Effective Detuning of a Three-Level Atom Interacted with the Laguerre-Gaussian Laser Beam

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Abstract—We derive the general analytic expression of the effective detuning of the linearly polarized light field from the Λ-configuration three-level atomic mode. The detuning amending term resulting from the three-level atomic mode is not related with the type of light field, which is determined by the two lower level split and the ratio of the two lower level dipole matrix element to the total dipole matrix element in the three-level atomic mode. For a linearly polarized Laguerre-Gaussian laser beam, we show that there is same frequency amending for the axial, radial and azimuthal frequency detuning. We define an effective two-level atomic mode to substitute the three-level atomic mode, in which it can be more practical and convenient in consideration of the light detuning and Doppler shift from atomic transition frequency.

Keywords-three-level atomic mode, laguerre-Gaussian laser beam, light detuning, Doppler shift

I. INTRODUCTION

In 1936, Beth first experimentally observed the deflection of a quartz wave plate suspended from a thin quartz fiber with circularly polarized light passing through it [1]. In 1992, Allen et al. observed the torque on suspended cylindrical lenses arising from the reversal of the helicity of a Laguerre-Gaussian mode with well-defined orbital angular momentum [2]. Recently, the transfer of angular momentum from a light beam carrying angular momentum to absorbing macroscopic particles trapped in an optical tweezer causing them to rotate has been experimentally and theoretically studied [3-6]. Furthermore, an atom moving in a Laguerre-Gaussian beam experiences a torque and an azimuthal shift in its resonant frequency in addition to the usual axial Doppler shift and recoil shift [7-8]. Where they considered a two-level atom which is interacted with the Laguerre-Gaussian beam and obtained the azimuthal Doppler shift as a complementarity for the total detuning of the beam frequency from the atomic resonant frequency.

In this paper, we extend the two-level atomic mode to the three-level atomic mode because the three-level atom has been received considerable attention for creating many new quantum effects [9-10]. First we consider a Λ-configuration three-level atom in a light beam and derive the general expression of the detuning of the beam from the three-level atom. Then taking a linearly polarized Laguerre-Gaussian for an example, we derive the total detuning of the Laguerre-Gaussian beam from the three-level atom. Finally, for the three-level atomic mode we derive an effective detuning and an effective two-level atomic mode.

II. CALCULATION AND DISCUSSION

We consider a Λ-configuration three-level atom with one upper level [3] and two lower levels [1], [2] with frequency split Δ, as shown in Fig.1, which is interacted with an arbitrary light field with the frequency ω. The atomic transition between [1] and [3], [2] and [3] are assumed to be optically allowed, while the transition between [1] and [2] is forbidden, which can be resulted from the D₁, D₂ line of alkali-metal atoms.

Figure 1. The Λ-configuration three-level atom diagram

The Hamiltonian of our system is written as [9]

\[ H = \frac{\mathbf{p}^2}{2m} + U(\mathbf{r}) + \hbar \omega_1 |1 \rangle \langle 1 | + \hbar \omega_2 |2 \rangle \langle 2 | + \hbar \omega_3 |3 \rangle \langle 3 | + \hbar \omega_4 a^\dagger a + H_{\text{int}} \] 

(1)

Where P and R are the momentum vector and the position vector of the atomic center of mass with atomic mass m, and U(R) is trapping potential. Where \( \hbar \omega_1 \), \( \hbar \omega_2 \), \( \hbar \omega_3 \) are eigenenergys of levels [1], [2], [3], respectively, and \( \omega_{13} = \omega_3 - \omega_1 \), \( \omega_{23} = \omega_3 - \omega_2 \) are atomic transition frequencies. \( a^\dagger \) (a) is the creation (annihilation) operator of light field \( \omega_4 \). The detuning of the laser frequency
from atomic transition \( \omega_{13} \) is \( \Delta_o \). The last part \( H_{int} \) is the interaction Hamiltonian, taking electronic dipole and rotating wave approximating, which in the interaction picture may be given by

\[
H_{int} = -i\hbar \left\{ e^{-i\omega t} \hat{\sigma}^+ a f_1(\mathbf{r},t) - f_1^*(\mathbf{r},t) a^\dagger e^{i\omega t} \right\} - i\hbar \left\{ e^{-i\theta_{13} t} \sigma^+ a f_2(\mathbf{r},t) - f_2^*(\mathbf{r},t) a^\dagger e^{i\theta_{13} t} \right\}
\]

