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RELIABILITY GROWTH PREDICTION BASED ON AN IMPROVED GREY PREDICTION MODEL

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Abstract

As limits of time, labors and expenses, observed data usually have the characteristic of small sample sizes in development test program. Redesigns or corrective actions can result in changes of reliability for equipments. We propose an improved GM(1,1) model to predict reliability growth in this paper. First, a newly initial condition in time response function is set in this improved GM(1,1) model. The newly initial condition is comprised of the first item and the last item of a sequence which is generated from applying the first-order accumulated generation operator to a sequence of raw data. Then the improved model can express the principle of new information priority well and improve prediction precision through fully applying new information in raw data. Secondly, we make use of the improved model to predict reliability growth in a numerical example. The comparison of predicted reliability growth curve from the improved GM(1,1) model and that from the Lloyd-Lipow model indicates that the improved GM(1,1) model is much better than the Lloyd-Lipow model for the reliability growth prediction.

Keywords: reliability growth, GM(1,1) model, the initial condition, first-order accumulated generation operator.

1. Introduction

Reliability growth often comes of some appropriate redesigns or corrective actions for detected faults of system. Each system modification determines a new change to the system and results in a new reliability level.[1] Reliability growth studies are necessary to insure that the reliability goals are met by acceptance. It is important for a project manager to use a reliability growth model on project completion data. If the predicted reliability value from the reliability growth model can meet or beyond requirements of specified goal, then the reliability growth testing can be realized carrying out well. Otherwise, the project manager should have to reassess the reliability growth prediction technologies or modify designs and take corrective actions of the equipment to meet reliability goals.[2] So reliability growth prediction technologies are essential to reliability testing of equipments. And a suitable reliability growth prediction technology chosen to predict reliability growth of equipment can also save a number of labors, time and expenses.

There are a large number of models can be utilized to predict reliability growth of equipment. Fard presents

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that reliability growth model can be deterministic or probabilistic. The deterministic reliability growth model refers mainly to the maximum likelihood estimations or least square estimations; the later refers mainly to the Bayesian estimation method.[3] Robinson[4] divides reliability growth model into discrete reliability growth model and continuous reliability growth model. And he also presents a continuous parametric model to analyze the failure rate of a system that is undergoing development testing. Discrete reliability growth models emphasize on improvements in the probability of a success as a function of total test trials rather than the failure rate as a function of total test time in continuous reliability growth models. We will focus mainly on the discrete reliability growth model in this paper.

As the importance for discrete reliability growth model， there are a large number of models proposed to predict reliability growth of equipment. Barlow and Scheuer assume that a test program is carried out in K stages and similar items are tested within each stage. They utilize maximum likelihood estimation to obtain probabilities of inherent failure and assignable cause failure in a trinomial model.[5] Lloyd and Lipow consider an N-stage test program and utilize least square method to obtain parameters of reliability growth model.[6] However, estimators obtained from Lloyd and Lipow's method will result in positively high s-biased when N is not large enough and the actual reliabilities of equipment are not increasing rapidly.[1] Read finds that the method proposed by Lloyd and Lipow is incomplete and then presents a supplementary method to deal with that problem.[7] Lloyd predicts reliability growth using a binomial model when reliability requirements are high and testing data are limited. The author also assumes that when a corrective action of a fault system is implemented and the failures should not be carried as full failures in the subsequent reliability estimates.[8] The Gompertz equation to predict reliability growth of a product is used by Vlrene[9] and the modified Gompertz reliability growth model is presented by Kececiodlu to predict a reliability growth model with s-shaped curve.[10] However, an approximate fixed proportion of maximum attainable reliability at inflection point of a curve with s-shaped and a highly initial reliability value setting may be not impractical for some equipment. In addition to these models mentioned above, there are still other ones such as Bayesian methods[4, 11-13] and logical growth curve[2, 14], artificial neural network[15-16] etc.

