Design and simulation of a software robustness testing model
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Abstract. The software robustness is affected by many factors, in order to improve the detection accuracy of software robustness, multiple probability analysis of computer control system is introduced, and a software robustness detection model based on probability robustness analysis is put forward. The computer control system of multiple probability analysis is utilized to detect the overall trend of software robustness, and achieve accurate detection of software robustness. The simulation results show that, compared to the traditional detection method, the software robustness detection model based on probability robustness analysis improves the detection accuracy of software robustness, provides some scientific, constructive suggestions for the work of software development, and improves the quality of software.

Introduction
Software robustness is regarded as an important index of the quality of the software, the accuracy of detection results has important significance on the software market decision [1-2]. Aiming at the problem of software robustness detection, domestic and foreign scholars have carried out extensive research, great achievements have been made [3]. The traditional software robustness detection model mainly include the method of multiple linear regression, time series analysis method and grey model [4-6], however, the traditional single model have the problem of low detection accuracy. In recent years, combination optimization theory has been widely used, but hasn’t been applied to software robustness detection.

In order to improve the detection accuracy of software robustness, with computer control system can describe the advantages of overall system variation trend, at the same time, using the probability robustness analysis to describe advantages of system dynamic change, a software robustness detection model based on probability robustness analysis is put forward, through the complementary advantages of these two to fully excavate the information in software robustness original data, and the performance of the combination model is tested through simulation experiments.

Computer control system robustness detection with the introduction of multiple probability analysis

Problem description. Large computer control system with uncertain linear time-varying can be described as:
\[ \dot{x} = (A + \Delta A)x + (B + \Delta B)u \]  (6)

The name of large computer control system can be described as:
\[ \dot{x} = Ax + Bu \]  (7)

In the formula, \( A \in \mathbb{R}^{n \times n} \), \( B \in \mathbb{R}^{n \times m} \), \( \Delta A \) and \( \Delta B \) are used to describe the perturbation of same dimension dynamic, and all belong to an uncertainty set \( U \), i.e.:
\[ U = \{ (\Delta A, \Delta B): \| \Delta A \| = \delta \alpha, \| \Delta B \| = \delta \beta \} \]  (8)

Among them, \( \| \cdot \| \) is used to describe the spectral norm of matrix. Assuming that the equivalent system after discretization on nominal control system is described by (9):
\[ x[k+1] = \Phi x(kT) + \Gamma u(kT) \]  (10)

In the formula, \( T \) is used to describe the sampling period, at the same time:
\[ \Phi = e^{\Phi t} \]  
\[ \Gamma = \int_{a_0}^{t} e^{\Phi \tau} d\tau B \]  

Assuming that the discrete time state feedback controller designed for the nominal control system is as follows:

\[ u(kT) = Kx(kT) \]  

On the basis of the controller designed with formula (13) to do the robustness detection for control system.

\[ x[(k+1)T] = e^{d\Delta t(T)} + \int_{0}^{d\Delta t(T)} e^{(d\Delta t(T))} d\tau (B + \Delta B)Kx(kT) \]  

Robustness detection method based on probability robustness analysis

The introduction of GM (1,1) model. Based on the relevant theories to shape GM (1,1) model, so as to analyze the large computer control system, and according to the obtained reference factors and evaluation results to provide the basis for the robustness detection of large computer control system. \( X^{(0)} = \{ x^{(0)}_t, t = 1,2,\ldots,n \} \) is used to describe the original time series.

A cumulative operation can obtain 1-AGO sequence \( X^{(0)} = \{ x^{(0)}_t, t = 1,2,\ldots,n \} \), \( x^{(0)} = \sum_{t=1}^{n} x^{(0)}_t \), generating sequence \( Y = \{ y_0, y_1, \ldots, y_n \} \), according to the sequence to derive the following first-order linear differential equations:

\[ \frac{dX(t)}{dt} + \alpha X(t) = \beta \]  

Among them, \( \alpha, \beta \) are used to describe the developing gray and inner generated control gray respectively. After completing infinitesimal element discretization, the matrix form of differential equation parameter estimation can be described as:

\[ \begin{bmatrix} \hat{\alpha} \\ \hat{\beta} \end{bmatrix} = (B^T B)^{-1} B^T Y \]  

\[ B = \begin{bmatrix} -\left(x^{(0)}_t + x^{(0)}_1\right) / 2 & 1 \\ \vdots & \vdots & \vdots \\ -\left(x^{(0)}_t + x^{(0)}_n\right) / 2 & 1 \end{bmatrix} \]

After the differential equation is solved, the time response sequence can be obtained:

\[ x^{(0)}_{\tau+1} = x^{(0)}_t - \frac{\hat{\beta}}{\alpha} \exp\left(-\hat{\alpha} \tau\right) + \frac{\hat{\beta}}{\alpha} \]  

The restore operation is applied to acquire the fitting sequence:

\[ \hat{Y}(t+1) = \hat{x}^{(0)}_{\tau+1} - x^{(0)}_{\tau+1} \]  

The robustness value of different control system sub unit is described accurately.

