

Synchronization of Light-Emitting Diodes Networks via Adaptive Neural Network Control

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Abstract

As ideal candidates for the implementation of a complex network on a chip, light-emitting diodes (LEDs) with optoelectronic feedback loop can display complex sequences of periodic mixed mode oscillations and chaotic spiking. This paper presents a neural network based feedback control scheme for synchronization of coupled LEDs networks. Based on Lyapunov stability theory, the controller can stabilize the synchronization error dynamics around the origin point, thus robust synchronization can be obtained. The effectiveness of the proposed control scheme is illustrated by a numerical example.

Keywords: chaos synchronization; light-emitting diode; complex dynamics; neural network control.

Introduction

Synchronization is ubiquitous for a population of dynamically interacting units and plays a very important role in various fields [1]. Various kinds of synchronization phenomena have been observed and studied, such as complete synchronization [2], phase synchronization [3], lag synchronization [4] and projective synchronization [5]. Recently, synchronization in complex networks has attracted an increasing attention [6]. In the real world, synchronization of complex networks can not only explain many natural phenomena, but also have many applications [6]. Therefore, various synchronization methods for complex networks have been presented, such as adaptive pinning control [7], and observer based control [8] and adaptive-impulsive control [9].

Recently, a GaAs light-emitting diode (LED) with ac-coupled nonlinear optoelectronic feedback has been shown to exhibit complex dynamics including mixed mode oscillations and chaos [10]. The effects of noise on the chaotic attractor in coupled LED systems have also been studied [11]. The goal of this paper is to achieve robust synchronization between two LED networks. In each network, LEDs are coupled bi-directionally through the bias current with sufficiently weak coupling. We propose a controller based on radius basis

function (RBF) neural networks which deal with the unknown terms or uncertainty of the nonlinear systems due to their approximation ability [12]. Based on Lyapunov stability theory, stability of closed-loop error system is proven. Then the synchronization is guaranteed. Simulation results demonstrate the validity of the method.

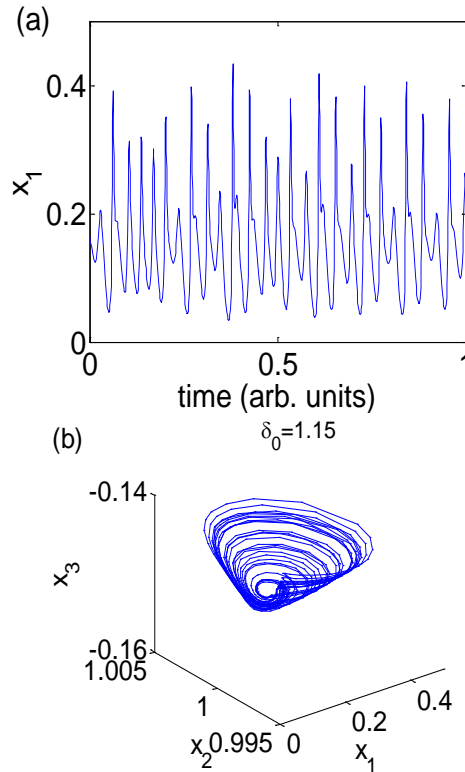
LED System Model and Its Complex Dynamics

For numerical and analytical purposes, the LED system dynamics is written in dimensionless form (see [8] for details):

$$\dot{x}_1 = x_1(x_2 - 1), \quad \dot{x}_2 = \gamma(\delta_0 - x_2 + \alpha(x_3 + x_1)/(1 + s(x_3 + x_1)) - x_1x_2), \quad \dot{x}_3 = -\varepsilon(x_3 + x_1) \quad (1)$$

where δ_0 , γ , ε , α and s are system parameters.

The (a, c) panels in Fig. 1 show some typical patterns with different values of δ_0 obtained by numerical integration of Eq. (1). The (b, d) panels are the corresponding phase portraits in $x - y - \omega$. We observe chaotic spiking with $\delta_0 = 1.15$ and periodic oscillation with $\delta_0 = 1.1$.



