An Improved Adaptive Federal Kalman Filter Algorithm For Integrated Navigation

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Abstract. Integrated navigation system has become an important navigation system. On the basis of previous research about the federal Kalman filter, this paper mainly studied the key technologies of low-cost SINS/GPS/Magnetic compass integrated navigation system, an adaptive federal filter method of vector-form information distribution coefficients is proposed and compared with ordinary federal Kalman filter and the adaptive federal filter based on filter covariance matrix eigenvalue decomposition. The simulation results show that the proposed algorithm can get higher accuracy of navigation.

Introduction

With the development of modern technology, single navigation system is difficult to meet the needs of people. Strapdown inertial navigation system neither needs external information nor radiates any electromagnetic waves, but its error accumulates over time [1]. GPS can provides high precision navigation information all-weather and all the time, but it has a low output frequency, and it is easy to lose signals when there are lots of obstacles [2]. Compass has a wide measuring range, high stability and anti-jamming characteristics, it can provides high-precision heading information with accelerometer. However, it is easy to be disturbed by external magnetic field [3]. Therefore, SINS/GPS/Magnetic compass are integrated to increase the precision of the navigation system and meet the demands of navigation system.

Federal Kalman filter has a great advantages such as fast calculation, strong fault tolerance and convenient for the fault detection and isolation of subsystems, but it is still based on a single model of filter. With the change of system or environment, estimate error will increase and even diverge. An adaptive federal filter method of vector-form information distribution coefficients is proposed and compared with ordinary federal Kalman filter and the adaptive federal filter based on filter covariance matrix eigenvalue decomposition through navigation experiment.

Principle of Adaptive Federal Kalman Filter

In recent years, the application of Kalman filter techniques in integrated navigation system achieved great development [4]. The emergence of new technologies include: the federal extended Kalman filter, the unscented Kalman filter and the adaptive Kalman filter technique, etc [5, 6].

Federal Kalman filter theory is a kind of special form of distributed Kalman filter method, its general principle and basic design process are as seen in [7, 8]. Analysis shows that performance of federal Kalman filter is depend on sub-filter’s information distribution coefficient value $\beta_i$ and main filter’s information distribution coefficient value $\beta_m$. In fact, performance of navigation equipment and sub-filter may be affected by environmental factors, which will affect the estimation precision of the whole system.

In order to improve the performance of federal Kalman filter, a variety of dynamic information distribution methods are proposed, such as method based on filter covariance matrix eigenvalue decomposition, method based on observable matrix condition number [9, 10], etc. Information distribution coefficients of the above researches are based on scalar form. They treat each element
of the state variables as the same. In fact, different sensors have different functional properties and accuracy, state equation and observation equation established by sub filters are also different.

Accordingly, existing methods with scalar distribution coefficients are difficult to timely reflect dynamic characteristics of each state variable. In view of the above situations, an adaptive federal filter method of vector-form information distribution coefficients is proposed.

1. Solution of $M_i$

Covariance matrix of subsystem $P_i$ is decomposed with characteristic values:

$$P_i = L_i \Lambda_i L_i^T$$

where $\Lambda_i = diag \{ \lambda_{i1}, \lambda_{i2}, \cdots, \lambda_{in} \}$. $\lambda_{i1}, \cdots, \lambda_{in}$ are the characteristic values of $P_i$, $n$ is the order number of matrix $P_i$, $L_i$ is the matrix that decomposed by eigenvalues of matrix $P_i$, $X_y$ is the j-th element of the error state in the i-th sub filter. Information distribution coefficients are calculated as follows:

$$m_{ij} = \frac{1}{\lambda_{ij}}$$

where $i = 1, 2, \cdots, N$, $j = 1, 2, \cdots, n$, $\lambda_{ij}$ is the characteristic value that corresponded with the state variable $X_{ij}$, $N$ is the number of subsystem.

