Speckle Noise Reduction in SAR image based on K-SVD

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Abstract.

This paper presents a method based on K-SVD (Singular Value Decomposition) to despeckle Synthetic Aperture Radar (SAR) image. First, we get sparse representation of a noisy image *f* over a fixed complete dictionary *D* by using Orthogonal Matching Pursuit (OMP). Then, train *D* on the noisy image to get a updated dictionary *D'* with K-SVD algorithm. Finally, reconstruct image with the new dictionary *D'*. Compared with traditional methods, this method is better in despeckling effect and image fidelity.

Keywords: SAR, Despeckle, Sparse Representation, Dictionary, OMP, K-SVD

Introduction

SAR images have been widely used nowadays, as the SAR system has great potential due to its all-weather, all-time and strong penetrating capabilities. However, SAR images are inevitably accompanied by speckle noise, which will degrade the image quality significantly. In recent decades, scholars have designed a lot of algorithms to remove speckle noise, representative conventional methods are Lee filter [1], Frost filter [2], Kuan filter [3] and wavelet [4,5], and so on.

S. Mallat and Z. Zhang first proposed the concept of over-complete dictionary in 1993 [6]. The image can be decomposed in an over-complete dictionary, and then one can get the sparse representation of image. Due to the better effect, image denoising (including SAR image despeckling) based on sparse representation becomes a new research hotspot recently [7].

This paper presents a method based on K-SVD to despeckle SAR images. Compared with traditional methods, this method can produce better despeckling effect and image fidelity.

Sparse representation of image

We often decompose the signal with Fourier Transform, Shot-Time Fourier

Transform, and Wavelet Transform so that we can do research more easily. Decomposition of the signal is generally expressed as:

$$f_s = \sum_{k=1}^{N} c_k p_k \tag{1}$$

In the expression, f_s is the signal to be decomposed, p_k is basis function, and c_k is decomposition coefficient.

This decomposition is based on a complete orthogonal basis, which has some defects on a detailed description of the signal. So scholars were interested in decomposition the signal on an over-complete dictionary. The element in the dictionary is equivalent to the basis, which is referred to as atom. The over-complete dictionary has redundancy, which ensures that the representation of signal is sparse. Extending the theory to the two-dimensional image, sparse decomposition of the signal is choosing the linear combination of the best atom form the over-complete dictionary to represent an image, and the coefficient of decomposition of the image on the over-complete dictionary is not the only one. Let the image being decomposed be *f*, the size of which is N_1 x N_2, where N_1 and N_2 are the high and wide of the image respectively. *D* is the over-complete dictionary which is known:

$$\mathrm{D} = \{g_\gamma\}_{\gamma\in\Gamma} \tag{2}$$

g_γ is atom, and must be normalized$\|g_\gamma\| = 1$. Because the dictionary is over-complete, if *P* represents the number of atoms in *D*, $P \gg N_1 \times N_2$.We can build a model of sparse representation of the image *f* :

$$\min\|a\|_0 \qquad s.t \qquad \|f - Da\| \leq \varepsilon \tag{3}$$

Solution of sparse representation is equivalent to the question of minimization norm L_0 of decomposition coefficient *a*., in other words, minimization the number of nonzero element. ε is sparse approximation error. Because of the non-convexity of norm L_0, it is a problem of NP to solve the unique solution of *a*. We generally use the greedy approximation algorithm such as MP (Matching pursuit), OMP in order to achieve *a*.

Dictionary training based on K-SVD algorithm

In sparse representation, adaptive dictionary describes characteristics of the image is more detailed than the fixed dictionary. K-SVD is a good adaptive algorithm in training dictionary.

Let$D \in \mathbb{R}^{n\times K}$is a fixed over-complete dictionary, $f \in \mathbb{R}^n$is a block of noisy image, $x \in \mathbb{R}^K$ is the coefficient vector of sparse representation, and $\mathrm{F} =$

$\{f_i\}_{i=1}^{N}, X = \{x_i\}_{i=1}^{N}$. The algorithm is as follows [8]:

(1) Sparse decomposition: we use an efficient pursuit algorithm for solving the following equation to get X:

$$\min\{\|f_i - Dx_i\|_2^2\} \quad s.t \quad \forall i, \|x_i\|_0 \le T_0, i = 1,2,\dots,N \tag{4}$$

T_0 is the upper limit of the number of non-zero coefficients.

