An Adaptive Constant Scallop-height Tool-Path Planning Method For Turn-Milling Machining of NURBS free-form surfaces

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Abstract. Because of its flexible topology and robust data structure, the NURBS surface has become the trend of free-form surfaces representation in the realm of CAD design. Yet its application in manufacturing has not been fully explored due to the limit of tool-path planning method. In this work, an Adaptive Constant Scallop-height Tool-Path planning algorithm has been proposed to exploit the advantage of turn-milling on the NURBS free-form surfaces, as well as to overcome the disadvantages of inefficient and poor-precision by tradition turning or milling method. This paper firstly illustrates the freeform surface by mathematic model. Then, generation tool path for turn-milling machining of NURBS free-form surfaces by three steps: generation initial tool path based on the adaptive constant scallop-height, tool path interval calculation for turn-milling machining, and adjacent tool path calculation based on an adaptive parameter. Finally, an example is given to prove the proposed approach is feasibility.

Introduction

Turn-milling is a relatively new process in manufacturing technology, where both the workpiece and the tool are given a rotary movement simultaneously. Turn-milling is a new prospective technology for the production of precise rotationally symmetrical work pieces [1]. The cost, time, and quality are the most important factors in production. Although the turn-milling method has many advantages in machining NURBS free-form surfaces, which is difficult to machine with conventional milling or turning [2], because of the limit of tool-path planning method, turn-milling method has not been used efficiently.

At present, there are in-depth researches on the method of NC machining tool path planning [3-5], yet these algorithms cannot effectively solve the problem of tool path planning on free surfaces using turn-milling. To solve this problem, this article proposed an adaptive Constant Scallop-height Tool-Path Planning Method. An accurate offset method based on curves on the curved surface will be firstly given as a basis, and the boundary contours will be offset with identical residues to generate precise tool paths which will be consistent with boundaries.

The mathematic model of freeform surface

Blade is typical of NURBS freeform surface part, the twist blade surface of a blade is difficult to machining. Therefore, the tool axis should be kept in a certain amount of space and angle range when processing blades and flow channels, in order to facilitate the tool-path algorithm, the freeform surface must be precise expression. It is known that the blade surface is divided into the suction surface and pressure surface.

The pressure surface and suction surface of the blade both are ruled surfaces which can be -formed with a line (generatrix) swept along a curve (guide curve) in the space. Therefore, parametric equations of ruled surface can be expressed as \( S(u, v) = (1-v)p(u) + vq(u) \). where \( u \) ’means one parameter along the wire direction and \( v \) ’ means another one along the bus direction, \( 0 \leq u \leq 1, 0 \leq v \leq 1, p(u) \) is cover curve and \( q(u) \) is shaft plate curve, One point on the surface can be
considered to be obtained by interpolating \( p(u) \) and \( q(u) \) with the same \( u \). Obviously, when \( v=0 \), the ruled surface change to the cover curve \( p(u) \), while, when \( v=1 \), the ruled surface become to the shaft plate curve \( q(u) \).

The pressure surface curve and the suction surface curve of the blade, in this passage, are all 3 B-spline curves. Equation of the cover curve can be expressed as \( p(u) = \sum_{i=0}^{n} d_i N_{i,3}(u) \). Use \( U = [u_0, u_1, \ldots, u_{n+1}] \) to mean the knot vector and \( d_i \) (\( i = 0, 1, \ldots, n \)) the controlling vertices. \( K \) times B-spline basis can be expressed as \( N_{i,k}(u)(i = 0, 1, \ldots, n) \), which can be obtained by recursions from the formula as follows:

\[
N_{i,0}(u) = \begin{cases} 
1 & \text{if } u_i \leq u \leq u_{i+1} \\
0, & \text{other}
\end{cases}
\]

\[
N_{i,k}(u) = \frac{u - u_i}{u_{i+k} - u_i} N_{i,k-1}(u) + \frac{u_{i+k+1} - u}{u_{i+k+1} - u_{i+1}} N_{i+1,k-1}(u)
\]

Where \( k = 3 \), then the first derivative of \( p(u) \) by \( u \) can be got \( p'(u) = \sum_{i=0}^{n} d_i N'_{i,3}(u) \) in which \( N'_{i,3}(u) \) can be computed by recursions from \( N'_{i,3}(u) = k \left( \frac{N_{i,k-1}(u)}{u_{i+k} - u_i} \frac{N_{i+1,k-1}(u)}{u_{i+k+1} - u_{i+1}} \right) \), similarly, the second derivative of \( p(u) \) can obtained.

