The Mathematical Model of Gear Reducer Build and Optimize

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Abstract— With the lightest weight as the optimization goal is established the mathematical model for optimal design of planetary gear reducer, and put forward the solving method. When a given power P is constant, according to the input speed and other technical conditions and requirements, find out a set of economic and technical index of the optimal design parameters and to achieve the optimal design of gear reducer.

Keywords-reducer;design variables; constraint functions; mathematics

I. SINGLE SPUR GEAR REDUCER OPTIMIZATION DESIGN

The optimization design of the reducer, generally refers to the given power P, number of teeth than u, input speed n_1 and other technical conditions and requirements, find out a set of uesd a economic and technical index hit the optimal design parameters. Cylindrical gear reducer type and structure form has a lot of kinds, the working conditions and the design requirements are various, it is difficult to use unified mathematical model to describe different types, different structure and different conditions and the design requirements of the optimal design of the reducer. Usually, for different types of speed reducer, the selection of design variables are different.

In the Fig .1,The single stage cylindrical gears reduction gear ratio $u = 5$, the input power $P = 75 + 5 \times 44 = 295kW$, input shaft speed $n = 980r/\min$. Requirements in guarantee gear bearing capacity conditions, so that the quality of the reducer minimum.

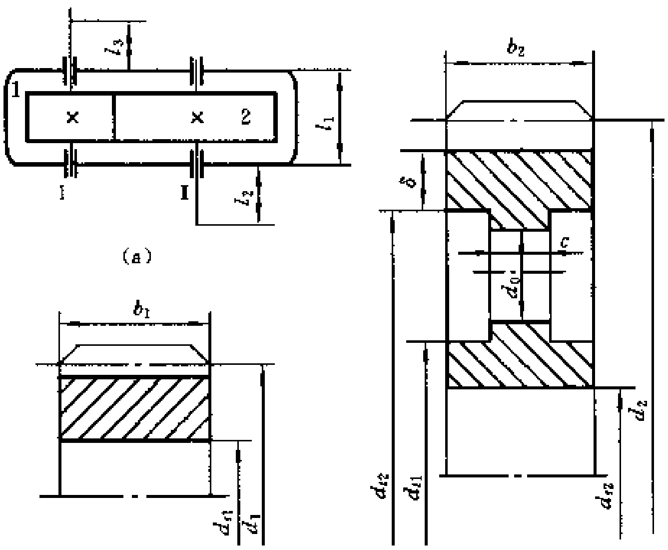

Figure 1. single stage cylindrical gear reducer

II. TO DETERMINE WHETHER THE DESIGN VARIABLES

The volume of gear reducer is mainly dependent on the size of the internal gear and shaft, according to the gear geometry size and the size of the structure, the calculation formulas of the monopole cylindrical gear reducer housing internal gear and shaft overall product v can approximate expressed as

$$v=\frac{\pi}{4}\left(d_1^2-d_{s1}^2\right)b_1+\frac{\pi}{4}\left(d_2^2-d_{s2}^2\right)b_2+\frac{\pi}{4}d_{s1}^2\left(l_1+l_3\right)$$
$$+\frac{\pi}{4}d_{s2}^2\left(l_1+l_2\right)-\frac{\pi}{4}\left(D_2^{'2}-D_1^{'2}\right)\left(b_2-c\right)-4\left(\frac{\pi}{4}d_0^2c\right) \quad (1)$$

The type, single standard spur gear reducer optimization design design variable desirable for

$$X=[x_1\ x_2\ x_3\ x_4\ x_5\ x_6]^T=\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ X_5 \\ X_6 \end{bmatrix}=\begin{bmatrix} b \\ z_1 \\ m \\ l_1 \\ d_1 \\ d_2 \end{bmatrix}$$

This approximate take $b_1=b_2=b_0$

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III. DETERMINE A GOAL FUNCTION