(2) Training dictionary: we have known *X*, let d_k be column *k* of the dictionary matrix, and *k*=1, 2…*K*, so:

$$\|\mathrm{F} - \mathrm{DX}\|_\mathrm{F}^2 = \left\|\left(\mathrm{F} - \sum_{j \ne k} \mathrm{d_j x_T^j}\right) - \mathrm{d_k x_T^k}\right\|_\mathrm{F}^2 = \left\|\mathrm{E_k} - \mathrm{d_k x_T^k}\right\|_\mathrm{F}^2 \tag{5}$$

x_T^k is row *k* of the coefficient matrix, and E_k is the error of *F* without the effects of d_k.

Dictionary is updated column by column. In general, the steps of updating d_k are as follows:

①Define $w_k = \{i | 1 \le i \le K, x_T^k(i) \ne 0\}$ as the group of indices pointing to f_i that use the atom d_k;

②Compute the total error matrix $E_k = F - \sum_{j \ne k} d_j x_T^j$;

③Select the number of columns associated with w_k constraint E_k to get E_K^R;

④ E_K^R can be decomposed via SVD. SVD decomposes it to $E_K^R = U\Delta V^T$, where the first column of U is $\widetilde{d_k}$ which is the updated result of d_k, and the updated result of x_R^k is the first column of *V* multiplied by $\Delta(1,1)$.

Repeat above steps, we can get a new dictionary *D'*. Figure 1 shows a result that a DCT dictionary has been trained with K-SVD algorithm.

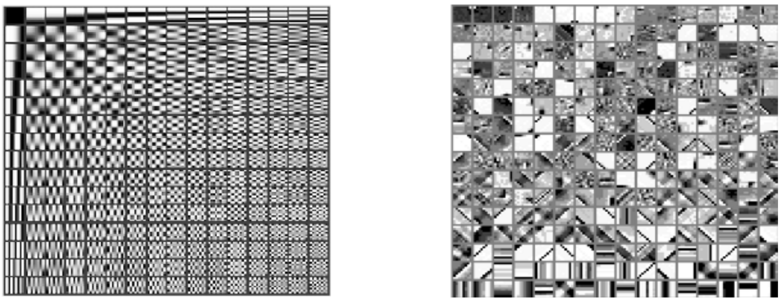


Fig. 1 DCT Dictionary (left) and Dictionary training by K-SVD (right)

De-noising process of SAR image

Mentioned in the introduction, SAR image has inevitable speckle noise. This noise is different from ordinary additive noise, it is caused by a coherent superposition of waves. Goodman proposed the concept of fully developed speckle noise in 1976 [9]. Depending on the nature of speckle noise, multiplicative noise model of SAR image was established as [10, 11]:

$$f = SX \tag{6}$$

Where, *f* is the intensity of the noisy image which is received, *S* is a random variable of speckle noise, and *X* is a radar scattering properties of the target. To make it easier to remove speckle, we do logarithmic transformation for this model, so speckle noise is modeled as additive noise:

$$\tilde{f} = \log(f) = \tilde{S} + \tilde{X} \tag{7}$$

Assuming *L* is the number of looks, and then the mean and variance of the noise $\tilde{S}$ are given mathematically by [12]:

$$\mu_{\tilde{s}}^2 = -0.577215 - \log(L) + \sum_{m=1}^{L-1} \frac{1}{m} \qquad var_{\tilde{s}} = \frac{\pi^2}{6} - \sum_{m=1}^{L-1} \frac{1}{m^2} \tag{8}$$

Standard deviations of the noise can be defined as the square root of the variance σ which will be used as the parameter of K-SVD algorithm.

We know the filtering equation of SAR image [7]:

$$\hat{X} = \left(\lambda I + \sum_{i,j} R_{ij}^T R_{ij}\right)^{-1} \left(\lambda Y + \sum_{i,j} R_{ij}^T D\, \hat{a}_{ij}\right) \tag{9}$$

Where, I is unit matrix.

In fact, the noise model of SAR image was logarithmic transformed before, so we must to be exponential transform $\hat{X}$ to get the final result.

The general idea of SAR image de-noising is: First, decompose SAR image *Y’* which was logarithmic transformed by OMP algorithm on DCT dictionary *D* to get the sparse coefficient matrix *a* [13]. Second, train the dictionary *D* by K-SVD algorithm to get the Adaptive dictionary *D’* and updated sparse coefficient matrix *a’*. Finally, calculate the result $\hat{X}$ of de-noising image by filtering equation, and do exponential transform to $\hat{X}$ to get the final result. The flow diagram is shown in Figure 2.