Information distribution coefficients matrix corresponded to $X_i$ is:

$$M_i = diag \{ m_{i1}, m_{i2} \cdots m_{in} \}$$

2. Solution of $Y_i$

If $T \ (T \in \mathbb{R}^{p \times q})$ is the observation matrix of system, then it can be decomposed with singular values as follows:

$$T = USV^T$$

where $U$ is the left singular matrix of $T$, and it is the orthogonal matrix with order of $p \times p$, $U = [u_1 u_2 \cdots u_p]$, $u_1 u_2 \cdots u_p$ are the left singular column vectors. $S = \begin{bmatrix} \Lambda_{rr} & 0 \\ 0 & 0 \end{bmatrix}$, $\Lambda_{rr} = diag(\sigma_1 \sigma_2 \cdots \sigma_r)$, $\sigma_i (i = 1, 2, \cdots, r)$ respectively represent the rank and singular values of matrix $T$. $V$ is the right singular matrix of $T$, and it is the orthogonal matrix with order of $q \times q$, $V = [v_1 v_2 \cdots v_q]$, $v_1 v_2 \cdots v_q$ are the right singular column vectors.

Through the analysis of matrix $V$, this paper proposes that the right singular vector $v_i$ is corresponded with $\sigma_i$, and $\sigma_i$ is a singular value that corresponds with the maximum absolute values of the state variables which are gotten from $v_i$. From singular value decomposition of the subsystems’ observation matrix, each element’s information distribution coefficients of $X_i$ are calculated as:

$$Y_y = \begin{cases} \sigma_y & \sigma_y \neq 0 \\ \frac{\sigma_i + \sigma_j + \cdots + \sigma_{nj}}{\sigma_y} & \sigma_y = 0 \end{cases}$$

where $\sigma_y$ is the singular value that corresponds with the state variable $X_y$, information distribution coefficients corresponded with matrix $X_i$ are:

$$Y_i = diag \{ y_{i1} y_{i2} \cdots y_{in} \}$$

3. Solution of $B_i$

$$B_i = \frac{1}{2} (M_i + Y_i) = diag \{ \beta_{i1} \beta_{i2} \cdots \beta_{in} \}$$
Meanwhile, the federal Kalman filter information distribution coefficients satisfy the information conservation principle:

$$\sum_{i=1}^{N} B_i = I_{w,n}$$  (8)

According to the federal filter information distribution coefficients of vector form, $Q(k)$ and $P(k)$ are distributed to each sub filter as follows:

$$P_i^{-1}(k) = \sqrt{B_i P_i^{-1}(k) B_i^T}$$

$$Q_i^{-1}(k) = \sqrt{B_i Q_i^{-1}(k) B_i^T}$$

$$\hat{X}_i(k) = \hat{X}_f(k)$$  (9)

**System Models**

SINS has many advantages such as strong autonomy, high reliability and comprehensive output information, so it is selected as the public reference system. GPS is selected as observation systems 1, and then SINS/GPS are combined to get the position sub filter and velocity sub filter. Magnetic compass is selected as observation systems 2, and then SINS/Magnetic compass are combined to get the posture sub filter [11]. System integration scheme is seen as figure 1.

![Fig.1. Structure of the integrated navigation system](image)

1. SINS/GPS
   (1) SINS/GPS position sub filter
   The state equation of position sub filter is:
   $$X_i = F_i X_i + G_i W_i$$  (11)
   State vector is selected as follows:
   $$X_i = [\delta \phi, \delta \phi, \delta \theta, \delta \psi, \delta V_x, \delta V_y, \delta V_z, \delta L, \delta \delta \epsilon_b, \epsilon_b, \epsilon_b, \epsilon_b, \epsilon_b, \epsilon_b, \epsilon_b, \delta L, \delta \lambda, \delta h_G]^T.$$  
   where $F_i, G_i, W_i$ respectively represent the state transition matrix, the noise driving matrix and the system noise. $W_i = \left[ w_{g_x}, w_{g_y}, w_{g_z}, w_{e_x}, w_{e_y}, w_{e_z}, w_{\phi_g}, w_{\phi_g}, W_{AG}, W_{AG}, W_{AG}, W_{AG} \right]^T$, state transition matrix:
   $$F_i = \text{diag}\{ F_G, F_G \}, F_G = \text{diag} \left[ -\frac{1}{\tau_{LG}}, -\frac{1}{\tau_{LG}}, \frac{1}{\tau_{LG}} \right].$$  
   Noise driving matrix: $G_i = \text{diag}\{ G_i, 1_{3 \times 3} \}$.

