Study on Solution of Non-homogeneous Linear Equation based on Ordinary Differential Equation Driving

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Abstract. This paper introduces two orders Runge-Kutta ordinary differential solution drive in the process of solving the non-homogeneous, and uses MATLAB software to program the algorithm. This paper uses ordinary differential solution method of *M* function to solve non-homogeneous equation. Through the calculation plane and space solution of homogeneous equation are obtained, finally the characteristic curve is drawn. Through calculation, calculation speed is fast and the accuracy is high using MATLAB language, which provides a theoretical basis for non-homogeneous equation solution.

Introduction

Theory of linear differential equations is complete, and the application scope is very extensive. Especially the two order linear differential equation with constant coefficients, it is widely applied in mechanical, electrical engineering, etc.. The solution is generally through investigation multiplicity of corresponding characteristic equation characteristic roots, then determine according to the non-homogeneous term by using the method of undetermined coefficients or variation parameters [1, 2]. This paper uses ordinary differential equations to solve non-homogeneous linear equation, and the solving process is as shown in Figure 1.

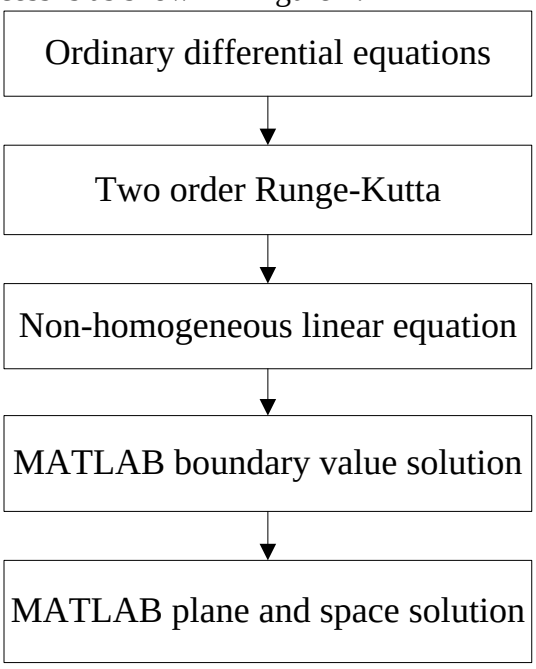

Fig.1: Ordinary differential equation solving algorithm of non-homogeneous equation

Figure 1 shows the ordinary differential equation solving algorithm of non-homogeneous equation [3, 4]. From the chart it can be seen, this paper uses ordinary differential driving method of two orders Runge-Kutta to solve the non-homogeneous equation, and uses MATLAB software to program the algorithm, finally boundary value solution and the plane space solution of non-homogeneous differential equation are obtained.

The Theory Foundation of Non-homogeneous Differential Equation Driven Solution

The solution of constant coefficient linear differential equation plays an important role in ordinary differential equations; it is one of the basic content of ordinary differential equation system, with complete theory and rich practical background, especially the two order constant coefficients linear differential equation is more prominent focus [5-7]. This paper discusses the following solving method of the two order constant coefficient linear differential equation.

$$y'' + k_1 y' + k_2 y = f(x) \tag{1}$$

The corresponding homogeneous linear equation is:

$$y'' + k_1 y' + k_2 y = 0 \tag{2}$$

Using two order Runge-Kutta ordinary differential equations driving method solves the non-homogeneous linear equation as shown in formula (3).

$$\begin{cases} y_{n+1} = y_n + h(c_1 K_1 + c_2 K_2) \\ K_1 = f(x_n, y_n) \\ K_2 = f(x_n + \lambda_2 h, y_n + \mu_{21} h K_1) \end{cases} \tag{3}$$

Where, c_1, c_2, λ_2 and μ_{21} are determined, the local truncation error is:

$$T_{n+1} = y(x_{n+1}) - y_n - h[c_1 f(x_n, y_n) + c_2 f(x_n + \lambda_2 h, y_n + \mu_{21} h f_n)] \tag{4}$$

The coefficient can be obtained through the formula (5):

$$\begin{cases} c_1 + c_2 = 1 \\ c_2 \lambda_2 = \dfrac{1}{2} \\ c_2 \mu_{21} = \dfrac{1}{2} \end{cases} \tag{5}$$

Using four orders Runge-Kutta method solves the differential equations as shown in the formula (6).