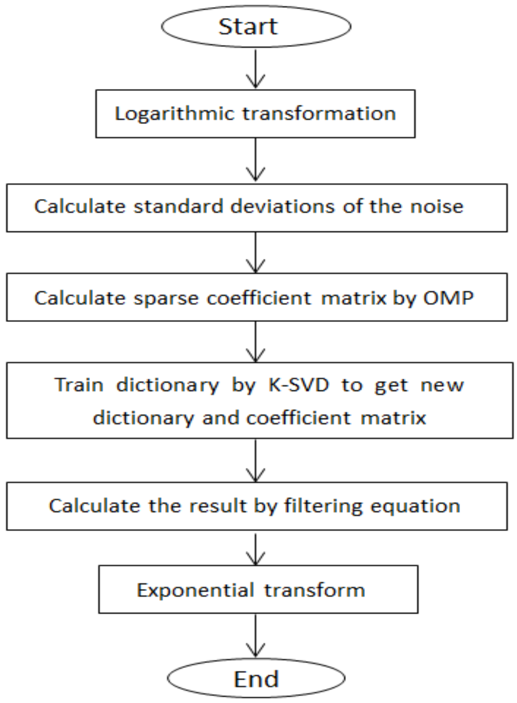


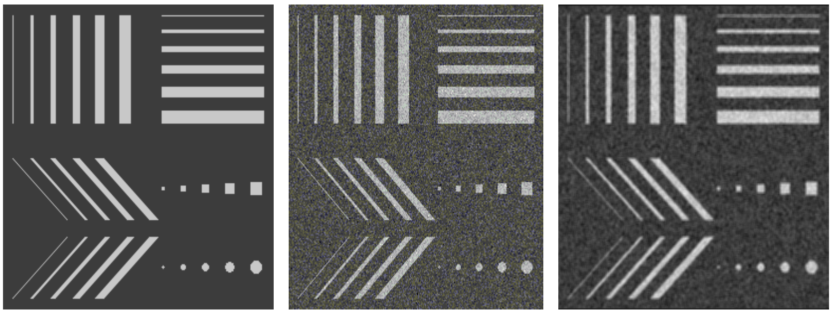
Fig. 2 The flow diagram of SAR image de-noising

Experimental results and conclusion

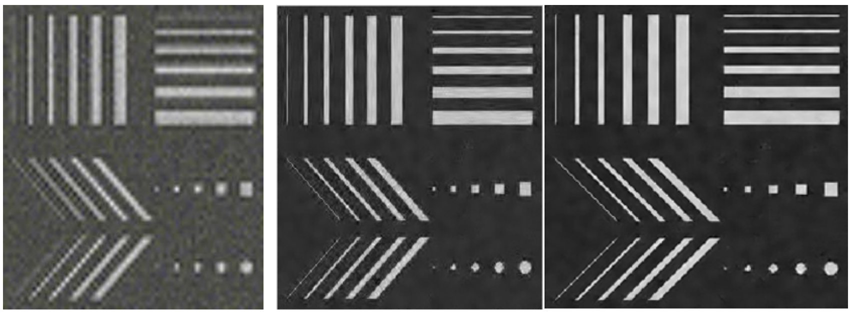
In order to test the despeckling ability of K-SVD algorithm for SAR image, this paper introduces the other three methods to despeckle a given image. These three methods are improved Lee algorithm [14], sym4 wavelet decomposition, sparse decomposition despeckling based on DCT dictionary.

We chose two experimental SAR images. The first one is the artificially simulated image, which is obtained by adding random noise with standard deviation σ 25 to a clean image. The second one is a real SAR image.

AS is shown in figure 3, the method of sparse representation based on a dictionary has better despeckling effect, which makes the details of the image clear, and retains the information intact. Improved Lee filter and wavelet despeckling smooth the noise to a certain extent, but both of them are at the expense of more image information, so that the despeckled image becomes blurred. The method of DCT despeckling is not as good as K-SVD algorithm for retaining information which is the details of the image including edge, texture, and so forth. In order to describe the effect of despeckling objectively, we introduce Peak Signal-to-Noise Ratio (PSNR). The calculation formula of PSNR is $10\log_{10}\frac{255\times255}{MSE}$, where MSD is mean square error. PSNR represents the ratio between the image signal and noise. The larger the value of PSNR, the smaller the ratio of noise and the better the effect of despeckling are. The results are shown in figure 4.