As natures of equipment vary from each other and then their reliability growth models may appear to distinct increasing trends. So it is difficult for us to find a reliability growth model fitting increasing trends well under all situations. However, it is common that because of expense, resources, schedule and other considerations[17] the data observed from a development testing program may be resulted in a characteristic of small sample size mostly. It will be difficult for traditional statistic methods to obtain good results of parametric estimators and reliability growth prediction values. In this paper we will present a new prediction method for reliability growth prediction based on an improved GM(1,1) model. This model can obtain a better prediction results especially for data of small sample sizes and without requirement of prior distribution assumption for observed reliability growth data in advance.

The remaining of this paper is organized as follows. A brief introduction to the original GM(1,1) model and the improved GM(1,1) model will be presented in section 2. Section 3 will be utilized to illustrate the method of applying the improved GM(1,1) model to predict reliability growth by a numerical example. Conclusions and future work focused mainly will be given in section 4.

2. Reliability Growth Prediction Method Based On an Improved Grey Model

Grey systems theory was proposed first in 1982 by Professor Julong Deng.[18] From then on this theory has been applied in a wide range of fields. Grey systems theory focuses on dealing with problems of uncertain systems such as systems with partial information known and partial information unknown or poor information systems. Grey systems theory can implement correct description and effective monitor of uncertain systems mainly by some calculations of sequence generation which can extract valuable information from partially known information.[19] GM(1,1) model is one of the important models in grey models group. And this type of models have been applied in a wide range of scientific fields such as natural science[20] and social science[21] etc. However, the time response function of original GM(1,1) model can not make full use of new pieces of information emphasized in grey systems theory. Then a newly initial condition is set in time response function of the improved GM(1,1) model to express the principle of new information priority fully in this paper.

2.1. ***The original GM(1,1) model***

Assume that

$$X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n)) \quad (1)$$

is a non-negative sequence of raw data, then

$$X^{(1)} = (x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(n)) \quad (2)$$

is a sequence generated from applying the first-order accumulated generation operator (1-AGO) to $X^{(0)}$, where,

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i),\ k = 1,2,\cdots,n. \quad (3)$$

In grey systems theory the following equation,

$$x^{(0)}(t) + az^{(1)}(t) = b \quad (4)$$

is called a grey differential equation, also denoted as GM(1,1) model.[18-19] Where,

$$z^{(1)}(t) = 0.5x^{(1)}(t) + 0.5x^{(1)}(t-1),$$

$$t = 2,3,\cdots,n. \quad (5)$$

If we assume that

$$Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}, B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix} \quad (6)$$

then we can obtain the parametric estimators according to least square method as following,

$$\hat{u} = [a,b]^T = (B^T B)^{-1} B^T Y \quad (7)$$

$$\begin{cases} a = \dfrac{\sum_{k=2}^{n} x^{(0)}(k) \cdot \sum_{k=2}^{n} z^{(1)}(k) - (n-1) \cdot \sum_{k=2}^{n} x^{(0)}(k) \cdot z^{(1)}(k)}{(n-1)\sum_{k=2}^{n} [z^{(1)}(k)]^2 - [\sum_{k=2}^{n} z^{(1)}(k)]^2} \\ b = \dfrac{1}{(n-1)} \cdot [\sum_{k=2}^{n} x^{(0)}(k) + a \cdot \sum_{k=2}^{n} z^{(1)}(k)] \end{cases}$$

(8)

Proof is omitted.[19] □

The equation

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b \quad (9)$$

is called the whitened equation of the GM(1,1) model. And the time response function of the whitened equation yields,

$$x^{(1)}(t) = (x^{(1)}(1) - \frac{b}{a})e^{-a(t-1)} + \frac{b}{a} \quad (10)$$

Then the time response function of the GM(1,1) model is given below[21,22],

$$\hat{x}^{(1)}(k+1) = (x^{(0)}(1) - \frac{b}{a})e^{-ak} + \frac{b}{a} \quad (11)$$