Probability robustness analysis. This paper takes a control system of a large computer control system as an example for study. Figure 1 is a description of a typical closed-loop control system.

Figure 1 closed-loop control system

In the formula, \( C(s) \) is used to describe the controller, \( P(s,q) \) is used to describe the object, \( s \) is used to describe the nonlinear measurable variable function, \( q \) is used to describe the uncertain parameters of the historical definition in \( Q \) collection, \( q = (q_1,q_2,\ldots,q_n) \).
The random vector in multivariate probability density function \( f(q) \) is described by \( q \), the sampling interval is described with \( N \), the sampling points of independent and identical distribution is described with \( q \), and \( f(q) \) is used to describe the probability density function, assuming the sensitivity function is described as follows:

\[
S(s,q) = \frac{1}{z + P(s,q)C(s)}
\]  

(12)

Supposing that \( S(s,q) \) is a stable transfer function, that is:

\[
u(q) = \|S(s,q)\|_p = \sup_{\omega} |S(j\omega,q)|
\]  

(13)

Assuming that the performance index \( \gamma > 0 \), the probability is made to satisfy the following conditions:

\[
u(q) \leq \gamma
\]  

(14)

Give an indication function \( I(q) \)

\[
I(q) = \begin{cases} 1 & \nu(q) \geq \gamma \\ 0 & \nu(q) < \gamma \end{cases}
\]  

(15)

The probability of unacceptable performance can be described as:

\[
P = P(\nu(q) \geq \gamma) = \int_{u} f(q)I(q) dq
\]  

(16)

Through the Monte Carlo to simulate and calculate \( P \), which is based on the given \( f(q) \) to randomly access \( N \) sub samples, then arithmetic average value of statistics \( I(q) \) can be acted as estimated amount of \( P \), i.e.:

\[
P = \frac{1}{N} \sum_{i=1}^{N} I(q_i)
\]  

(17)

If the estimation accuracy \( \varepsilon \in (0,1) \) and confidence level \( \delta \in (0,1) \) are known, \( \hat{P} \) with a certain probability values converges to \( P \), which makes the:

\[
P\left[P - \hat{P} \leq \varepsilon\right] \geq 1 - \delta
\]  

(18)

The reasonable minimum number of Monte Carlo random trials is:

\[
N_{\text{min}} \geq \frac{\ln\frac{1}{\delta}}{\ln\frac{1}{1-\varepsilon}}
\]  

(19)

The probability metric of system performance index reliability is called the degree of robustness, the formula can be described as follows:

\[
\gamma_i = \left[t_\omega, \delta_i, t_\omega, \eta_i, x_i \right]^T o_i^T, i = (1,2,3,4,5)
\]  

(20)

In the formula, \( t_\omega, \delta_i, t_\omega, \eta_i, x_i \) are used to describe the adjustment time, overshoot, time back to zero, stability time and probability values of controllable degree, \( o_i \) is used to describe the corresponding weight, through the hierarchy analyzing method it can be known that, \( i = (1,2,3,4,5) \) is used to describe the number of evaluation index needs to be analyzed analysis in the calculation process of robustness, which can be increased based on the need.

Simulation experiment

Data sources. The experimental data is from robustness test data in a supermarket management system, the data is divided into training set and test set, the last 10 data is viewed as the test set, the rest are training sets, which can be used to establish software reliability prediction model.

Data preprocessing. Because the changing range of software reliability data is large, equation (9) is carried on for pretreatment, to make the range between \([0, 1]\).

\[
x^i = \frac{x - x_{\text{min}}}{x_{\text{max}} - x_{\text{min}}}
\]  

(9)

In the formula, \( x \) is the original data, \( x_{\text{max}} \) and \( x_{\text{min}} \) is the maximum value and the minimum value.
**Results of robustness detection.** Using computer control system on the training set to learn the software reliability, software robustness detection model is established, and then use the model to detect the test set, test results obtained as shown in table 1.