According to the structure design of experience formula, $\delta=5m$ 、 $D'_2=d_2-2\delta$ 、 $D'_1=1.6d_{s2}$ 、 $d_0=0.25\left(D'_2-D'_1\right)$ 、 $c=0.2b$, And take $l_2=32mm$ 、 $l_3=28mm$ And to take these experience formula and data substitution type (1), and use the design variable to said that finish to the objective function for the expression

$$f(x)=0.78539815\left(4.75x_1x_2^2x_3^2+85x_1x_2x_3^2-85x_1x_3^2+0.92x_1x_6^2-x_1x_5^2\right.$$
$$\left.+0.8x_1x_2x_3x_6-1.6x_1x_3x_6+x_4x_5^2+x_4x_6^2+28x_5^2+32x_6^2\right) \quad (2)$$

IV. SURE CONSTRAINT FUNCTION

1) in order to avoid root cutting, should have $z_1\geq z_{\min}=17$, constraint function

$$g_1(x)=17-x_2\leq 0 \quad (3)$$

2) according to the technology and equipment conditions, the big gear diameter d_2 no more than $1500mm$, the pinion diameter d_1 should not be more than $300mm$ namely $mz_1\leq 30cm$, constraint function

$$g_2(x)=x_2x_3-30\leq 0 \quad (4)$$

3) to ensure the carrying capacity of gear at the same time avoid load along the width of tooth distribution serious inequality for tooth width coefficient $\Phi_m=b/m$ meet $16\leq\frac{b}{m}\leq 35$, which have

$$g_3(x)=x_1x_3^{-1}-35\leq 0 \quad (5)$$

and

$$g_4(x)=16-x_1x_3^{-1}\leq 0 \quad (6)$$

4) to transfer power gear, modulus can't is too small, general $m\geq 2$ mm, and take the standard series value, it is

$$g_5(x)=0.2-x_3\leq 0 \quad (7)$$

5) according to the experience, Lord, driven shaft diameter value range for $10cm\leq d_{s1}\leq 15cm$、$13cm\leq d_{s2}\leq 20cm$ Thus, it is

$$g_6(x)=10-x_5\leq 0 \quad (8)$$

$$g_7(x)=x_5-15\leq 0 \quad (9)$$

$$g_8(x)=13-x_6\leq 0 \quad (10)$$

$$g_9(x)=x_6-20\leq 0 \quad (11)$$

6) according to the structure relationship, shaft bearing span meet: $l_1 \ge b + 2\Delta + 0.5d_{s2}$, including for housing wall to bearing center distance, we take $\Delta = 2cm$, it is constraint function

$$g_{10}(x) = x_1 + 0.5x_6 + 4 - x_4 \le 0 \quad (12)$$

7) according to the gear contact fatigue strength and bending fatigue strength condition, should be:

$$\sigma_H = \frac{336}{a}\sqrt{\frac{KT_1(u+1)^3}{bu}} \le [\sigma_H] \quad (13)$$

$$\sigma_{F_1} = \frac{2KT_1}{bd_1 mY_{F_1}} \le [\sigma_{F_1}] \quad (14)$$

$$\sigma_{F_1} = \frac{\sigma_{F_1} Y_{F_1}}{Y_{F_2}} \le [\sigma_{F_2}] \quad (15)$$

Type, a the standard of gear transmission center distance, the unit is cm, $a = 0.5mz_1(u+1)$; K For load coefficient, here take $K = 1.3$; T_1 For small gear transmission torque, the unit is, $N \bullet cm$

$$T_1 = 955000P/n_1 = 95500 \times 295/980 N \bullet cm \approx 287474 N \bullet cm$$