(a) The clean image (b) The noise image (c)Improved Leefilter denoising



(d) Wavelet de-noising (e) DCT de-noising (f) K-SVD de-noising

Fig. 3 De-noising effect of simulated SAR image

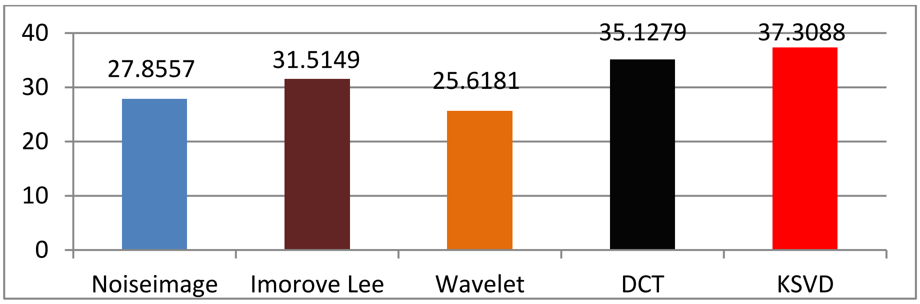
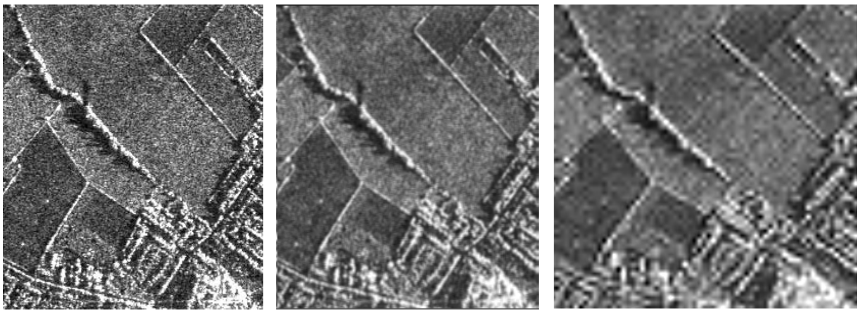
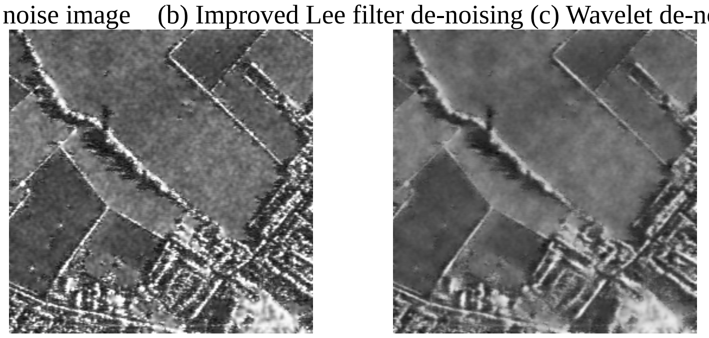


Fig. 4 Comparison table of PSNR

Next, we will despeckle a real SAR image.



(a) The noise image (b) Improved Lee filter de-noising (c) Wavelet de-noising



(d) DCT de-noising (f) K-SVD de-noising

Fig. 5 Effect of de-noising for a real SAR image

The same as the intuitive feelings of despeckling for simulated SAR image, the effect of sparse decomposition despeckling method is much better, which can be seen in figure 5. For a real SAR image, because there is not noiseless image which could be compared, we can't use PSNR to evaluate the effect of de-noising. Another metric available is equivalent number of looks (ENL). It is defined as the square of the ratio of image mean and standard deviation. The larger the value of ENL, the better the effect of despeckling is. The results are shown in figure 6.

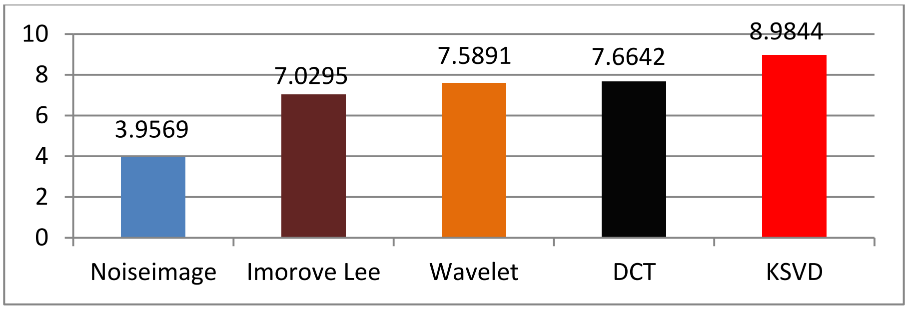


Fig. 6 Comparison table of ENL

Through the experiment, we've demonstrated that K-SVD algorithm shows great advantages. Because sparse representation can best distinguish between the useful information and noise information in image, and K-SVD algorithm is better than the algorithm of fixed over-complete dictionary. However, because of the impact of the iterative calculation and convergence speed, running time of K-SVD algorithm is longer than the other methods mentioned in the text, so there is a big research space on the adaptive method of convergence the termination condition.

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