Simulation of the Effective Refractive Index of the Cladding Mode in LC-LPFG
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Keywords: LPFG; Temperature Sensitivity; Liquid Crystal

Abstract. This paper uses four-layer structure model based on the coupled-mode theory and presents the simulation analysis of the effective refractive index of the cladding mode in LC-LPFG. Conclusions are drawn that, when the refractive index of the liquid crystal (temperature of environment), the incident wavelength or the radius of the liquid crystal (thickness of the film) changes, the effective refractive index of the cladding mode changes monotonically. In addition, choosing higher order cladding mode can enhance the sensitivity of the temperature sensor.

Introduction
Long-period fiber grating (LPFG) is a kind of component with advantages of compact construction, resistance to harsh environments, immunity to electromagnetic interference. The grating causes attenuation at certain wavelengths in the transmission spectrum. The resonant peak of the wavelengths will have a large scale drift by changing the parameters in the process of coupling [1]. LPFG with an “active” external layer used in the field of sensing became a hot research area recent years. Liquid crystal, of which structure properties are between liquid and anisotropic crystal, is easily affected by external stimulus [2]. This paper uses LPFG coated with liquid crystal (LC-LPFG) in the temperature sensor due to the thermal light effect. As the nature of the sensor is the shift of the effective refractive index of the cladding mode, a four layer structure model is established to investigate this parameter.

Theoretical analysis
The effect of environment on the core mode can be ignored based on coupled-mode theory. So this paper only simulates the cladding mode. The structure model of LC-LPFG is presented in Figure 1.

Fig.1. The structure model of LC-LPFG

n₁, n₂, n₃, n₄ stand for the refractive index of the core mode, the cladding mode, the LC layer and the environment. a₁, a₂, a₃ stand for the radius of the core, the cladding and the LC layer. The resonant wavelength is expressed by \( \lambda_{LP} = (n_{eff}^{co} - n_{eff}^{cl})\Lambda \), where \( \Lambda \) stands for the period of the grating. \( n_{eff}^{co} \) stands for the effective refractive index of the core mode, which can be calculated by HE linear polarization equation [3]. \( n_{eff}^{cl} \) stands for the effective refractive index of the cladding mode, which can be expressed by [4][5]:
\[
\left[ \frac{\left( J_{n_2}/n_2^2 \right)}{\left( \Gamma_n/n_2^2 + K_n u_1 \Gamma_n + n_2^2 \right)} - \left( 1/n_2^2 \right) \right] \left( \Gamma_n/n_2^2 + K_n u_1 \Gamma_n + n_2^2 \right) \\
\left[ \frac{\left( J_{n_2}/n_2^2 \right)}{\left( \Gamma_n/n_2^2 + K_n u_1 \Gamma_n + n_2^2 \right)} - \left( 1/n_2^2 \right) \right] = \frac{\left( \xi_1^2 + \xi_2^2 \right)}{\left( \xi_1^2 + \xi_2^2 \right)} \left( \xi_1^2 + \xi_2^2 \right) \left( \xi_1^2 + \xi_2^2 \right) P_{n_2^2} + \frac{\xi_1^2 + \xi_2^2}{\xi_1^2 + \xi_2^2} P_{n_2^2} \\
+ \frac{2 \xi_1^2 \xi_2^2 u_1 \Gamma_10}{r_3 r_1 n_2 u_1 \pi^2 + 2 \xi_1^2 \xi_2^2 u_1 \pi^2} P_{n_2^2} \\
+ \frac{2 \xi_1^2 \xi_2^2}{r_3 r_1 n_2 u_1 \pi^2 + 2 \xi_1^2 \xi_2^2 u_1 \pi^2} P_{n_2^2} \\
+ \frac{\xi_1^2 + \xi_2^2}{\xi_1^2 + \xi_2^2} \left( \Gamma_1 + K_n u_1 \Gamma_1 \right) \left( \Gamma_1/n_2^2 + K_n u_1 \Gamma_1/n_2^2 \right) + \frac{\xi_1^2 + \xi_2^2}{\xi_1^2 + \xi_2^2} P_{n_2^2} \\
+ \frac{\xi_1^2 + \xi_2^2}{\xi_1^2 + \xi_2^2} \left( \Gamma_1 + K_n u_1 \Gamma_1 \right) \left( \Gamma_1/n_2^2 + K_n u_1 \Gamma_1/n_2^2 \right) + \frac{\xi_1^2 + \xi_2^2}{\xi_1^2 + \xi_2^2} P_{n_2^2} \\
+ \frac{\xi_1^2 + \xi_2^2}{\xi_1^2 + \xi_2^2} \left( \Gamma_1 + K_n u_1 \Gamma_1 \right) \left( \Gamma_1/n_2^2 + K_n u_1 \Gamma_1/n_2^2 \right) + \frac{\xi_1^2 + \xi_2^2}{\xi_1^2 + \xi_2^2} P_{n_2^2} \\
+ \frac{\xi_1^2 + \xi_2^2}{\xi_1^2 + \xi_2^2} \left( \Gamma_1 + K_n u_1 \Gamma_1 \right) \left( \Gamma_1/n_2^2 + K_n u_1 \Gamma_1/n_2^2 \right) + \frac{\xi_1^2 + \xi_2^2}{\xi_1^2 + \xi_2^2} P_{n_2^2} \\
+ \frac{\xi_1^2 + \xi_2^2}{\xi_1^2 + \xi_2^2} \left( \Gamma_1 + K_n u_1 \Gamma_1 \right) \left( \Gamma_1/n_2^2 + K_n u_1 \Gamma_1/n_2^2 \right) + \frac{\xi_1^2 + \xi_2^2}{\xi_1^2 + \xi_2^2} P_{n_2^2} \\
+ \frac{\xi_1^2 + \xi_2^2}{\xi_1^2 + \xi_2^2} \left( \Gamma_1 + K_n u_1 \Gamma_1 \right) \left( \Gamma_1/n_2^2 + K_n u_1 \Gamma_1/n_2^2 \right) + \frac{\xi_1^2 + \xi_2^2}{\xi_1^2 + \xi_2^2} P_{n_2^2} \\
+ \frac{\xi_1^2 + \xi_2^2}{\xi_1^2 + \xi_2^2} \left( \Gamma_1 + K_n u_1 \Gamma_1 \right) \left( \Gamma_1/n_2^2 + K_n u_1 \Gamma_1/n_2^2 \right) + \frac{\xi_1^2 + \xi_2^2}{\xi_1^2 + \xi_2^2} P_{n_2^2} \tag{1}
\]