**Tool path generation**

Tool path planning is a critical task in the machining of NURBS free form surfaces. Specific constraints are applied in path planning for different machining stages to achieve the optimal time and quality. For example in finish machining, the machining time should be minimized while the scallop height must be maintained below the specified level. An ideal tool path should generate uniformly distributed scallops across the whole surface [6]. Smaller scallop size doesn't necessarily mean a better tool path, since it is achieved at the cost of increased machining time. On the other hand, the minimum machining time will be achieved when the scallop height is set to the maximum allowable measure. Toolpath planning is composed of 2 aspects: path topology and path parameters. The former is defined by the pattern that the cutter moves to produce the surface, and the latter is modeled by the tool side step between successive paths and the tool’s forward step in each path. Many researches have been carried out on the optimization of the tool path in these two areas [5-7]. Hence the tool path generation problem will be converted into the following sub-problems:

**Based on the adaptive Constant Scallop-height generation initial tool path.** Tool path means the routes of tools generated in the processing of surfaces of a workpiece. Besides, Cutter Location Data is the data which can accurately express each position of the tool in the workpiece coordinate system during the processing of the workpiece, e.g., cutter contacts, cutter location points, cutter shaft unit vectors.

Supposing that there were \( n+1 \) ordered cutter sites \( P_j \) (\( j = 0, 1, \ldots, n \)), and using the sites to NURBS curve fit, we get a \( k \) times NURBS curve \( r(u) = \sum_{j=0}^{n} R_{i,j}(u) C_j \), in which \( R_{i,j}(u) \) means a rational B-spline basis function and \( C_j \) (\( i = 0, 1, \ldots, n \)) is the control polygon vertices of the curve. Simultaneously, the deviations of the curve and points satisfy a certain accuracy requirement.

As mentioned above, the weight of control vertexes can be set to 1 at the beginning of the curve fitting, when \( R_{i,j}(u) = N_{i,j}(u) \). In order to get fairing NURBS curves which at least second order and continuous curves, set the order \( k \) to 4. The number of initial control vertexes can be set to \( n+1 \), with the shape features of tool path and the cutter location points being considered.
The arm of optimization for control vertices is to minimize the deviations between cutter location points and the fitting curve which can be expressed by the sum of distances between cutter location points and corresponding points on the curve \( \delta = \| p_i - P(u_i) \| \).

Because the adjustment of every control vertex on the \( k \) times NURBS curve will affect the change of the corresponding curve of tools, the deviation of all cutter location points are involved in the optimization of control points. The sum of square of cutter location points’ deviations is chose to as the object function which can be expressed as

\[
f(c_0, c_1, \ldots, c_n) = \sum_{i=0}^{n} \| p_i - P(u_i) \|^2 + \sum_{i=0}^{n} p_i \cdot \sum_{j=0}^{n} N_{ij}(u_j) \cdot c_j
\]

Each control vertex on the curve can be expressed by three coordinate components in Cartesian coordinates.

**Cutter path interval calculation.** Efficiency and precisions of surface machining can be determined by the shortest distance between adjacent two rows of the cutter path called Feeding Space, which is closely related to Residual Height generally. Oversized space will distinctly increases the surface residual height, which means adding the processing capacity of the next step. Conversely, undersized space will waste working time by increasing feeding times.

In Multi-Axis Machining planning, the processing steps and the line spaces are two factors with greatest impact on surface quality and processing efficiency. However, the impact on surface quality brought by processing steps can be controlled by enlarging the number of cutter location points. Namely, the impact has been getting smaller and smaller with the improvement of processing efficiency of CNC machine. Similarly, the number of toolpaths and Residual Height of processing surface are directly determined by line spacing. Therefore, when feeding space is not reasonable, uneven distribution of the cutting tool path will occur on the surface and then the distribution of Residual Height will also be non uniform. In order to improve productivity and meet the requirements of surface roughness and machining accuracy, reasonable Feeding Space ought to be chosen.

There are 3 kinds of calculating methods of path interval \( L \) according to the local features of curved surface, and more detailed expressions are shown in Table 1. Where “\( h \)” express path interval, “\( R \)” express the radius of the tool, “\( R_b \)” express Radius of curvature of surface.