   The difference of position information between SINS and GPS is selected as the measurement quantity, the measurement model is:
   $$Z_i = \left[ \begin{matrix} L_i - L_G \\ \lambda_i - \lambda_G \\ h_i - h_G \end{matrix} \right] = H_i X_i + V_i$$  (12)
   where $H_i = \left[ 0_{3 \times 1}, \text{diag}(R_i \cos \phi, R_i, 1) \right] \cdot 0_{3 \times 3}, V_i = \left[ N_e, N_n, N_z \right]^T$ is zero-mean white measurement noise of GPS.
(2) SINS/GPS velocity sub filter
State vector is selected as follows:
\[
X_2 = [\delta \phi, \delta \phi, \delta \theta, \delta \phi, \delta \theta, \delta \phi, \delta \theta, \delta L, \delta \lambda, \delta \phi, \delta \phi, \delta \phi, \delta \phi, \delta \phi, \delta \phi, \delta \phi, \delta \phi, \delta \phi, \delta \phi] \top.
\]
The state equation of velocity sub filter is:
\[
\dot{X}_2 = F_2 X_2 + G_2 W_2
\]
where \(F_2 = F_1, \quad G_2 = G_1, \quad W_2 = \begin{bmatrix} w_{xz} & w_{yz} & w_{zz} & w_{xw} & w_{yw} & w_{zw} \end{bmatrix} \).

The measurement vector \(Z_2\) is the velocity difference, which is written as:
\[
Z_2 = \begin{bmatrix} V_{le} - V_{Gx} \\ V_{ln} - V_{Gn} \\ V_{ls} - V_{Gs} \end{bmatrix} = H_2 X_2 + V_2
\]
where \(H_2 = [0, 0, 0], V_1 = [M, M, M] \top\) is zero-mean white measurement noise of GPS.

2. SINS/Magnetic compass
The state equation of attitude sub filter is:
\[
X_3 = F_3 X_3 + G_3 W_3
\]
where \(X_3 = X_2, \quad F_3 = F_2, \quad G_3 = G_2, \quad W_3 = W_2.\)

The measurement vector \(Z_3\) is the attitude difference, which is written as:
\[
Z_3 = \begin{bmatrix} \phi_{le} - \phi_{re} \\ \phi_{ln} - \phi_{re} \\ \phi_{ls} - \phi_{re} \end{bmatrix} = H_3 X_3 + V_3
\]
where \(H_3 = [I, 0, 0], V_3 = [P, P, P] \top\) is zero-mean white measurement noise of Magnetic compass.

Matlab Simulation Experiment
In order to verify the effectiveness and accuracy of the proposed algorithm, field vehicle experiment was carried out. Three filter methods are compared through MATLAB simulation. In the process of experiment, sample frequency of SINS, GPS and Magnetic compass are set to 10Hz. Filter frequency of the main filter is 1Hz and the total filter time is about 35 minutes. Initial state of the system is as follows, velocity: 0m/s, pitch angle: -0.6354deg, roll angle: 0.5631deg, course angle: 89.92deg, longitude: 116.3552deg, latitude: 39.9901deg, altitude: 41.5938m. In order to compare the performance of the integrated navigation system, a high precision gyroscope is selected as the benchmark, and the errors between them are shown in the figures below.

![Fig.2. Attitude error](image1)

![Fig.3. Position error](image2)

![Fig.4. Velocity error](image3)

According to above figures, black line represents ordinary federal Kalman filter, blue line represents the adaptive federal filter based on filter covariance matrix eigenvalue decomposition, red line represents the adaptive federal filter method of vector-form information distribution coefficients. It is obvious that the errors of attitude, position and velocity from the proposed algorithm is more smooth and lower than ordinary federal Kalman filter. When compare the improved adaptive federal Kalman filter with the adaptive federal Kalman filter based on filter...
covariance matrix eigenvalue decomposition, the overall performance improvement is not obvious, but its convergence is better. GPS lose signals when vehicle entered into the floor shielding zone in the twentieth minute and left in the twenty-fifth minute, but the integrated navigation system is not affected by it, and still remain high navigation accuracy. It reflects a good fault tolerant performance and improves the system's dynamic response ability and anti-interference ability.

Conclusion

Inertial navigation, magnetic compass and GPS have become an important part of the integrated navigation system. An adaptive federal filter method of vector-form information distribution coefficients used in integrated navigation system can improve the system’s accuracy effectively. Reliability, stability and anti-interference ability of the integrated navigation system are improved.

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