Where transition operators \( \hat{\sigma}^+ \) and \( \hat{\sigma}^\dagger \) represent transitions from \([1]\) to \([3]\) and \([2]\) to \([3]\) respectively, while transition operators \( \hat{x} \) and \( \hat{\sigma} \) represent transitions from \([3]\) to \([1]\) and \([3]\) to \([2]\) respectively. The coupling parameters \( f_1(\mathbf{r},t) \) and \( f_2(\mathbf{r},t) \) can be written as

\[
f_1(\mathbf{r},t) = G_1(\mathbf{r}) e^{-i[\theta_{13}(\mathbf{r})/\Delta_o] t}, \quad (3.a)
\]

\[
f_2(\mathbf{r},t) = G_2(\mathbf{r}) e^{-i[\theta_{13}(\mathbf{r})/\Delta_o] t}. \quad (3.b)
\]

Where \( G_1(\mathbf{r}) = \frac{1}{\hbar} D_{13} \cdot \mathbf{e}(\mathbf{r}) \) and \( G_2(\mathbf{r}) = \frac{1}{\hbar} D_{23} \cdot \mathbf{e}(\mathbf{r}) \) correspond to position dependent Rabi frequency, \( \mathbf{e} \) and \( \theta_{13}(\mathbf{r}) \) are the light field amplitude and phase respectively, \( D_{13}(\mathbf{D}_{23}) \) is the dipole matrix element between levels \([1]\) and \([2]\) respectively. The total coupling parameter \( f(\mathbf{r},t) \) can be written as

\[
f(\mathbf{r},t) = f_1(\mathbf{r},t) + f_2(\mathbf{r},t). \quad (4)
\]

The time evolution of the total coupling parameter is derivable from the Heisenberg equation of motion, which is given as [7]

\[
df(\mathbf{r},t)/dt = i/\hbar [H, f(\mathbf{r},t)] = \frac{1}{2m} \left\{ \mathbf{P}, \mathbf{V} f(\mathbf{r},t) + \mathbf{V} f(\mathbf{r},t) \cdot \mathbf{P} \right\}. \quad (5)
\]

Here we have assumed that the coupling parameters \( f_1(\mathbf{r},t) \) and \( f_2(\mathbf{r},t) \) commute with \( U(\mathbf{R}) \).

From Eq.(5), we may derive the frequency detuning operator \( \delta \) including Doppler and recoil frequency shift, etc, which by considering the leading order is given by [7]

\[
\delta = -i \left\{ \frac{\mathbf{P} \cdot \nabla f(\mathbf{r},t) + f(\mathbf{r},t) \cdot \mathbf{P}}{f(\mathbf{r},t)} \right\}_0. \quad (6)
\]

Here the subscript zero denotes operators at the initial time \( t=0 \). Eq.(6) can be further written as

\[
\delta = -i \left\{ \frac{\mathbf{V} f(\mathbf{r},t) \cdot \mathbf{V} - \hbar \nabla^2 f(\mathbf{r},t)}{2mf(\mathbf{r},t)} \right\}_0. \quad (7)
\]

For the case of a linearly polarized along the x-axis plane wave, we assume \( D_{13} = D_{13} \hat{x} \), \( D_{23} = D_{23} \hat{x} \), \( \mathbf{e}(\mathbf{r}) = \hat{x} \mathbf{e} \), \( \theta(\mathbf{r}) = -k \cdot \mathbf{R} \), and obtain

\[
\left[ -i \frac{\mathbf{V} f(\mathbf{r},t) \cdot \mathbf{V}}{f(\mathbf{r},t)} \right]_0 = \Delta_0 + k \cdot \nabla + \frac{D_{23}}{D_{13} + D_{23}} \Delta_g \quad (8.a)
\]

\[
\left[ -i \frac{h \nabla^2 f(\mathbf{r},t)}{2mf(\mathbf{r},t)} \right]_0 = \frac{\hbar k^2}{2m} \left\{ \frac{\hbar k \Delta_0}{mv} \frac{D_{23} \Delta_g}{D_{13} + D_{23}} + \frac{\hbar \Delta_g^2}{2mv^2} \right\} + \frac{h(2\Delta_0 \Delta_g + \Delta_g^2)}{2mD_{13} + D_{23}}. \quad (8.b)
\]

We know from Eqs.(8.a), (8.b) that there are amending terms resulting from three-level atomic mode and related with two lower level split \( \Delta_g \), \( D_{13} \) and \( D_{23} \) for the Doppler shift \(-k \cdot \mathbf{v}\) and recoil shift \(\hbar k^2 / 2m\).

