Abstract—A 2-DOF model with the coupling of vertical and torsional motions is considered to study galloping of transmission lines in this paper. Based on Lagrange equation, the equation of motion is established. Dynamic analysis is obtained by using Newmark-β method in an example. Finally, anti-galloping VED is used to investigate its effect of vibration control.

Keywords—transmission lines; galloping; 2-DOF model; anti-galloping VED; numerical analysis

I. FOREWORD

Galloping of iced transmission line is a typical low-frequency (about 0.1 ~ 3Hz), large amplitude (about 5 ~ 300 times of the diameter of the wire) of the self-excited vibration of gas-solid coupling phenomenon. When galloping, all-speed wire do directional wave undulating movement. Due to large amplitude, long-time vibration is easy to cause flashover, seriously causing fitting damage, line tripped, tower failure, etc[1-2].

Since the 1920s, national science and technology workers have carried out extensive research and achieved fruitful results about galloping in the theoretical analysis and experimental verification. In galloping mechanism, Den Hartog wave vertical excitation mechanism[3] of the United States and O. Nigol torsional excitation mechanism[4] of Canadian are still the mainly ones, the studies of other mechanism and anti-galloping measures are based on the two theories; In Experiment, wind tunnel test is mainly used to test the aerodynamic coefficients of iced conductors, wind tunnel simulation test of galloping with iced line is considerable difficulty carried out. To study test line is undoubtedly the most direct method for galloping, but the observation time long and expensive. Numerical simulation research has been an important means of galloping problem. Currently, there are single-degree-of-freedom model of vertical, 2-dof model of vertical and torsional coupling, as well as 3-dof model of vertical, horizontal and torsional coupling[5] used to study galloping. The 2-dof model, not only taking into account the Den Hartog vertical galloping theory and Nigol torsional excitation mode, galloping inspire is dynamic instability phenomenon caused by negative damping energy accumulation of wire system, so the conductor structural damping is one of the important characteristic parameters of galloping, the source of galloping in the inherent characteristics of the structure. Adding wire damping structure and dissipating wind power of wire system is a fundamental way to inhibit galloping. In this paper, the anti-galloping of VED, with the energy dissipation capability of viscoelastic materials and the TMD effect[10-12], increase the damping of the wire and effectively inhibit galloping of wire.

In this paper, the galloping of wire is studied with 2-dof degree of vertical and torsional coupling and Newmark-β dynamic analysis methods, the anti-galloping VED is joined, the mathematical model is proposed, and damping effect is theoretically analyzed.

II. 2-DOF MODEL OF THE WIRE

A. Dynamic Balance Equation of Wire without Anti-galloping VED

The 2-dof model is described by the use of the whole fixed coordinate system Y-O-Z and local moving coordinate system Ya-a-Za, as shown in Figure 1. The origin of Ya-a-Za is located in the center of rotation a, the rotation angle of the moving coordinate system around a is described by \( \theta_t \).
The coordinate of any point S on iced wire (Figure 1) in the local moving coordinate system is \((z_a, y_a)\), the rotation angle of its is \(\theta\), then the coordinate of its in the whole fixed coordinate system is,

\[
\begin{align*}
\cos \theta &+ \sin \theta \cos \theta = z_o \\
\cos \theta - \sin \theta &- z_o \sin \theta = y_o
\end{align*}
\]  
(1)

When \(\theta\) is small enough, equation (1) can be approximated by the value of

\[
\begin{align*}
z_{oa} &\approx z_a + y_a \theta \\
y_{oa} &\approx y + y_a - z_a \theta
\end{align*}
\]  
(2)

The derivative to Equation (2) on both sides is \(z_{oa}\) and \(y_{oa}\), so the per unit length kinetic-energy of the iced wire is,

\[
T = \frac{1}{2} \int_A (y_{oa}^2 + z_{oa}^2) \rho dA
\]  
(3)

The per unit length potential-energy of the iced wire is,

\[
V = \frac{1}{2} (k_y y^2 + k_\theta \theta^2)
\]  
(4)

Where,  \(k_y\) is the wire vertical stiffness,  \(k_\theta\) is the wire torsional stiffness.