<table>
<thead>
<tr>
<th>Processing Method</th>
<th>Processing Plane</th>
<th>Convex Surface of Machining</th>
<th>Concave Surface of Machining</th>
</tr>
</thead>
<tbody>
<tr>
<td>Formula</td>
<td>( L = 2\sqrt{R^2 - (R-h)^2} = 2\sqrt{2Rh - h^2} )</td>
<td>( L \approx 2\sqrt{3hR} \frac{R_b}{R_b + R} )</td>
<td>( L \approx 2\sqrt{3hR} \frac{R_b}{R_b - R} )</td>
</tr>
</tbody>
</table>

**Legend**

**Adjacent tool path Calculation based on adaptive parameters.** Identical Residual Height is a method making the residue height the same between paths by controlling the distance between adjacent paths. Judging from the principles of Identical Residual calculation, the total length of Identical Residual Height paths is shortest and accordingly the efficiency should be highest. Nevertheless, it also has its weaknesses because errors accumulate and transfer during the process of path generations, since each a tool path generation is based on the former one. In this research the Modified Newton method is putted forward, namely a variable factor is used to replace the constant step length in traditional methods to keep optimizing along the Newton direction in the process of optimization. This study sets the number of initial control points to \( n \). The Adjacent tool path solving process is shown in Fig. 1, where derivation formula of corresponding cutter contacts is as:
\[
\begin{align*}
(P^*_0 - P_0) & \left( P_0 \frac{du}{dt} + P_0 \frac{dv}{dt} \right) = 0, \\
\left\| P^*_0 - P_0 \right\| &= W
\end{align*}
\]

The path \( i \) \hspace{1cm} \text{The path } i+1

(a) Corresponding Relation \hspace{1cm} (b) Solution Procedure

Fig. 1 Adjacent tool path solving process diagram

Suppose \( P_0 \) and \( P'_0 \) are the current point of tangency and corresponding cutter contact respectively. Correspondingly, \( P_0u \) and \( P_0v \) are tangential directions of \( u \) and \( v \). Expand \( P'_0 \) to \( P'_0 = P_0 + P_0u \Delta u + P_0v \Delta v \), and to get equations:

\[
E \Delta u \frac{du}{dt} + F \left( \Delta u \frac{du}{dt} + \Delta v \frac{dv}{dt} \right) + G \Delta v \frac{dv}{dt} = 0, \quad E \left( \Delta u \right)^2 + 2F \Delta u \Delta v + G \left( \Delta v \right)^2 = W^2
\]

After solving the equations above, the available parameter increments are as follows:

\[
\Delta u = \pm W \left( \frac{F \frac{du}{dt} + G \frac{dv}{dt}}{\sqrt{EG - F^2 \left( \frac{du}{dt} \right)^2 + 2F \frac{du}{dt} \frac{dv}{dt} + G \left( \frac{dv}{dt} \right)^2}} \right), \quad \Delta v = \pm W \left( \frac{F \frac{du}{dt} + G \frac{dv}{dt}}{\sqrt{EG - F^2 \left( \frac{dv}{dt} \right)^2 + 2F \frac{du}{dt} \frac{dv}{dt} + G \left( \frac{du}{dt} \right)^2}} \right)
\]

Coordinates of corresponding cutter contacts can be obtained after getting parameters increments, and then the adjacent tool path corresponding with current cutter contact can also be achieved:

\[
P'(i) = P\left[ u(t) + \Delta u(t), v(t) + \Delta v(t) \right]
\]

Then \( P'_0 = P[ u(t) + \Delta u(t) - \Delta u_c, v(t) + \Delta v(t) - \Delta v_s ] \). Where \( \Delta u_c \) and \( \Delta v_s \) is the speed of the spindle.

Example

To verify the adaptive constant scallop-height tool-path planning method, this paper select blade as the test part. The blade model is shown in Fig. 2. The proposed method and techniques have been implemented in NX UG 7.5. Fig. 3 shows the tool path by our approach. In this paper, compare the machining accuracy with different tool path planning for three cases, which is shown in Fig. 4. Such as machining accuracy of milling method based on constant scallop-height tool path planning in Fig. 4-a, machining accuracy of turn-milling method based on constant scallop-height tool path planning in Fig. 4-b, machining accuracy of turn-milling method based on adaptive constant scallop-height tool path planning in Fig. 4-c. The result shows that the proposed method have the high accuracy.

Fig. 2 Blade model \hspace{1cm} Fig. 3 generate tool path

Conclusions

In This paper an adaptive Constant Scallop-height Tool-Path Planning Method is proposed, this method is effectively solved the difficult of tool path in NURBS free-form surfaces with Turn-Milling. In this research after calculation each iterative, the deviations between corresponding points on NURBS curve and cutter location points are judged to ensure product processing precision, the Modified Newton method not only keep the convergence of the algorithm, but effectively avoid
rising of the objective function in the process of iteration. Ensure the deviation between fitter curve and cutter location points are greater than the error precision. Correspondingly, when the lower bound of the error is less than a given value, reduce the number of control points to simplify the calculation process of the fitted curve. Finally, Simulation experiment results have shown the effectiveness of this algorithm.

![Fig. 4 Compare the machining accuracy with different tool path planning](image)

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**References**