<table>
<thead>
<tr>
<th>The total number of errors</th>
<th>The predicted value</th>
<th>The prediction accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>347</td>
<td>395</td>
<td>86.17%</td>
</tr>
<tr>
<td>389</td>
<td>375</td>
<td>96.40%</td>
</tr>
<tr>
<td>342</td>
<td>331</td>
<td>96.78%</td>
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<tr>
<td>377</td>
<td>386</td>
<td>97.61%</td>
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<td>424</td>
<td>393</td>
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<td>93.44%</td>
</tr>
<tr>
<td>475</td>
<td>501</td>
<td>94.53%</td>
</tr>
<tr>
<td>401</td>
<td>415</td>
<td>96.51%</td>
</tr>
</tbody>
</table>

The average prediction accuracy is 94.64%.

**Modification of probability robustness analysis on detection results.** According to the relative value and the actual situation, the system is divided into 3 states respectively, $E_1$, $E_2$, $E_3$, including: $E_1 \in (86.17\%, 93.44\%)$, $E_2 \in (94.53\%, 95.44\%)$, $E_3 \in (96.40\%, 97.61\%)$, and then according to the state corresponding to the detection point, the state transition probability matrix of each step can be acquired:

$$
\begin{bmatrix}
0 & 1 & 0 \\
1/2 & 0 & 0 \\
1 & 0 & 1/2
\end{bmatrix}
\begin{bmatrix}
0 & 1/2 & 1/2 \\
0 & 1/4 & 1/4 \\
1/2 & 1/2 & 0
\end{bmatrix}
\begin{bmatrix}
1/2 & 1/2 & 1/4 \\
1/4 & 1/8 & 1/8 \\
0 & 1/2 & 1/2
\end{bmatrix}
$$

The first data point being detected as an example, because the state is divided into 3, so 3 data close to the first data point are selected to make the prediction table, transfer steps are 1, 2, 3, in the transfer probability matrix corresponding to transfer step number, corresponding line vector of initial state is obtained to get the first data point detection table.

The maximum value of the sum of transfer probability is 1/4, the corresponding state is $E_1$, total number of discovered errors of the first data point in this software system is at the state $E_1$, according to the total number of discovered errors of one data point in this software system to obtain the detection value 395, according to equation (10) can get the detection value of probability robustness analysis 361, predicted detection accuracy is 95.97, compared to the single grey model, the detecting precision of the proposed method has been improved, in the same way, next detection value of other points can be calculated, the detection results are shown in table 2.
Testing results of probability robustness analysis. The obtained results are shown in table 3. From table 3 we can see, the prediction accuracy of a single Markov chain model is significantly lower than that of the proposed method.

Results and analysis. The detection residual error of grey model, Markov chain model, probability robustness analysis detection model is calculated, the comparison graph of testing value and real value of three kinds of models as shown in Figure 2 and the absolute value of the detection residual error, as shown in figure 3.

![Figure 2 The test data in a supermarket management system](image-url)

### Table 2 test results of probabilistic robustness analysis

<table>
<thead>
<tr>
<th>The total number of errors</th>
<th>The test value</th>
<th>The test accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>347</td>
<td>351</td>
<td>98.86%</td>
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<tr>
<td>389</td>
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<td>419</td>
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<td>97.34%</td>
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<tr>
<td>401</td>
<td>410</td>
<td>97.80%</td>
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<tr>
<td><strong>The average test accuracy</strong></td>
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<td><strong>97.79%</strong></td>
</tr>
</tbody>
</table>

### Table 3 testing results of probability robustness analysis detection model

<table>
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<tr>
<th>The total number of errors</th>
<th>The test value</th>
<th>The test accuracy</th>
</tr>
</thead>
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<td>389</td>
<td>385</td>
<td>98.97%</td>
</tr>
<tr>
<td>342</td>
<td>338</td>
<td>98.83%</td>
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<tr>
<td>377</td>
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<tr>
<td>401</td>
<td>392</td>
<td>97.76%</td>
</tr>
<tr>
<td><strong>The average test accuracy</strong></td>
<td></td>
<td><strong>98.42%</strong></td>
</tr>
</tbody>
</table>
Conclusions

In view of the features of software robustness, like many affecting factors, strong uncertainty, great stochastic volatility, first using the computer control system to detect the software robustness, then probability robustness analysis model is utilized to modify test results. The simulation results show that, comparing to the single model or the traditional model, system and the combined model of probability robustness analysis and computer control system realizes the complementary advantages, can reveal the overall change trend of the robustness, and also overcome the influence of randomly fluctuated data on the detection accuracy, the defection accuracy of software robustness is higher, so as to achieve the goal of scientifically detecting software robustness, and provides the powerful technical support for the detection of software development.

References