The per unit length non-conservative force of the iced wire is,

\[
\begin{align*}
Q_y &= F_y - 2m \varphi_y \omega_y \dot{y} \\
Q_\theta &= M_\theta - 2J \varphi_\theta \omega_\theta \dot{\theta}
\end{align*}
\]  
(5)

Equation (3), (4), (5) into the Lagrange equation,

\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial \dot{q}_i} = Q_i
\]
(6)

The dynamic equilibrium equation of the 2-dof model of the iced wire is,

\[
\begin{align*}
m \ddot{y} + 2m \varphi_y \omega_y \dot{y} - S \ddot{\theta} + k_y y &= F_y \\
J \ddot{\theta} + 2J \varphi_\theta \omega_\theta \dot{\theta} - S \ddot{y} + k_\theta \theta &= M_{\theta}
\end{align*}
\]  
(7)

This model is mainly used to study the coupling problem between vertical freedom and torsional freedom of the conductor. The bundle conductors are mainly studied in this paper, the whole span conductors are simplified to be a particle in the middle of the span, the amplitude and the dynamic properties of this point adding anti-galloping VED are studied for the studies of future multi-particle calculations. In Equation (7), \(m\) is the equivalent mass of the bundle conductor at its midpoint of the span, \(J\) is the equivalent moment of inertia, \(S\) is equivalent eccentric moments, \(\varphi_y\), \(\varphi_\theta\) is respectively the vertical and the torsional damping ratio, \(\omega_y\), \(\omega_\theta\) is respectively the vertical and the torsional natural frequency, \(F_y\), \(M_{\theta}\) is respectively the vertical and the torsional aerodynamics force, it can be expressed as

\[
F_y = \frac{n}{2} \rho U^2 D C_y L
\]
\(M_{\theta} = \frac{n}{2} \rho U^2 D^2 C_\theta L
\]  
(8)

Where, \(\rho\) is the air density, \(U\) is the wind speed, \(D\) is the height of the conductor vertical to the direction of flow, \(L\) is the length of the conductor, \(C_y\), \(C_\theta\) is respectively the vertical and the torsional aerodynamic coefficient, so, based on the wind tunnel test datas, it can be expressed as [5] [13]

\[
\begin{align*}
C_y &= \alpha y \theta + \alpha_2 \theta^3 \\
C_\theta &= \beta_1 \theta + \beta_2 \theta^3
\end{align*}
\]  
(9)

Where, \(\alpha_1 = -1.6478, \beta_1 = 0.1761, \beta_2 = 0.3592, \beta_2 = -0.0434, \theta = \theta_0 - \dot{y} / U\), \(\theta_0\) is the Horizontal angle of the icing.

B. Dynamic Balance Equation of Wire with Anti-galloping VED

With the above Lagrange equation, the dynamic balance equation of wire with anti-galloping VED can be derived with reference to the dynamic balance equation of Structure with TMD [10] [14],

\[
\begin{align*}
m \ddot{y} + 2m \varphi_y \omega_y \dot{y} + (k_y + k_d) y + c_d (\dot{y} - \dot{y}) &+ \ddot{\theta} = F_y \\
J \ddot{\theta} + 2J \varphi_\theta \omega_\theta \dot{\theta} + (S + k_\theta) + k_\theta \theta = M_{\theta}
\end{align*}
\]  
(10)
Where, \( m_d \), \( c_d \), \( k_d \) is respectively the mass, the damping and the stiffness of the anti-galloping mechanism, \( \ddot{y}_d \), \( \dot{y}_d \), \( y_d \) is respectively the acceleration, the velocity and the displacement of the anti-galloping mechanism, the rest coefficients are the same with the ones without anti-galloping mechanism.