The restored values of raw data can be given by,

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k)$$

$$= (1-e^{a})(x^{(0)}(1) - \frac{b}{a})e^{-ak} \quad (12)$$

This Eq. (12) can be used to simulate or predict the sequence of raw data. It should be noted that we can utilize the first-order accumulated generation operator several times in accordance with the smooth condition of raw data to increase prediction precision of GM(1,1) model. However, we should restore the predicted values at corresponding times for obtaining prediction values of raw data. Moreover, we can find that the initial condition in time response function of the original GM(1,1) model is the first item in a sequence generated from applying the first-order accumulated generation operator to $X^{(0)}$. And this type of initial condition in time response function can not take full advantage of new pieces of information in raw data. In addition, the principle of new information priority can not be expressed fully by this type of initial condition. Then we propose a newly initial condition which can express this principle well in time response function in next section.

2.2. ***An improved GM(1,1) model based on the newly initial condition***

From the construction procedure of the original GM(1,1) model we can find that the time response function of the whitened equation can be expressed as following,

$$x^{(1)}(t) = ce^{-at} + \frac{b}{a} \tag{13}$$

where c is a constant, and $t = 1,2,\cdots,n$; parameters a and b can be derived from the least square estimation as mentioned above.

For Eq. (13), if we let $t = 1$, then Eq. (14) can be obtained,

$$x^{(1)}(1) = ce^{-a} + \frac{b}{a} \tag{14}$$

And if we let $t = n$, then Eq. (15) can be derived,

$$x^{(1)}(n) = ce^{-an} + \frac{b}{a} \tag{15}$$

To make full use of new pieces of information in raw data and also reserve the initial condition in time response function of the original GM(1,1) model, then we set a newly initial condition in time response function equaling to $0.5(x^{(1)}(1) + x^{(1)}(n))$. And the constant c can be derived in accordance with (14) and (15) as shown in Eq. (16),

$$c = 2(e^{-a} + e^{-an})^{-1}(\frac{x^{(1)}(1) + x^{(1)}(n)}{2} - \frac{b}{a}). \tag{16}$$

Then the time response function of the improved GM(1,1) model is given by,

$$x^{(1)}(t) = \frac{2}{1+e^{-a(n-1)}}(\frac{x^{(1)}(1) + x^{(1)}(n)}{2} - \frac{b}{a})e^{-a(t-1)} + \frac{b}{a} \tag{17}$$

And the restored value of raw data yields,

$$\hat{x}^{(0)}(t) = 2(1-e^{a})(1+e^{-a(n-1)})^{-1} \times (\frac{x^{(1)}(1) + x^{(1)}(n)}{2} - \frac{b}{a})e^{-a(t-1)} \tag{18}$$

We can utilize Eq. (18) to simulate or predict values of raw data. This newly initial condition in time response function is comprised of the first item and the last item from a sequence derived from applying 1-AGO to the sequence of raw data. This type of initial condition can preserve not only the format of initial condition in original time response function but also make full use of new pieces of information by the expression of $x^{(1)}(n)$.

3. Numerical Example

In this section we will demonstrate an application of the

Table 1. Predicted reliability from the Lloyd-Lipow model and the improved GM(1,1) model.

Number of test stage, k	Number of tests in k[th] states, N_k	Number of successful tests in k[th] stages, S_k	Reliability calculated from raw data, S_k/N_k, R	Predicted reliability from Lloyd-Lipow model, $\hat{R}_1$	Predicted reliability from improved GM(1,1) model, $\hat{R}_2$
1	10	5	0.500		
2	8	5	0.625	0.637	0.6391
3	9	6	0.667	0.700	0.6522
4	9	7	0.778	0.732	0.6656
5	10	6	0.600	0.751	0.6792
6	10	7	0.700	0.764	0.6932
7	10	8	0.800	0.773	0.7074
8	10	7	0.700	0.780	0.7219
9	10	6	0.600	0.785	0.7367
10	11	7	0.636	0.789	0.7518
11	10	9	0.900	0.793	0.7672
12	11	10	0.909	0.795	0.7829
13	12	9	0.750	0.798	0.7990
14	10	8	0.800	0.800	0.8150
15	10	7	0.700	0.802	0.8321
16	10	8	0.800	0.803	0.8492
17	10	9	0.900	0.805	0.8666
18	10	9	0.900	0.805	0.8843
19	10	10	1.000	0.807	0.9025
20	10	9	0.900	0.808	0.9210

