Delivery lead time decision under multi-commodity influence with competition
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Abstract. This paper studies under the retailer’s competition a supplier how to make an optimal decision about the delivery lead time. With continuous development of the financial market and the trade market, more and more companies are choosing to have a make to order production plan as to save money from storage, and considering the process of production orders, product quality and the level of order, production planning, etc. are all needed to take into account. How to make the optimal pricing strategy and save cost must be considered in the development of the enterprises. Pricing which related to the allocation of market share and to the development of corporate profits is existed in all aspects of production planning, during the production planning in the development process, we often have to consider the delivery problems in production planning and the impact on prices, delivery all these issues under different conditions are all we need to analysis. The main purpose of this paper is to consider products planning under market complementary conditions, the retailer optimal decision of price and delivery time, according to different situation of market. How to set up delivery lead time in order to maximize service levels meeting all consumer’s satisfaction. The next is to satisfy the price, the delivery and ordering supplements link between these three shipments. In the course of the analysis, we found that the price of the entire decision-making model on delivery is part of a concave model, there is an optimal value, which we call it the best result. At the same time there is a close relationship between the price and delivery time means that this research between model and the fact in some kind has similar part. Description the study has some certain reference value and the significance.

Introduction

About the study of financial markets and delivery lead time there has a lot of directions, most of them are based on the early news-boy production model, and “So” in 2000 considered the punishment system and the inventory costs in other situation to the delivery time model. Most of the delivery time model has to consider strictly on delayed costs and inventory costs. Yano’s (1987) earlier study proposed due date as decision variables, the study is to investigate the procurement time in random conditions and how to develop a reasonable commitment to the expected total cost to achieve the minimum delay problem. Hopp and Spearman’s (1993) study based on several companies investment were randomly procurement and fixed assembly time period set in advance of delivery problems. With further research, many scholars began to focus on the sensitive of customer about due date and support on announcing to all customers the due date, the customer can corporate announcements and insured delivery to determine their own requirements. On this basis, the development of the basic assumptions of the model is the customers are time-and price-sensitive ; 2 for short delivery time we need to increase investment. So and Song (1998), Palaka et al. (1998) and Hill et al. (2000) study about by considering the setting about limited production capacity, inventory costs and delivery delay penalty cost in order to maximize their own profit. Yan haoyun based on the coordination supply chain considered the delivery time model with the impact of load on the negotiated price and delivery time. Yang wensheng make an double-level analysis for...
the delivery with market forecasts and the actual supply chain delivery different situations were. Shen chenglin also carried out the delivery mode under competitive market and the impact of competition on the supply chain. 2009 someone make a integration of existing commitments and delivery model, based on the original model but relaxed the restrictions, pointing out that the size of extension penalty fee will have a certain impact on demand. In addition to these study almost all these aspects about competition or marketing are all considered, just like the price volatility preference factor, or the market complexity and what we all know is that the factors determines the delivery time is difficult to get an exact optimal solution for all the situation. Our work is just add a new factor to discuss this various new complementary situation, it is on the basis of previous.

**Model assumption**

There are two complementary products existed, the variety of complementary products exists a unified price, The variety of complementary products in the form of a single commodity has an impact on the other product.

A retailer and a manufacturer's model, the manufacturer does not influence market, These products directly lead to the retailers, the market demand affected by retailer's price production lead time and the complementary goods' price.

For vendors, its revenue comes from the price of goods, and it consists of three parts, respectively, expenditures for the production cost, delivery delays and the requirement to the cost of the delay time before the delivery is inventory costs incurred. Supplier delivery according to the literature shows a negative exponential distribution.

\[ f(x) = \theta e^{-\theta x}, \quad F(x) = 1 - e^{-\theta x} \]

**Symbol hypothesis**

- \( P_1 \) Retailer’s production price
- \( P_2 \) Complementary production price
- \( a \) The max demand of the market
- \( b \) Retailer’s production price-sensitive factor
- \( K \) Delivery-sensitive factor
- \( d \) Complementary production price-sensitive factor
- \( T \) The delivery lead time, \([0, T]\)
- \( t \) The actual lead time
- \( \Pi \) Revenue of the retailer
- \( C \) Unit cost of the production
- \( S \) Service level
- \( g \) Unit penalty cost for unsatisfied demand
- \( y \) Unit inventory cost
After the assumption of the symbols, we can build a model related to the revenue, as we all know the market demand changes with price and delivery time now get these factor in the function of the revenue, then we may get the revenue equation:

\[(P_1-c)(a-bP_1-KT+dP_2)-\int_0^T(t-T)(a-bP_1-KT+dP_2)g(t)dt-\int_0^T(t-T)\gamma(a-bP_1-KT+dP_2)f(t)dt\]

**Model analysis**

This revenue model on delivery is a part of concave function.

Given by the revenue function, we can see that this is a very complex function on T equation, no matter to prove that T is a concave function, or to prove that they have a maximum value we must derivative it.

