Retrieval of wave information using nautical radar images based on the 2D CWT and 1D FFT algorithm
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Abstract. It is a useful way to gain ocean wave information using X-band nautical radar images. In this paper, an algorithm of 2-D CWT in spatial domain and 1-D FFT in time domain is presented for extracting the wave information. Firstly, the basic theory of 2-D continuous wavelet transform for space domain and its application for wave field analysis is introduced. Then, a scheme of analysis of the radar image sequence with this algorithm is given. By implementing this algorithm, the ocean wave spectrum and mean wave direction are extracted from the X-band radar images and compared with the in-situ data.

Introduction

The information of wave measurement plays great role for safety and quality in the routing of marine traffic and researching of scientific. As one of useful means to gain such information, a lot of investigations have been done by applying ocean remote sensing. Wave height measurement, as one of important part of wave characteristics evaluation, plays great role in these activities. The application of 3-D fast Fourier transform (FFT) on a series of radar images is one of the traditional methods. This method has been proved successfully used by the WaMoS (wave monitoring system) radar [1,2,3].

In the method of 3-D fast Fourier transform, the ocean wave was assumed homogeneity and temporal[10]. However, the wave may be inhomogeneous and non-stationary affected by the topography. Some researchers have used a 2-D continuous wavelet transform (CWT) to resolve this problem[1,4,5]. Wu and Chuang have used a 2-D continuous wavelet transform (CWT) to process X-band nautical radar data[6]. An et al have applied the 2-D continuous wavelet transform to extracting wave information from X band nautical radar images and obtained good agreement of the results with the in-situ data [9]. However, the proposed method could not eliminate the energy offset due to the surface current because of without the band-pass filtering process and could not remove the wave direction ambiguity. Carrying out a 3-D CWT on a sequence of radar images is a solution to resolve this problem. But the 3-D CWT will spend a lot of time to obtain a result. In this paper, 2-D CWT in spatial domain and 1-D FFT in time domain are used to obtain the wave information, which can reduce the time of wave analysis comparing the 3-D CWT and eliminate the wave direction ambiguity. We choose Morlet wavelet to study a series of radar images and the results from radar data are compared with in-situ data.

Theory of the Wavelet Transform and Its Application for Wave Field Analysis

The 2-D continuous wavelet transform algorithm.

Wavelet transform is similar to the Fourier transform, the CWT breaks signals into various wavelets, which then can be scaled, shifted and rotated of the pre-chosen mother wavelet. In the study of Antoine et al. (2004), the 2-D CWT was given and applied of image. If an image function is given as f(x) = f(x, y), the 2-D CWT can be defined as[6]:

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\[ w(\theta, \alpha) = \psi(\theta, \alpha) = \frac{1}{a} \int_{-\infty}^{\infty} \psi(x) f(x) \text{d}x \quad (1) \]

\[ C_\psi = \int_{-\infty}^{\infty} |\psi(\omega)|^2 \omega^{-1} d\omega < \infty \quad (2) \]

Where \( \psi^* \) is the complex conjugate of the mother wavelet \( \psi \). The factor \( a^{-1} \) is a normalization that gives all dilated versions of the mother wavelet the same energy. The shifting parameter \( \vec{b} \) indicates the position of the wavelet as it shifts in the space domain. The rotation matrix \( \tau_{-\theta} \) is related to a rotation angle \( \theta \), which rotates the wavelet in spatial coordinates, usually it is given as:

\[ r_{-\theta} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \quad 0 \leq \theta < 2\pi \quad (3) \]

We choose a mother wavelet function before implying the Eq. (1). In this article the Morlet wavelet is chosen as the mother wavelet for detecting the dimensional wave information from radar image. The Morlet wavelet function can be defined as (4) in space domain.

\[ \psi(x) = \exp(-0.5|A^2x|^2) \exp(i k_0 x) - \exp(-0.5|A^2x|^2) \exp(-0.5|A^{-1}k_0|^2) \quad (4) \]

Where, \( A = \text{diag}[\epsilon^{-0.5}, 1] \), \( \epsilon \geq 1 \) is an anisotropic diagonal matrix. The parameter \( k_0 \) is a vector constant which satisfies the admissibility condition. Commonly, it is negligible for \( |k| \geq 5.6 \) [7]. To apply the equation (1) to radar image analysis, it should be described discretely. It is possible to reduce the algorithm execution time by performing the CWT in the Fourier space. From the theory of 2-D CWT, we know that the transformed wavelet could be gotten through the scaling, shifting, and rotating the mother wavelet. If \( \vec{k}_0 \) is given as the peak wavenumber of the mother Morlet function in the Fourier space, after dilation and rotation with parameters in dimension space, the peak wavenumber is \( \vec{k}' \):

\[ \vec{k}' = \frac{\vec{k}_0}{a_{\theta_{0,\alpha}}} \quad (5) \]

