Dynamic Output Feedback Consensus Control of Multi-agent Systems with Time-delay

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Abstract. This paper considers the consensus problem of a multi-agent system with time delay and proposes a distributed dynamic output feedback protocol. A sufficient condition in terms of linear matrix inequalities (LMIs) is given to ensure consensus of the multi-agent system.

Introduction

Over the past few decades, the consensus problem in multi-agent systems has received considerable attention from many fields such as physics, biology and control theory. And numerous results have been obtained for consensus problems of multi-agent systems. For example, a simple model is proposed in [1] for phase transition of a group of self-driven particles and complex dynamics of the model are demonstrated numerically. Jadbabaie et al.[2] studied the observed behavior reported in [1] and analyzed the alignment of an undirected network of agents with switching topologies which are periodically connected. Moreau[3] researched the nonlinear discrete-time multi-agent systems with time-dependent communication links, and introduced a novel method based on the notion of convexity. Olfati et al. [4] investigated a systematical framework of consensus problems with directed networks and undirected networks with fixed and switching topologies and time-delay and fixed topology, respectively. Later, Ren et al. [5] extended the conclusions of [2] and [4] and presented more loose conditions for consensus of information under dynamically changing interaction topologies.Owing to constraints on measurement or economic costs in practice, it is sometimes hard to measure the relative information of all states directly. However, only the relative information of all outputs is available. Thus, consensus protocols with dynamic output feedback (dynamic compensator) case are adopted in this paper. Because of each agent has limited capability of collecting information, the consensus protocol for each agent in descriptor multi-agent systems is distributed and only depends on the output information of the agent itself and its neighbours. Some researchers have investigated the consensus problem of the multi-agent systems through dynamic output-feedback approach[6]-[7]. However, the results about the consensus problem of the multi-agent systems with time delay through dynamic output-feedback approach are much less. In fact, time delays are ubiquitous phenomenon in multi-agent systems due to the congestion of communication channels and the finite transmission speed, which makes the dynamical behaviors of a network much more complicated. So it is necessary to consider the consensus problem of uncertain multi-agent systems with nonlinear dynamics and time delays.

Preliminaries

Let $\ell(\nu,\varepsilon,\rho)$ be an undirected graph of order n with the set of nodes $\nu=\{s_1,\cdots,s_n\}$, the set of undirected edges $\varepsilon\subseteq\nu\times\nu$, and a symmetric adjacency matrix $\rho=[a_{ij}]$, with nonnegative elements a_{ij}. The node indexes belong to a finite index set $\{1,\cdots,n\}$. The adjacency elements associated with edges of graph are positive, that is, $e_{ij}\in\varepsilon$ or $e_{ji}\in\varepsilon$ if and only if $a_{ij}=a_{ji}>0$. Moreover, it is

assumed that $a_{ii}=0$ for all $i\in\{1,\cdots,n\}$. The Laplacian corresponding to the graph ℓ is defined as $L=\lfloor l_{ij}\rfloor$, where $l_{ii}=\sum_j a_{ij}$, and $l_{ij}=-a_{ij}$, $i\neq j$.

Lemma 1[8]: If an undirected graph ℓ is connected, then its Laplacian L has the following properties:

1. Zero is a simple eigenvalue of L, and 1_n is the corresponding eigenvector, that is, $L1_n=0$.

2. The rest $n-1$ eigenvalues are all positive.

Lemma 2[9]: Let $L_c=\lfloor L_{c_{ij}}\rfloor\in R^{n\times n}$ be a symmetric matrix with $L_{c_{ij}}=\begin{cases}(n-1)/n, & i=j\\ -1/n, & i\neq j\end{cases}$

Then the following statements hold:

1. The eigenvalues of L_c are 1 with multiplicity $n-1$ and 0 with multiplicity 1. The vectors 1_n^T and 1_n are the left and the right eigenvectors of L_c associated with the zero eigenvalue, respectively.

2. There exists an orthogonal matrix $U=\begin{bmatrix}U_1 & \overline{U}_1\end{bmatrix}\in R^{n\times n}$ with $\overline{U}_1=\frac{1}{\sqrt{n}}1_n$, such that $U^TL_cU=\begin{bmatrix}I_{n-1} & 0\\ 0 & 0\end{bmatrix}$ holds. Let $L\in R^{n\times n}$ be the Laplacian matrix of any given undirected graph, then $U^TLU=\begin{bmatrix}L_1 & 0\\ 0 & 0\end{bmatrix}$ holds, where $L_1\in R^{(n-1)\times(n-1)}$ is positive definite if and only if the corresponding graph is connected.