improved GM(1,1) model by predicting reliability growth curves. The observed data and predicted reliability ones from Lloyd-Lipow model citing from Kececioglu[2] are shown as table 1.

According to the reliability values calculated from raw data we can obtain parameter estimators and time response function of the improved GM(1,1) model as following,

$$a = -0.0203, b = 0.6225, t = 2,3,\cdots,20.$$

$$x^{(1)}(t) = 31.1640e^{0.0203(t-1)} - 30.6650 \quad (19)$$

From the time response function (19) we can derive the restored values of raw data easily as shown in table 1. In table 1 the predicted reliability values from the Lloyd-Lipow model[2] are also shown for the aim of comparison with the predicted reliability values from the improved GM(1,1) model. The reliability growth curves derived from the Lloyd-Lipow model and the improved GM(1,1) model are depicted in Fig. 1 respectively.

For the reliability growth curve from the Lloyd-Lipow model in Fig. 1 we can find that reliability values in initial testing stages are overestimated mostly. However, reliability values in later testing stages increase slowly which can not fit the increasing trend of reliability values from raw data well. Furthermore, for the reliability growth curve from the improved GM(1,1) model we can find that reliability values in the whole testing stages can fit the increasing trend of reliability values from raw data better than those from the Lloyd-Lipow model. This reliability growth curve may be seen as a good fit to the actual reliability growth data. To compare with predicting efficiency of the Lloyd-Lipow model and the improved GM(1,1) model more intuitively, relative errors of predicted reliability values in all testing stages are calculated in table 2 and depicted in Fig. 2, respectively.