$$\begin{cases} y_{n+1} = y_n + \dfrac{h}{6}(K_1 + 2K_2 + 2K_3 + K_4) \\ K_1 = f(x_n, y_n) \\ K_2 = f(x_n + \dfrac{h}{2}, y_n + \dfrac{h}{2} K_1) \\ K_3 = f(x_n + \dfrac{h}{2}, y_n + \dfrac{h}{2} K_2) \\ K_4 = f(x_n + h, y_n + h K_3) \end{cases} \tag{6}$$

In order to realize the constant micro decomposition method of non-homogeneous linear equation, according to the above formulas, this paper uses MATLAB numerical calculation method to design the algorithm. The program is as follows:

```
syms x y;
f=y*(y- 1);
a=18;b=18;x1=- 9;y1=- 9;
m=80;n=80;h1=a/ m;h2=b/ n;
```

```
hold on
for i=1:m
x=x1+(i- 1)*h1;
for j=1:n
y=y1+(j- 1)*h2;
d=eval(f);
y2=y+2/ 3*h1*d;
if(abs(y2- y)>2/ 3*h2）
x1=x+1/ d*h2*2/ 3;
……
```

Research on Ordinary Differential Numerical Drive for Non-homogeneous Equation

In order to verify the validity and reliability of ordinary differential drive numerical solution for non-homogeneous equation, this paper uses the ordinary differential equation *M* function solution of MATLABT to solve non-homogeneous equation, and the calculated the boundary value solution is as shown in Figure 2.

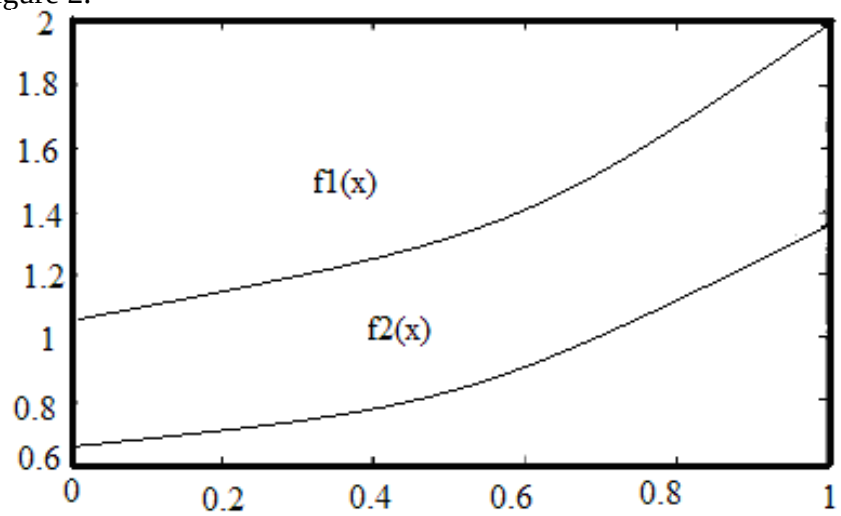

Fig.2: Homogeneous equation boundary value solution using ordinary differential equation driving

As shown in Figure 2, the description of differential equation is completely consistent with the initial value, and the boundary value is described [8]. It will call the bvp5c () function, and they can also be through the inline () function or anonymous functions directly.

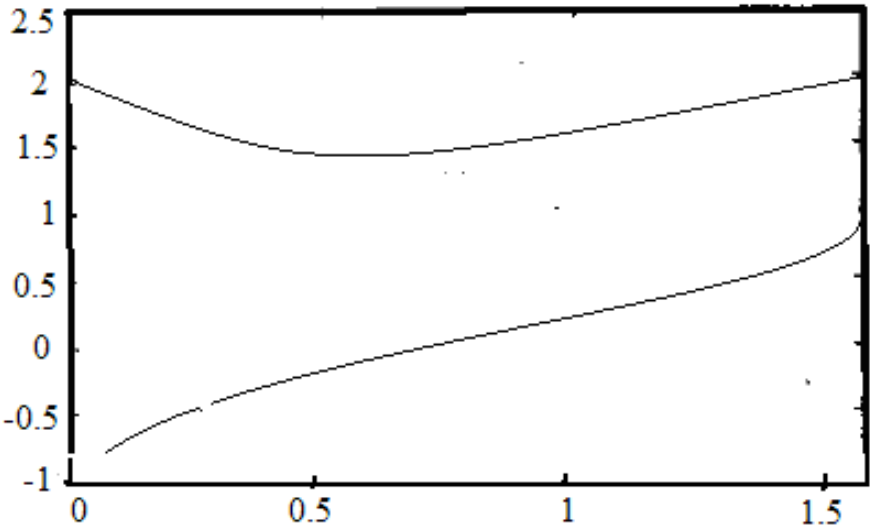

Fig.3: Non-homogeneous boundary value solution using ordinary differential equations driving

As shown in Figure 3, non-homogeneous boundary value solution is obtained using ordinary differential equations driving [9]. By setting the boundary values, the graphic solution of original

equation is obtained. Non-homogeneous equation solution is using the ordinary differential equation to decrease order. The plane solution is obtained as shown in Figure 4.