For gear of the allowable contact stress, the unit is MPa ,Here, take; $\lfloor\sigma_{F_1}\rfloor$、$\lfloor\sigma_{F_2}\rfloor$ Respectively with the great pinion gear allowable bending stress, the unit is MPa, here take $\lfloor\sigma_{F_1}\rfloor = 261MPa$, $\lfloor\sigma_{F_2}\rfloor = 213MPa$; Y_{F_1}、Y_{F_2} Respectively, for small gear, gear tooth shape coefficient, standard gear:

$$Y_{F_1} = 0.169 + 0.006666z_1 - 0.0000854z_1^2 \quad (16)$$

$$Y_{F_2} = 0.2824 + 0.0035399z_1 - 0.0000015769z_2^2 \quad (17)$$

To the above formula for substitution, operation and finishing, satisfied gear contact strength and bending strength conditions of constraint function: :

$$g_{11}(x) = 45002x_2^{-1}x_3^{-1}x_1^{-\frac{1}{2}} - 855 \le 0 \quad (18)$$

$$g_{12}(x) = 7474/\left[x_1 x_2 x_3^2\left(0.169 + 0.6666\times10^{-2}x_2 - 0.854\times10^{-4}x_2^2\right)\right] - 261 \le 0 \quad (19)$$

$$g_{13}(x) = 7474/\left[x_1 x_2 x_3^2\left(0.2824 + 0.177\times10^{-2}x_2 - 0.394\times10^{-4}x_2^2\right)\right] - 213 \le 0 \quad (20)$$

According to the driving shaft (in this case is pinion shaft) stiffness conditions, shaft maximum bending deflection $y_{\max}$ Should be less than allowable value $[y]$, namely

$$y_{\max} - [y] \le 0 \quad (21)$$

And took $[y] = 0.003l_1$; $y_{\max}$ The next type computation.

$$y_{\max} = F_n l^3/(48EJ) \quad (22)$$

Type, F_n For action in the small gear tooth surface load method, the unit is N, $F_n = 2T_1/(mz_1\cos\alpha)$, α For gear pressure Angle, $\alpha = 20°$; E For shaft of materials of the modulus of elasticity, $E = 2\times10^5 MPa$; J For axial moment of inertia, the unit is cm^4, the circular cross section, $J = \pi d_{s1}^4/64$.

Similarly, to the above formula for substitution, operation and finishing, can get satisfaction shaft bending rigidity conditions of constraint function

$$g_{14}(x) = 0.01298 x_2^{-1} x_3^{-1} x_4^3 x_5^{-4} - 0.003 x_4 \le 0$$

(23)

8) according to the axis of the bending strength condition, there is

$$\sigma_b = \frac{\sqrt{M^2 + (\alpha' T)}}{W} \le [\sigma_b] \qquad (24)$$

Type, T for shaft by torque, $T = T_1$; M For shaft by bending moment, the unit is $N \bullet cm$, $M = F_n l_1 / (m z_1 \cos\alpha) = 26444 l_1 / (m z_1)$; d For consider torque and bending moment function properties surprised coefficient, here take $d = 0.58$; $[\sigma_b]$ For shaft allowable bending stress, $[\sigma_b] = 55MPa$; W For shaft bending section modulus of solid shaft, $W = 0.1 d_{z1}^3$.

Thus, for small gear and gear shaft, can write respectively satisfy bending strength conditions of constraint function

$$g_{15}(x) = 27310 x_2^{-1} x_3^{-1} x_4 x_5^{-3} \left(1 + 0.29709 x_2^2 x_3^2 x_4^{-2}\right)^{\frac{1}{2}} - 55 \le 0$$

(25)

And

$$g_{16}(x) = 27310 x_2^{-1} x_3^{-1} x_4 x_5^{-3} \left(1 + 7.42727 x_2^2 x_3^2 x_4^{-2}\right)^{\frac{1}{2}} - 55 \le 0$$

(26)

V. SUMMARY

Single stage standard spur gear reducer with minimum volume as optimal objective optimization design problem, is a has sixteen inequality constraints six dimensional optimization problem, the mathematical model can be bsde for:

$$\min f(x) x = [x_1 x_2 x_3 x_4 x_5 x_6]^T \in R^6$$

$$S.t. \qquad g_j(x) \le 0 \qquad (j = 1, 2, \cdots, 16)$$

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