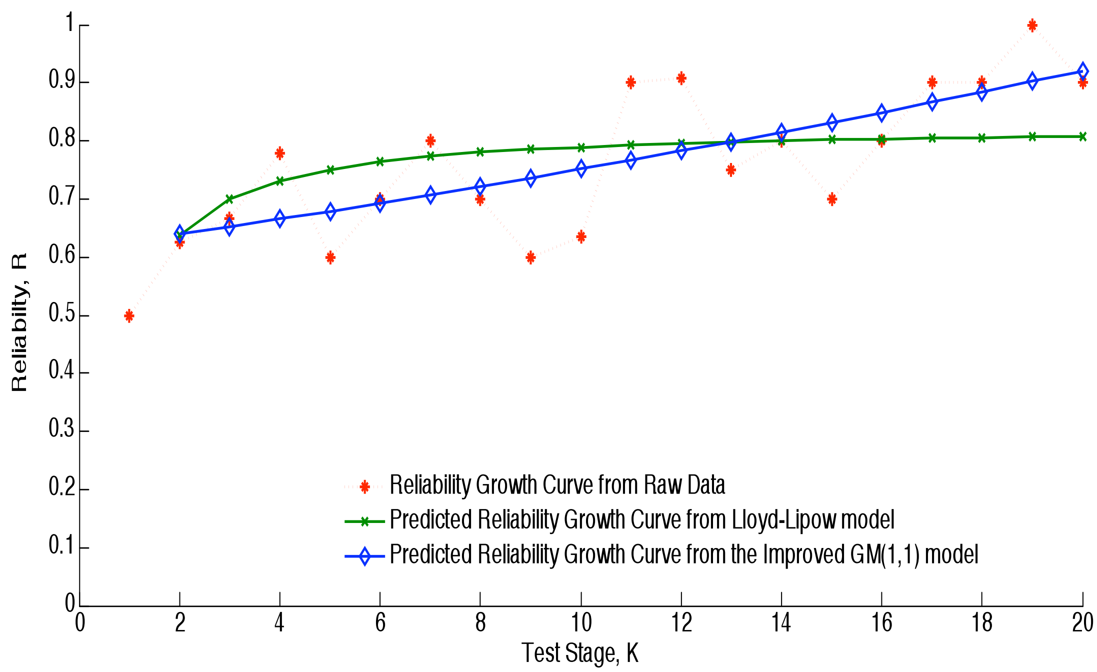

Fig. 1. Reliability growth curves predicted from the Lloyd-Lipow model and the improved GM(1,1) model.

Table 2. Relative errors of predicted reliability values from the Lloyd-Lipow model and the improved GM(1,1) model.

Number of test stage, k	Reliability calculated from raw data, R	Predicted reliability from Lloyd-Lipow model, $\hat{R}_1$	Residual error from Lloyd-Lipow model, ε_1	Relative error from Lloyd-Lipow model, r_1, %	Predicted reliability from improved GM(1,1) model, $\hat{R}_2$	Residual error from improved GM(1,1) model, ε_2	Relative error from improved GM(1,1) model, r_2,%
1	0.500						
2	0.625	0.637	-0.0120	-1.9200	0.6391	-0.0141	-2.2560
3	0.667	0.700	-0.0330	-4.9475	0.6522	0.0148	2.2189
4	0.778	0.732	0.0460	5.9126	0.6656	0.1124	14.4473
5	0.600	0.751	-0.1510	-25.1667	0.6792	-0.0792	-13.2000
6	0.700	0.764	-0.0640	-9.1429	0.6932	0.0068	0.9714
7	0.800	0.773	0.0270	3.3750	0.7074	0.0926	11.5750
8	0.700	0.780	-0.0800	-11.4286	0.7219	-0.0219	-3.1286
9	0.600	0.785	-0.1850	-30.8333	0.7367	-0.1367	-22.7833
10	0.636	0.789	-0.1530	-24.0566	0.7518	-0.1158	-18.2075
11	0.900	0.793	0.1070	11.8889	0.7672	0.1328	14.7556
12	0.909	0.795	0.1140	12.5413	0.7829	0.1261	13.8724
13	0.750	0.798	-0.0480	-6.4000	0.7990	-0.0490	-6.5333
14	0.800	0.800	0.0000	0.0000	0.8150	-0.0150	-1.8750
15	0.700	0.802	-0.1020	-14.5714	0.8321	-0.1321	-18.8714
16	0.800	0.803	-0.0030	-0.3750	0.8492	-0.0492	-6.1500
17	0.900	0.805	0.0950	10.5556	0.8666	0.0334	3.7111
18	0.900	0.805	0.0950	10.5556	0.8843	0.0157	1.7444
19	1.000	0.807	0.1930	19.3000	0.9025	0.0975	9.7500
20	0.900	0.808	0.0920	10.2222	0.9210	-0.0210	-2.3333

Where, residual error (ε_i) and relative error (r_i) are denoted as below,

$$\varepsilon_i = R_i - \hat{R}_i,\ r_i = \frac{\varepsilon_i}{R_i} \times 100\%,\ i = 1,2\,. \quad (20)$$