Through the Eq. (3), (5), we can derive that the two dimension continuous wavelet transform can be described by the shifting factor \( \vec{b} \) and the transformed wave-number \( \vec{k}' \) [9]. Once the shifting factor is specified as \( \vec{b}_0 \), the two dimension wavenumber spectrum at point \( \vec{b}_0 \) can be determined. When the scaling parameter is determined, all the other parameters of the two dimension continuous wavelet transform are determined.

The radar image to be processed for analyzing is in digital form, we need to choose the analyzing wavelet in order to use the continuous wavelets to analyze the discretely sampled data. In the physical space, the sampling space is \( \Delta x \), the total sample points of the wavelet is \( N \), the total length to be transformed is simple \( N \Delta x \). If the total non-dimensional space is \( 2T \), it’s mapped for \( N \) points. We know that the spatial frequency is inversely to the total length, it is clear that we can get the value scaling factor \( a \) [9].

In the study on [8], the minimum number of sample points of the Morlet should be sampled in the dimensional space as:

\[ N = \frac{2\pi}{\pi} \left( k_{0,\alpha} + \sqrt{-2\ln(\eta)} \right) \quad (6) \]
the parameter $\eta$ is commonly set to 0.01 which is the ratio of the wavelet value at the cutoff wavenumber and that at the peak wavenumber.

**The algorithm combining the 2-D CWT with 1-D FFT**

In the data processing, we choose 64 radar images in one image sequence and the interval time between two images is about 1.43s. The time of image sequence is short, so we may assume that it is homogenous in the time domain. Instead of 3-D CWT, the 1-D FFT for time domain firstly and then 2-D CWT for the spatial domain are used to analysis the radar image. Based on the analysis of image sequence with 3-D FFT [12,13], the scheme of analysis of the radar image sequence with 2-D CWT and 1-D FFT algorithm is shown in Fig.1.

![Fig.1. Scheme of analysis of the radar image sequence with 2-D CWT and 1-D FFT](image)

(a) 1-D frequency spectrum (b) Mean wave direction

**Fig. 2. Comparison of results from simulated radar about frequency spectra and mean wave direction**

![Fig.3. Typical radar image obtained by the X-band radar wave observation system from Sep. 10, 2010](image)
Results and Analysis

The algorithm is applied to extract the wave information from simulated radar images and typical X band nautical radar images. In the data processing, 128 sample points and the scaling factor $a \in [1,4]$ was chosen. The radar horizontal spatial resolution is 7.5m which is same as the simulation. The results from the simulated radar images are shown in Fig.2. The Fig.2 (a) gives the inversed 1-D wave spectrum and the input wave spectrum which is P-M spectrum[11] in simulation. Similarly, the frequency spectra in the range from $f= 0.15$ HZ to $f=0.25$ HZ has some discrepancies which may be caused by the shadowing effect and the harmonics [6]. Fig.2 (b) gives the mean wave direction spectrum, while the inversed mean wave direction agrees well with the input angle. Here, the wave direction ambiguity is eliminated by the algorithm.

A typical nautical radar image taken by the detecting software system is shown in Fig.3. The radar images of the sea surface are captured in September 2010 at Jinjiang China. The X-band radar wave observation system that developed by Ocean University of China and CSIC PRIDE (Nanjing) Atmosphere and Ocean Information System Co.Ltd was applied to collect the data. And the scalar wave-rider buoy was moored about 1.5km away from the radar. Fig. 4 contains a comparison of the spectra derived from radar with the buoy data. From Fig.4, we can see that there are some fluctuation in extracted wave spectrum and mean wave directional spectrum, however, the trend of the wave spectrum consists with the buoy.

Conclusion

An improved method to extract ocean wave information from the X-band nautical radar image based on 1-D FFT in the time domain and 2-D CWT in the space domain was proposed. The comparison of the frequency spectrum and wave direction between the simulation and field observed data has been done to verify this algorithm. In general, the wave information which is derived from this algorithm is encouraging.

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References