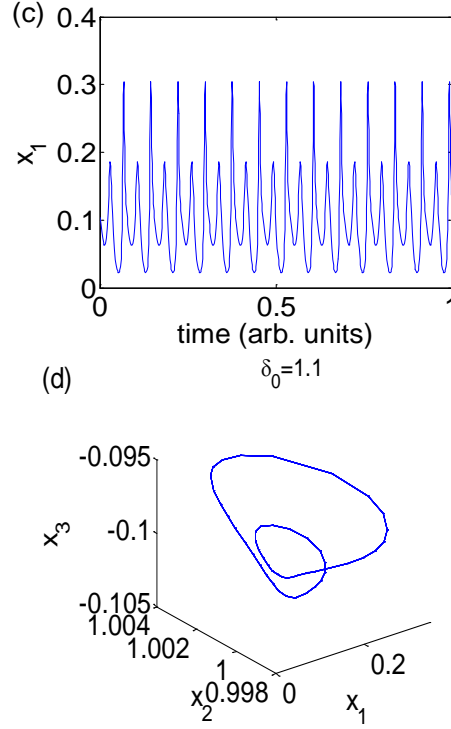


Fig. 1. Responses of the LED system with different values of parameter δ_0 : (a, c) Time series of the variable x_1 ; (b, d) the corresponding phase portraits in $x_1 - x_2 - x_3$.

Synchronization of Coupled LED Networks via Neural Networks Control

The drive network consists of N LEDs is given by:

$$\begin{aligned}\dot{x}_{i1} &= x_{i1}(x_{i2} - 1), \\ \dot{x}_{i2} &= \gamma(\delta_0 - x_{i2} + \alpha(x_{i3} + x_{i1})/(1 + s(x_{i3} + x_{i1}))) - x_{i1}x_{i2}, \\ \dot{x}_{i3} &= -\varepsilon(x_{i3} + x_{i1}) + \sum_{j=1}^N g_{ij}x_{j3}, \quad i = 1, 2, \dots, N\end{aligned}$$

where $G = (g_{ij}) \in R^{N \times N}$ is the outer-coupling matrix g_{ij} is the coupling strength.

The corresponding controlled LEDs network is given by:

$$\begin{aligned}
\dot{y}_{i1} &= y_{i1}(y_{i2} - 1) + u_{i1}, \\
\dot{y}_{i2} &= \gamma(\delta_0 - y_{i2} + \alpha(y_{i3} + y_{i1})/(1 + s(y_{i3} + y_{i1})) - y_{i1}y_{i2}) + u_{i2}, \\
\dot{y}_{i3} &= -\varepsilon(y_{i3} + y_{i1}) + \sum_{j=1}^N g_{ij}y_{j3} + u_{i3}, \quad i = 1, 2, \dots, N
\end{aligned}$$

where $u_i(t) = (u_{i1}, u_{i2}, u_{i3})^T$ is the control input to the i th node.

Subtract (2) from (3), one gets the error dynamics as

$$\begin{aligned}
\dot{e}_{i1} &= -e_{i1} + \Delta f_{i1}(\mathbf{x}_i, \mathbf{y}_i) + u_{i1}, \\
\dot{e}_{i2} &= -\gamma e_{i2} + \Delta f_{i2}(\mathbf{x}_i, \mathbf{y}_i) + u_{i2}, \\
\dot{e}_{i3} &= -\varepsilon(e_{i3} + e_{i1}) + \sum_{j=1}^N g_{ij}e_{j3} + u_{i3}, \quad i = 1, 2, \dots, N
\end{aligned}$$

where

$$\begin{aligned}
\Delta f_2(\mathbf{x}_i, \mathbf{y}_i) &= \gamma \alpha(y_{i3} + y_{i1})/(1 + s(y_{i3} + y_{i1})) - y_{i1}y_{i2} - \gamma \alpha(x_{i3} + x_{i1})/(1 + s(x_{i3} + x_{i1})) - x_{i1}x_{i2} \\
\text{and } \Delta f_1(\mathbf{x}_i, \mathbf{y}_i) &= y_{i1}y_{i2} - x_{i1}x_{i2}, \text{ The synchronization problem is how to} \\
\text{design the controller } u_i(t), \text{ which will stabilize (4) around zero.}
\end{aligned}$$