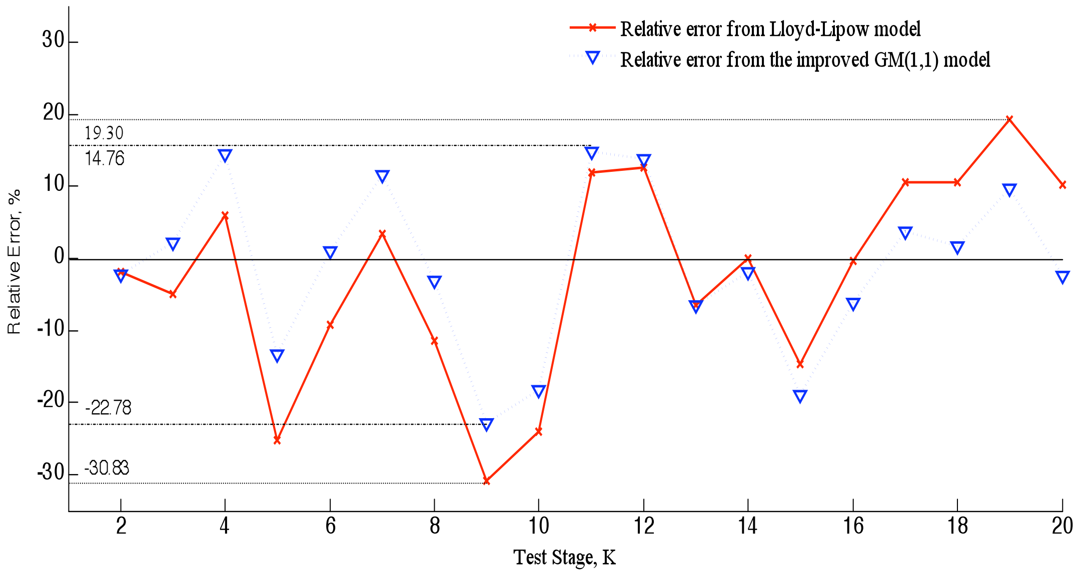

Fig. 2. Relative errors of predicted reliability values from the Lloyd-Lipow model and the improved GM(1,1) model.

Relative error is a good means to evaluate how well the predicted result is tracking raw datum in each testing stage. It is desirable for relative error to have error as close to zero as possible. From Fig. 2 we can find that relative errors from the improved GM(1,1) model fluctuate in the interval [-22.78%, 14.76%] and relative errors from the Lloyd-Lipow model fluctuate in the interval [-30.8333%, 19.30%]. The relative errors from the improved GM(1,1) model are much closer around zero than those from the Lloyd-Lipow model. In addition, if we take the absolute of relative error in table 2 and we can obtain the average absolute of relative error ($\bar{r}_1$) for the Lloyd-Lipow model is 11.2207% and the average absolute of relative error ($\bar{r}_2$) for the improved GM(1,1) model is 8.8624%, respectively. Where,

$$\bar{r}_i = \frac{1}{19}\sum_{j=1}^{19}\left|r_{ij}\right|, i = 1,2; j = 1,2,\cdots,19 . \quad (21)$$

From comparison of average absolute of relative error we can also find that the result of reliability growth curve predicted from the improved GM(1,1) model is much better than that from the Lloyd-Lipow model.

4. Conclusion and Future Work

In this paper we present an improved GM(1,1) model to predict a reliability growth curve of equipment. As the limit of time, labors and expenses, it is difficult for us to collect adequately large sample sizes to evaluate reliability level within each testing stage. This also limits some applications of classical mathematical models to predict reliability growth with small sample sizes. Then we try to present and apply an improved grey prediction model to predict reliability growth with small sample sizes. The improved GM(1,1) model can make full use of new pieces of information in raw data to increase prediction precision by setting a newly initial condition in time response function. And the newly initial condition is a combination of the first item and the last item in a new sequence generated from applying 1-AGO to the sequence of raw data. And this newly initial condition can emphasize more about the impact of new pieces of information in raw data. This is more consistent with the principle of new information priority emphasized in grey systems theory. From the result of a numerical example analysis we can see that the improved GM(1,1) model can predict the reliability growth curve better than the Lloyd-Lipow model.

From long-term studies we find that there are several types of causes resulting in errors in GM(1,1) model.[23] In this paper we just consider to improve one of causes to increase prediction precision of GM(1,1) model. In addition to keep a continuous study on optimizing the initial condition in time response function we will focus mainly on improvements of other causes such as optimization of background value, optimization of grey derivative and a combining optimization of several causes to improve prediction accuracy of GM(1,1) model in future work and we will also apply these improved models to reliability growth prediction in practice.

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