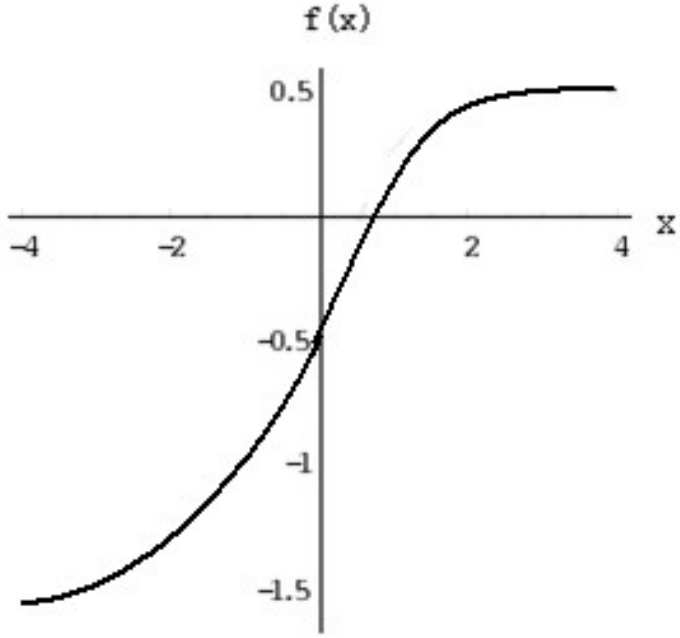

Fig.4: Plane solution graph

Figure 4 shows a characteristic curve of homogeneous equations. The curve can use MATLAB visual display function to solve, as shown in Figure 4 [10]. In the ordinary differential equation driving method a smooth curve is obtained, which is consistent with the characteristics of non-homogeneous equation.

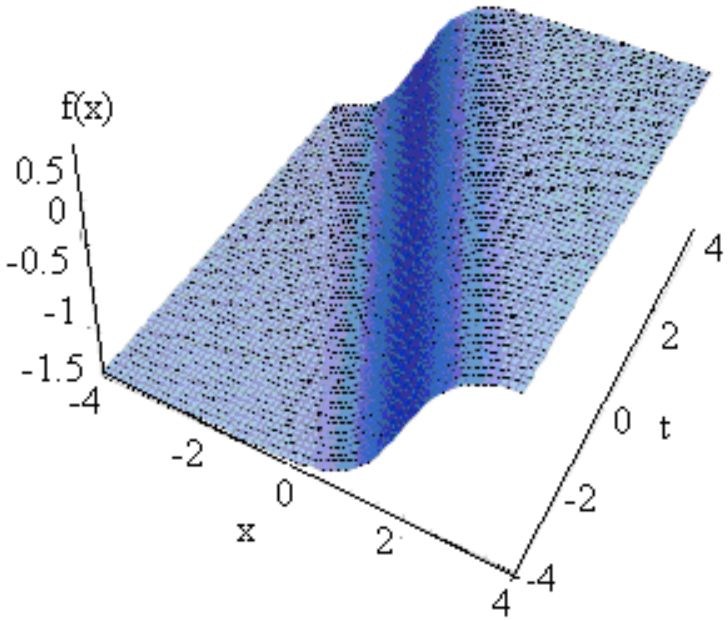

Fig.5: Space solution graph

In order to make the solution of the equation visualization, this paper uses the Plot function of MATLAB to draw the non-homogeneous linear equation solution set, as shown in Figure 5 [11]. The solution set is composed of a series of similar curve, and the screenshot consists with the 2D plane section.

Summary

This paper establishes two orders Runge-Kutta ordinary differential solving driving method of non-homogeneous linear equation, and uses MATLAB software to program the algorithm. In order to

verify the validity and reliability of the and reliability driving method, this paper uses the ordinary differential equation solving M function of MATLAB to solve non-homogeneous equation, and the characteristics solution curve of non-homogeneous linear equation is obtained. Through calculation, the general trend of MATLAB numerical solution is the same as analytical solution, and its calculation speed is faster, accuracy is higher.

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