Then we consider the case of atoms moving in a Laguerre-Gaussian beam which can be readily produced in the laboratory [10]. In the paraxial approximation, assuming the Laguerre-Gaussian beam traveling in the \(-z\) direction and polarized in the \(+x\) direction and \(D_{13} = D_{23} \hat{x} \), \( D_{23} = D_{23} \hat{x} \), we derive the total coupling parameter \( f(\mathbf{r},t) = f_1(\mathbf{r},t) + f_2(\mathbf{r},t) \), and \( f_1(\mathbf{r},t) \), \( f_2(\mathbf{r},t) \) are expressed as

\[
f_1(\mathbf{r},t) = -i \omega_1 D_{13} u_{\rho}(r,z) \exp(-ikz) \exp(-il\phi) \exp(i\Delta_g t), \quad (9.a)
\]

\[
f_2(\mathbf{r},t) = -i \omega_2 D_{23} u_{\rho}(r,z) \exp(-ikz) \exp(-il\phi) \exp[i(\Delta_g + \Delta_r) t], \quad (9.b)
\]

where the complex scalar function \( u_{\rho}(r,z) \) describing the distribution of the field amplitude is given by

\[
u_{\rho}(r,z) = (-i)^l \left\{ \frac{e^{ir\phi}}{w(z)} \frac{L_l(\rho^2/w^2(z))}{w(z)} \right\} \times \exp \left[ -i \frac{kr^2}{2w^2(z)} \right] \times \exp \left[ -i \frac{(l+l_1)\phi}{(l+l_1)\phi} \right]. \quad (10)
\]

Where \( L_l^m \) are associated Laguerre polynomials, \( z_R \) is the Rayleigh range, \( w^2(z) = 2\left( z_R^2 + z^2 \right)/kz_R \) is the beam width at distance \( z \) from the beam waist. The integer indices \( l \) and \( p \) are quantum numbers characterizing the mode and \( l \) is the orbital angular momentum quantum number. From Eqs.(9) and (10), we obtain the expression of \( \theta(\mathbf{R}) \)

\[
\theta(\mathbf{R}) = -\frac{kr^2}{2(z_R^2 + z^2)} - l\phi - (2p + l + 1) \tan^{-1}(z_R/z) - k\phi. \quad (11)
\]

In Eq.(7), the recoil shift (i.e. the second term) is so smaller than the Doppler shift in the first term that the recoil term can be not considered further. Therefore, we only consider the first term including the Doppler shift of the Laguerre-Gaussian beam from the three-level atom:

\[
\delta_{DG} = -i \frac{\mathbf{V} f(\mathbf{r},t) \cdot \mathbf{V}}{f(\mathbf{r},t)} \bigg|_0 = \delta_{DG}^{(1)} - i\delta_{DG}^{(2)} \quad (12)
\]

Where
\[ \delta_{LG}^{(1)} = \frac{f(R,t)\nabla[\theta(R) + \Delta_{am}] \cdot v + f(R,t)\nabla[\theta(R) + \Delta_{g}] \cdot v}{f(R,t)} \]

\[ \delta_{LG}^{(2)} = \frac{\nabla G(R,t) \cdot v \exp[\theta(R) + \Delta_{am}] + \nabla G(R,t) \cdot v \exp[\theta(R) + \Delta_{g}]}{f(R,t)} \]  

To discuss further quantitatively about the detuning amending term \( \Delta_{am} = D_{23} \Delta_{g} / (D_{13} + D_{23}) \), we consider the relation between \( \Gamma \) and \( D \) \[ \Gamma = \omega D^{2} (3\pi \varepsilon_{0} c h^{3}) \] 

In conclusion, we have derived the general analytic expression for every kind of the light field. The origin of the effective detuning is the three-level atomic mode.
Gaussian laser beam, we find that there is same frequency amending for the axial, radial and azimuthal frequency detuning. We define an effective two-level atomic mode in which the upper level is $|3\rangle$ and the lower level locates between levels $|1\rangle$ and $|2\rangle$ and has the frequency interval $\Delta_{\text{am}}$ from level $|1\rangle$. The effective detuning plays an important role in detecting the laser detuning and Doppler shift from atomic transition frequency.

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REFERENCES