### TABLE I. COEFFICIENT TABLE FOR ICED CONDUCTOR

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit Mass of the Conductor ( m_L )</td>
<td>3 kg/m</td>
</tr>
<tr>
<td>Conductor Diameter ( d )</td>
<td>28 mm</td>
</tr>
<tr>
<td>Conductor Tension ( T )</td>
<td>40000 N</td>
</tr>
<tr>
<td>Conductor Length ( L )</td>
<td>200 m</td>
</tr>
<tr>
<td>Wind Speed ( U )</td>
<td>10 m/s</td>
</tr>
<tr>
<td>Conductor Spacing ( a )</td>
<td>0.6 m</td>
</tr>
<tr>
<td>Vertical Damping Ratio ( \zeta_y )</td>
<td>0.06</td>
</tr>
<tr>
<td>Horizontal angle of Icing ( \theta_0 )</td>
<td>( \pi/4 )</td>
</tr>
<tr>
<td>Icing Density ( \rho_i )</td>
<td>900 kg/m³</td>
</tr>
<tr>
<td>Air Density ( \rho )</td>
<td>1.29 kg/m³</td>
</tr>
<tr>
<td>The Thickness of Icing ( h )</td>
<td>15 mm</td>
</tr>
<tr>
<td>Conductor Length ( L )</td>
<td>200 m</td>
</tr>
<tr>
<td>Conductor Spacing ( a )</td>
<td>0.6 m</td>
</tr>
<tr>
<td>Vertical Damping Ratio ( \zeta_y )</td>
<td>0.06</td>
</tr>
<tr>
<td>Horizontal angle of Icing ( \theta_0 )</td>
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<td>( \pi/4 )</td>
</tr>
</tbody>
</table>

In this paper, coefficients of bundled conductors can be expressed as [2] [3],

\[
\begin{align*}
    m_i &= \frac{\pi}{4} \rho_i h d \\
    m &= \frac{4nL(m_i + m_L)}{\pi^2} \\
    J &= \frac{4nL(m_i + m_L)La^2}{\pi^2} \\
    k_y &= \frac{4nT}{L} \\
    k_i &= \frac{4nL(d^2 + k_i)}{\pi^2} \\
    S &= \frac{2nLm_iL(d + h)}{\pi^2} \\
    d' &= \frac{a}{2\cos30^\circ} \\
    D &= d + h\sin\theta_0
\end{align*}
\]

Where, \( d' \) is the equivalent diameter of bundle conductors.

#### 3.2 The Results

According to the datas shown in Table 1, with the Newmark-\( \beta \) iterative analysis method [15-17], and joined the anti-galloping VED, \( \zeta_d = 16.13 N\cdot s / m \), \( k_d = 52000 N / m \), the results of dynamic characteristics of the conductor compared are shown in Figure 2 and 3.

It is showed that, in Figure 2 and 3, the use of 2-dof model, combined with Newmark-\( \beta \) method, vertical vibrations decrease of about 50% with anti-galloping VED as changes in displacement and rotational angle with time under the influence of particular wind speed. And because of the coupling of the vertical and torsional degree of freedom, the rotation angle reduced by about 70%. It is showed that anti-galloping VED has good damping effect.

![Figure 2. Vertical displacement comparison](image)

![Figure 3. Torsionall displacement comparison](image)
IV. CONCLUSION

In this paper, with 2-dof model of the vertical and torsional degree of freedom coupling, combined with Newmark-β dynamic analysis method, the galloping of the conductor is studied and the mathematical model is analyzed its damping effect theoretically with anti-galloping VED.

The 2-dof model of conductor galloping, not only ignoring the secondary level movement, but also reflecting the coupling of the degree of freedom of vertical and torsional, is relatively simple, it can simulate changes in displacement and rotation with time under the influence of particular wind speed.

In this paper, the theory of the anti-galloping VED is the energy consumption of the viscoelastic material and the TMD effect on the wire. It is showed that anti-galloping VED has good damping effect with vertical vibrations decreased of about 50% and the rotation angle reduced by about 70%.

REFERENCES