Make \(G(T) = a-bP_1-KT+dP_2\) shows the change in demand can play a simplified role in the derivation process.

\[
\frac{d\Pi}{dT} = G(T)[P_1-c-g(T-t)f(t)dt-\int_0^T(t-T)\gamma f(t)dt] - G(T)[\int_0^T f(t)dt-\gamma \int_0^T f(t)dt]
\]

Then we make:

\[
\beta = \int_0^T f(t)dt = e^{-\theta t} \quad \alpha = \int_0^T f(t)dt = 1 - e^{-\theta t}
\]

Then we must make a simplification or we will not find the right answer:

\[
\int_0^T tf(t)dt = Te^{-\theta t} + \frac{e^{-\theta t}}{\theta} = T\beta + \frac{\beta}{\theta}
\]

\[
\int_0^T tf'(t)dt = \frac{1-e^{-\theta t}}{\theta} - \frac{\theta e^{-\theta t}}{\theta} = \frac{\alpha}{\theta} - T\beta
\]

Then can get the function as:

\[-K(P-c-gT\alpha + g\frac{\alpha}{\theta} - gT\beta - yT\beta - y\frac{\beta}{\theta} + yT\beta) - G(T)(g\alpha + y\alpha - Y) =
\]

\[-K(P-c-gT + \frac{g\alpha + y\alpha - Y}{\theta}) - G(T)(g\alpha + y\alpha - Y)\]

Because the \([-K(P-c-gT + \frac{g\alpha + y\alpha - Y}{\theta}) = [P_1-c-gT(t-t)f(t)dt-\int_0^T(t-T)\gamma f(t)dt]\]

represent the unit revenue must over 0, if not this business will never happen, it doesn’t existed in the reality world. Now we discuss the first derivation of the situation after the ceremony:

1. If \(g\alpha + y\alpha - Y > 0\), then the first order derivative is less than zero, \(T \geq \frac{\ln(g + y) - \ln g}{\theta}\) we may find that when T entire earnings during this interval this is a decreasing function.

2. If \(g\alpha + y\alpha - Y < 0\), then the first derivative may has an zero solution, we were able to find the optimal solution for this function, now we must seek its second derivative to prove it concave.

\[
\frac{d^2\Pi}{dT^2} = 2K(g\alpha + y\alpha - Y) - gG(T)f(T) - yG(T)f(T)
\]
It is strictly <0, we can find when 
\[ 0 \leq T \leq \frac{\ln(g+y)-\ln g}{\theta} \], this function is concave. It has a max value, this is the optimal solution. \( T \) satisfy below equation:

\[
(g-e^{-\alpha T}y-e^{-\alpha T}g)(G(T)+\frac{K}{\theta})-kgT = K(e-P)
\]

\[ T < \frac{\ln(g+y)-\ln g}{\theta} \]

As we do the left formulation we can find

\[
\frac{\partial}{\partial T}[(g-e^{-\alpha T}y-e^{-\alpha T}g)(G(T)+\frac{K}{\theta})-kgT] > 0
\]

with delivery price increase the decision time would be reduced, more willing more time to prepare.

When consider the service level

Now we are considering get the service level in . How to describe it, the difference between the actual delivery time and the lead time means the service level is low. if the actual delivery is the same with the delivery lead time it shows the highest level of service. the customer almost has no satisfied points, and the actual delivery and pre-delivery instructions get the more farther away we get the lower level, where I use the countdown function to characterize the situation, then on service level get the expectation model:

\[
E(T) = \int_0^T f(t) dt + \int_{T-T}^T f(t) dt
\]

Derivate it:

\[ -\frac{1}{T^2}(1-e^{-\alpha T} - \theta Te^{-\alpha T}) + \frac{T^*}{(T^*-T)^2} e^{-\alpha T} - \frac{1}{(T^*-T)^2}(e^{-\alpha T} + \theta Te^{-\alpha T}) \]

if \( T^* > e^{\alpha T} + 1, \frac{T^*}{(T^*-T)^2} e^{-\alpha T} - \frac{1}{(T^*-T)^2} e^{-\alpha T} > \frac{1}{(T^*-T)^2} > \frac{1}{T^*} \)

but if \( T^* < 1, -\frac{1}{T^*}(1-e^{-\alpha T} - \theta Te^{-\alpha T}) + \frac{T^*}{(T^*-T)^2} e^{-\alpha T} - \frac{1}{(T^*-T)^2}(e^{-\alpha T} + \theta Te^{-\alpha T}) < 0 \)

The optimal T=0, From the calculate can konw when the T' is very small the service level function comes as decreasing, so when the delivery time set as 0 maybe better. In reality comes that if this is shortly needed you must get it quickly as you can, but in most situation the high limit comes huge, so you don’t need to consider this problem.
Since the intermediate term is about T 'higher-order function, but as a delivery timeliness ceiling T' should be quite large, then you can get that this formula is over zero. After done some numerical analysis and Taylor expansions can be able to get this is true. So on T is monotone increasing. It means that when we get the prelead time as the final date the service level we get better.,because the cost for the lose is more influence than the other situation..

Conclusion

Relationship between the market price and complementarity price and delivery time can be found in the above equations, there is an optimal value for the retail price , and this value decreases with the increase of delivery lead time T. That is if the delivery time is a little latter then the price must be lower so they can obtain higher profits. As for the complementary goods price, the higher the price the lower profit for the retailer, which point to the actual situation.

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