_{or}
\]

The first objective function, namely stroke volume, is:

\[
f_1(X) = V = 2S_1 \cdot h
\]

In order to reflect the material utilization rate of scroll plate, defined area utilization coefficient.

\[
r = \frac{4S}{\pi D^2}
\]

Where:
\( S \)— the cross-sectional area of the compression chamber
\( D \)— scroll meshing diameter

The length of the bus bar of second compression chamber is:

\[
L_2 = \int_{\varphi_1-2\pi}^{\varphi_1} \sqrt{\left( R_{g1} \cos \varphi + R_{s1} \sin \varphi \right)^2 + \left( R_{g1} \sin \varphi - R_{s1} \cos \varphi \right)^2} \, d\varphi
\]

\[
+ \int_{\varphi_1-4\pi}^{\varphi_1} \sqrt{\left( R_{g1} \cos \varphi + R_{s1} \sin \varphi \right)^2 + \left( R_{g1} \sin \varphi - R_{s1} \cos \varphi \right)^2} \, d\varphi
\]

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The cross-sectional area of single compression cavity of the second compression chamber is:

\[ S_2 = L_2 \cdot R_{or} + \left( R_g \left( \varphi_2 - 4\pi \right) - R_s \left( \varphi_2 - 2\pi \right) \right) \cdot R_{or} \]  \hspace{1cm} (9)

The cross-sectional area of all the compression cavities is:

\[ S = 2(S_1 + S_2) \]  \hspace{1cm} (10)

The second objective function, namely area utilization coefficient, is:

\[ f_2(X) = r = \frac{4S}{\pi D^2} \]  \hspace{1cm} (11)

This paper uses linear weighted method to group the two objective functions above as a system objective function \( F(X) \).

\[ F(X) = \sum_{i=1}^{2} \lambda_i f_i(X) \]  \hspace{1cm} (12)

The variables of the objective function are \( c_0, c_1, c_2, \varphi_1, \varphi_2 \). But \( \varphi_1, \varphi_2 \) are the function of \( c_0, c_1, c_2, c_3 \).  

So the final optimized parameters are \( \varphi_1, \varphi_2 \). They should meet the following constraints:

\[ \varphi_1 > \pi \quad 4\pi < \varphi_2 < 6\pi \quad \varphi_1 < \varphi_2 \]

**Optimization and contrast**

The greater of the value of objective function \( F(x) \), the higher of the efficiency of compressor. But the MATLAB genetic algorithm toolbox always takes the objective function’s minimum value as the result. Therefore, \(-F(x)\) is taken as the optimization function. Open MATLAB, write the M file of objective function, call up the optimization toolbox, choose genetic algorithm, invoke the target function and restrict it. Then begin the optimization and get the final result. The iterative process is shown in Fig3.

After optimization, get a set satisfactory results, their values are:

\[ \varphi_1 = 7.853, \varphi_2 = 15.709, -F(X) = -18506.3 \]

The dynamic and static scroll meshing is shown in Fig4.
Fig. 4  scroll meshing after optimization

Compare the parameters of before and after optimization, the results are shown in table 1.

<table>
<thead>
<tr>
<th></th>
<th>Stroke volume (mm³)</th>
<th>area utilization coefficient</th>
<th>the value of the objective function</th>
</tr>
</thead>
<tbody>
<tr>
<td>original design</td>
<td>6.283</td>
<td>44486</td>
<td>0.1962</td>
</tr>
<tr>
<td>optimized design</td>
<td>7.853</td>
<td>46262</td>
<td>0.2205</td>
</tr>
<tr>
<td>growth rate</td>
<td>3.99%</td>
<td>12.39%</td>
<td>4.00%</td>
</tr>
</tbody>
</table>

**Conclusion**

The parameters after optimization are greater than the parameters before optimization. Therefore, it achieves the aim of optimization. So it is an effective method to optimize combined scroll profile based on multi-objective genetic algorithm. It can improve the value of the objective function and provide a theoretical basis for improving the efficiency of scroll compressor.

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**References**