Let $\mathbf{e}(t) = [e_{11}, \dots, e_{N1}, e_{12}, \dots, e_{N2}, e_{13}, \dots, e_{N3}]^T \in R^{3N}$,
 $\Delta \mathbf{F} = [\Delta f_{11}, \dots, \Delta f_{N1}, \Delta f_{12}, \dots, \Delta f_{N2}, 0, \dots, 0] \in R^{3N}$, $w(t) \in R$, $K \in R^{3N}$
and $\mathbf{u}(t) = [u_{11}, \dots, u_{N1}, u_{12}, \dots, u_{N2}, u_{13}, \dots, u_{N3}]^T$. Then (4) becomes

$$\dot{\mathbf{e}} = \mathbf{A}\mathbf{e} + \Delta \mathbf{F} + \mathbf{u}(t)$$

$$\text{where } \mathbf{A} = \begin{bmatrix} -I & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\gamma I & \mathbf{0} \\ -\varepsilon I & \mathbf{0} & G - \varepsilon I \end{bmatrix}.$$

In this paper, we use RBFNNs to approximate the nonlinear function $\Delta \mathbf{F}$ in the following form:

$$\Delta \mathbf{F} = \Phi(\mathbf{x}, \mathbf{y})\Theta^* + \varepsilon$$

where ε is the approximation error vector and $\|\varepsilon\| \leq \varepsilon_N$ with ε_N a positive constant, and $\Theta^* = (\theta_{11}^*, \dots, \theta_{N1}^*, \theta_{12}^*, \dots, \theta_{N2}^*, 0, \dots, 0)^T \in R^{3N}$,
 $\Phi = \text{diag}\{\phi_{11}^T, \dots, \phi_{N1}^T, \phi_{12}^T, \dots, \phi_{N2}^T, 0, \dots, 0\}$ with $\theta_i^* \in R^{m \times 1}$ are the ideal

weights matrix which can't be obtained but can be on-line estimated by $\hat{\Theta}$ with the estimation error $\tilde{\Theta} = \hat{\Theta} - \Theta^*$. $\phi_i \in R^{m \times 1}$ are basis functions which are chosen as commonly used Gaussian function with fixed centers and widths.

Thus real controller is designed as

$$\mathbf{u}(t) = -\mathbf{B}\mathbf{e} - \Phi(\mathbf{x}, \mathbf{y})\hat{\Theta}$$

where the adaptive law of $\hat{\Theta}$ is designed as

$$\dot{\hat{\Theta}} = \Gamma \Phi^T \mathbf{e}$$

Then the error dynamics becomes

$$\dot{\mathbf{e}} = (\mathbf{A} - \mathbf{B})\mathbf{e} - \Phi(\mathbf{x}, \mathbf{y})\tilde{\Theta} + \boldsymbol{\varepsilon} = \mathbf{Q}\mathbf{e} - \Phi(\mathbf{x}, \mathbf{y})\tilde{\Theta} + \boldsymbol{\varepsilon}$$

Theorem 1. Consider the synchronization error dynamics (5) with the control law (7) and adaption law (8), by properly selecting the feedback gain matrix \mathbf{B} , one can make the error dynamical system (5) globally asymptotically stable at the origin, thus implying that the systems (2) and (3) are globally asymptotically synchronized.

Proof. Consider the following Lyapunov function candidate

$$V = \frac{1}{2} \mathbf{e}^T \mathbf{e} + \frac{1}{2\beta} \tilde{\Theta}^T \tilde{\Theta}$$

Differentiating (9) with respect to time and noting (6) and (7), one can obtain

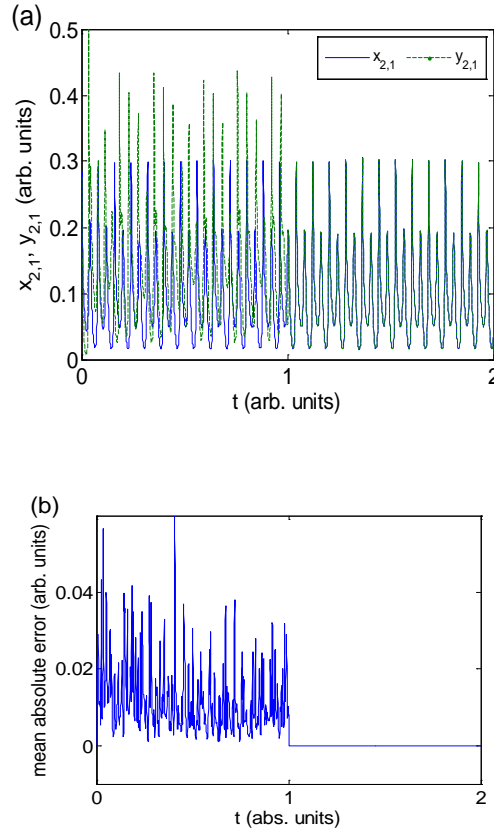
$$\dot{V} = \mathbf{e}^T \dot{\mathbf{e}} + \tilde{\Theta}^T \beta^{-1} \dot{\tilde{\Theta}} = \mathbf{e}^T \mathbf{Q}\mathbf{e} - \mathbf{e}^T \boldsymbol{\varepsilon} \leq -|\lambda_{\min}| \cdot \|\mathbf{e}\|^2 + \varepsilon_N \|\mathbf{e}\|$$