Consider the multi-agent system consisting of n identical agents. Suppose the ith agent has the following dynamics,

$$\begin{aligned}\dot{x}_i(t)&=Ax_i(t)+Bu_i(t)\\ y_i(t)&=Cx_i(t),\quad i=1,\cdots,n\end{aligned}\tag{1}$$

where $x_i(t)\in R^m$, $u_i(t)\in R^p$ and $y_i(t)\in R^q$ are the state, the protocol and the measured output of the agent i, respectively.

Define $z_i(t)=x_i(t)-\frac{1}{n}\sum_{j=1}^{n}x_j(t),\quad i=1,\cdots,n$ as controlled output fuction to measure the disagreement of $x_i(t)$ to the average state of all agents. It is obvious that consensus of the multi-agent system can be achived if and only if $\lim_{t\to+\infty}z_i(t)=0$, for all $i=1,\cdots,n$.

Owing to constraints on measurement, it is sometimes hard to measure the relative information of all states directly. However, only the relative information of all outputs is available. Thus, consensus protocols with dynamic output feedback case are adopted in this paper.

We can design a following distributed dynamic output feedback protocol with time delay.

$$\begin{aligned}\dot{v}_i(t)&=A_Kv_i(t)+B_K\sum_{j=1}^{n}a_{ij}\left(y_i(t-\tau)-y_j(t-\tau)\right)\\ u_i(t)&=C_Kv_i(t)+D_K\sum_{j=1}^{n}a_{ij}\left(y_j(t-\tau)-y_j(t-\tau)\right),\quad i=1,\cdots,n\end{aligned}\tag{2}$$

where $v_i(t)\in R^s$ is the state of the dynamic output feedback controller, and a_{ij} is the adjacency weights of the interaction graph ℓ.

Substituting protocol (2) into the system (1), we can obtain the following closed loop system

$$\begin{aligned}\overline{\xi}(t)&=\overline{A}\,\overline{\xi}(t)+\overline{A}_1\overline{\xi}(t-\tau)\\ z(t)&=\overline{C}\,\overline{\xi}(t)\end{aligned}\tag{3}$$

where $\overline{A}=\begin{bmatrix}I_n\otimes A & I_n\otimes B_1C_K\\ 0 & I_n\otimes A_K\end{bmatrix}$, $\overline{A}_1=\begin{bmatrix}L\otimes B_1D_KC & 0\\ L\otimes B_KC & 0\end{bmatrix}$, $\overline{C}=\begin{bmatrix}L_c\otimes I_m & 0\end{bmatrix}$

$v(t)=\left[v_1^T(t)\cdots v_n^T(t)\right]^T$, $\bar{\xi}(t)=\left[x^T(t)\quad v^T(t)\right]^T$, L is the Laplacian of graph ℓ.

We can see that L is a singular matrix, therefore the closed loop system is uncontrollable if the given matrix A is unstable. So we must make a model transformation of the closed loop system.

Let $\hat{x}(t)=x(t)-1_n\otimes\frac{1}{n}\sum_{j=1}^{n}x_j(t)=(L_c\otimes I_m)x(t)$, $\hat{v}(t)=(L_c\otimes I_s)v(t)$, $\hat{x}(t-\tau)=(L_c\otimes I_m)x(t-\tau)$.

Notice $L_c1_n=0$, $L1_n=0$. Then the closed loop system (3) can be rewrite the following form.

$$\begin{aligned}\dot{\hat{\xi}}(t)&=\hat{A}\hat{\xi}(t)+\hat{A}_1\hat{\xi}(t-\tau)\\ z(t)&=\hat{C}\hat{\xi}(t)\end{aligned}\tag{4}$$

where $\hat{A}=\begin{bmatrix}L_c\otimes A & L_c\otimes B_1C_K\\ 0 & L_c\otimes A_K\end{bmatrix}$, $\hat{A}_1=\begin{bmatrix}L_cL\otimes B_1D_KC & 0\\ L_cL\otimes B_KC & 0\end{bmatrix}$, $\hat{C}=\left[L_c\otimes I_m\quad 0\right]$

$U^TL_cU=\begin{bmatrix}I_{n-1} & 0\\ 0 & 0\end{bmatrix}$, $U^TLU=\begin{bmatrix}L_1 & 0\\ 0 & 0\end{bmatrix}$

Let $\tilde{x}(t)=(U^T\otimes I_m)\hat{x}(t)$, $\tilde{v}(t)=(U^T\otimes I_s)\hat{v}(t)$, $\tilde{z}(t)=(U^T\otimes I_m)z(t)$.

Then the closed loop system (4) can be rewrite the following form.

$$\begin{aligned}\dot{\tilde{\xi}}(t)&=\tilde{A}\tilde{\xi}(t)+\tilde{A}_1\tilde{\xi}(t-\tau)\\ \tilde{z}(t)&=\tilde{C}\tilde{\xi}(t)\end{aligned}\tag{5}$$

where $\tilde{A}=\begin{bmatrix}U^TL_cU\otimes A & U^TL_cU\otimes B_1C_K\\ 0 & U^TL_cU\otimes A_K\end{bmatrix}$, $\tilde{A}_1=\begin{bmatrix}U^TL_cLU\otimes B_1D_KC & 0\\ U^TL_cLU\otimes B_KC & 0\end{bmatrix}$, $\tilde{C}=\left[U^TL_cU\otimes I_m\quad 0\right]$

Main results

Then we can derive an reduced-order system as follows.

$$\begin{aligned}\dot{\xi}'(t)&=A'\xi'(t)+A_1'\xi'(t-\tau)\\ z'(t)&=C'\xi'(t)\end{aligned}\tag{6}$$

where, $A'=\begin{bmatrix}I_{n-1}\otimes A & I_{n-1}\otimes B_1C_K\\ 0 & I_{n-1}\otimes A_K\end{bmatrix}$, $A_1'=\begin{bmatrix}L_1\otimes B_1D_KC & 0\\ L_1\otimes B_KC & 0\end{bmatrix}$, $C'=\left[I_{n-1}\otimes I_m\quad 0\right]$

Theorem 1: Consider the network with an undirected interaction graph ℓ that is connected. The system (6) is asymptotically stable, if there exist a dynamic output feedback $u(t)$ and positive definite matrices $P_i\in R^{(m+s)\times(m+s)}$, $i=1,\cdots,n-1$, such that the following LMIs are satisfied for $i=1,\cdots,n-1$.

$$\Xi=\begin{bmatrix}\begin{bmatrix}A & B_1C_K\\ 0 & A_K\end{bmatrix}^TP+P\begin{bmatrix}A & B_1C_K\\ 0 & A_K\end{bmatrix}+S & P\begin{bmatrix}\lambda_iB_1D_KC & 0\\ \lambda_iB_KC & 0\end{bmatrix}\\ * & -S\end{bmatrix}<0$$

Proof: By Lemma 2, the matrix L_1 is positive definite. Therefore, there exists an orthogonal matrix $Q\in R^{(n-1)\times(n-1)}$ such that $Q^TL_1Q=diag\{\lambda_1,\cdots,\lambda_{n-1}\}$. Let $x''(t)=(Q^T\otimes I_m)x'(t)$, $v''(t)=(Q^T\otimes I_s)v'(t)$, $z''(t)=(Q^T\otimes I_m)z'(t)$. Then the system (6) can be rewrited in the following form.

$$\begin{aligned}\dot{\xi}''(t)&=A''\xi''(t)+A_1''\xi''(t-\tau)\\ z''(t)&=C''\xi''(t)\end{aligned}\tag{7}$$

where, $A'' = \begin{bmatrix} I_{n-1} \otimes A & I_{n-1} \otimes B_1 C_K \\ 0 & I_{n-1} \otimes A_K \end{bmatrix}$, $A_1'' = \begin{bmatrix} diag\{\lambda_1, \cdots \lambda_n\} \otimes B_1 D_K C & 0 \\ diag\{\lambda_1, \cdots, \lambda_n\} \otimes B_K C & 0 \end{bmatrix}$, $C'' = \begin{bmatrix} I_{n-1} \otimes I_m & 0 \end{bmatrix}$.

We can decompose the system (7) to the following $n-1$ subsystems.

$$\begin{aligned} \dot{\xi}_i''(t) &= A_i'' \xi_i''(t) + A_{1i}'' \xi_i''(t-\tau) \\ z_i''(t) &= C_i'' \xi_i''(t) \end{aligned}, \quad i = 1, \cdots, n \tag{8}$$

where, $A_i'' = \begin{bmatrix} A & B_1 C_K \\ 0 & A_K \end{bmatrix}$, $A_{1i}'' = \begin{bmatrix} \lambda_i \otimes B_1 D_K C & 0 \\ \lambda_i \otimes B_K C & 0 \end{bmatrix}$, $C_i'' = \begin{bmatrix} I_m & 0 \end{bmatrix}$

If for all $i = 1, \cdots, n$, system (8) is asymptotically stable, system (6) is asymptotically stable.

Consider the following Lyapunov-Kasovskii function,

$$V(\xi_i''(t)) = \xi_i''^T(t) P \xi_i''(t) + \int_{t-\tau}^{t} \xi_i''^T(s) S \xi_i''(s) ds$$

If $\Xi < 0$, we can obtain that $\dot{V}(\xi_i''(t)) < 0$.

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Summary

The consensus problem of a multi-agent system with time delay is dealt with through a distributed dynamic output feedback protocol. A sufficient condition in terms of linear matrix inequalities (LMIs) is obtained to ensure consensus of the multi-agent system.

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