which is negative as long as $\|\mathbf{e}\| > \varepsilon_N / |\lambda_{\min}|$, i.e. the error dynamics is stable around origin point, and the synchronization error has an upper bound $\varepsilon_N / |\lambda_{\min}|$, which can be as small as possible by proper choice of the control parameters. Here λ_{\min} is the minimum eigenvalue of \mathbf{Q} .

Simulation Results

To illustrate the effectiveness of the proposed control method for the synchronization of the LED networks, numerical simulations are carried out in this section.

The system parameters are set as $N = 5$, $\gamma = 0.0033$, $\varepsilon = 0.00004$, $\alpha = 1.002$. The values of δ_0 for the drive and response networks are different such that the networks display different dynamics before control. We consider two cases. In *Case 1*, the values of δ_0 are randomly distributed in $[1.14, 1.16]$ for the drive network, while those for the response network are randomly distributed in $[1.09, 1.11]$. In *Case 2*, the values of δ_0 switch. The coupling strengths are $g_{i,j} = 5 \times 10^{-5}$, $i, j = 1, \dots, 5$. The controller is switched on at time $t = 1$. The simulation results are shown in Fig. 2. In *Case 1*, the chaotic LED networks are controlled to follow the periodic ones as shown in Fig. 2 (a) and (b). In *Case 2*, the periodic LED networks are controlled to follow chaotic ones as shown in Figure 2 (c) and (d). Before the control is implemented, the master and slave LED systems exhibit their own original complex dynamical behaviors. After the controller is applied, the synchronization error converges to a very small neighborhood of the origin point and chaos synchronization is obtained.



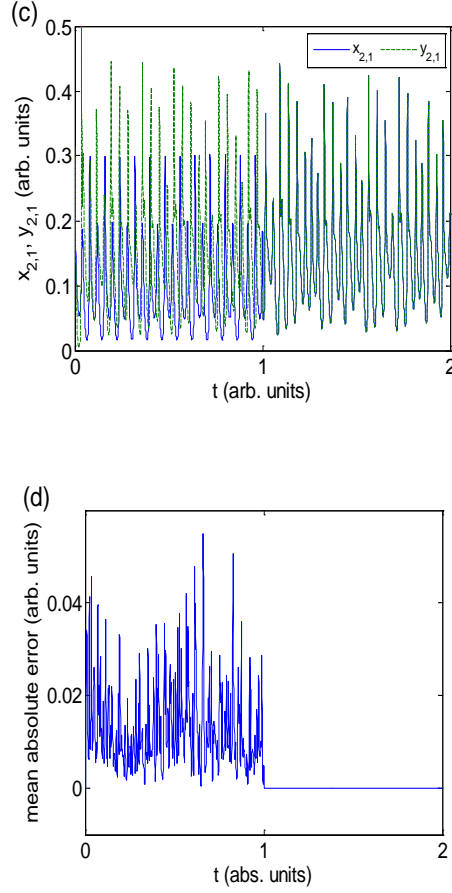


Fig. 2: Responses of waveforms of x_{21}, y_{21} (a, c) and the corresponding sum of absolute errors $\frac{1}{5} \sum_{i=1}^5 (y_{i1} - x_{i1})^2$ (b, d) for two cases: (a, b) Case 1 and (c, d) Case 2, whereas the control signal is switched on at time $t = 1$.

Conclusions

Adaptive synchronization of LED networks with complex dynamics has been investigated in this paper. A neural networks based control scheme has been proposed. Based on the Lyapunov stability theory, the stability analysis has been given. The proposed controller ensures stable synchronization between the drive LED network and the response one, regardless of the system dynamics. The effectiveness of the proposed control method has been demonstrated by simulation results.

Acknowledgments

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