Where: \( u_1 = k_0^2 (n_1^2 - n_{\text{eff}}^2) \), \( u_2 = k_0^2 (n_2^2 - n_{\text{eff}}^2) \), \( u_3 = k_0^2 (n_3^2 - n_{\text{eff}}^2) \), \( w_4 = k_0^2 (n_{\text{eff}}^2 - n_4^2) \)

\[
\nu_{12}^2 = k_0^2 (n_1^2 - n_2^2) \quad , \quad \nu_{12}^2 = k_0^2 (n_2^2 - n_1^2) \quad , \quad \nu_{12}^2 = k_0^2 (n_3^2 - n_1^2) \quad , \quad \nu_{12}^2 = k_0^2 (n_3^2 - n_2^2) \quad , \quad \nu_{12}^2 = k_0^2 (n_3^2 - n_4^2) \quad , \quad \nu_{12}^2 = k_0^2 (n_4^2 - n_1^2) \quad , \quad \nu_{12}^2 = k_0^2 (n_4^2 - n_2^2)
\]

\[
\xi_1 = \sigma_0 \nu_{12}^2 / r_1 n_1 u_2 \xi_1^2 \quad , \quad \xi_2 = \sigma_0 \nu_{12}^2 / r_1 n_2 u_2 \xi_2^2 \quad , \quad \xi_3 = \sigma_0 \nu_{12}^2 / r_3 n_3 u_2 \xi_3^2 \quad , \quad \xi_4 = \sigma_0 \nu_{12}^2 / r_3 u_2 \xi_4^2 \\
K_4 = K' (w_r 5) / w_4 K' (w_r 5) \quad , \quad J = J' (u_r 5) / u_1 J' (u_r 5) \quad , \quad K = 2 \pi / \lambda \quad , \quad \nu = 1
\]

\[
p_{n_2^2} = J_1 (u_2 r_1) Y_1 (u_2 r_1) = J_1 (u_2 r_1) Y_1 (u_2 r_1) = J_1 (u_2 r_1) Y_1 (u_2 r_1)
\]

\[
r_{n_2^2} = J_1 (u_2 r_1) Y_1 (u_2 r_1) = J_1 (u_2 r_1) Y_1 (u_2 r_1) = J_1 (u_2 r_1) Y_1 (u_2 r_1)
\]

\[
p_{n_2^2} = J_1 (u_2 r_1) Y_1 (u_2 r_1) = J_1 (u_2 r_1) Y_1 (u_2 r_1) = J_1 (u_2 r_1) Y_1 (u_2 r_1)
\]

\[
\Gamma = u_2 p_{n_2^2} q_{n_2^2} - u_3 r_{n_2^2} q_{n_2^2}, \quad \Gamma = u_2 p_{n_2^2} q_{n_2^2} - u_3 r_{n_2^2} q_{n_2^2}, \quad \Gamma = u_2 p_{n_2^2} q_{n_2^2} - u_3 r_{n_2^2} q_{n_2^2}
\]

\[
\Gamma = u_2 p_{n_2^2} q_{n_2^2} - u_3 r_{n_2^2} q_{n_2^2}, \quad \Gamma = u_2 p_{n_2^2} q_{n_2^2} - u_3 r_{n_2^2} q_{n_2^2}, \quad \Gamma = u_2 p_{n_2^2} q_{n_2^2} - u_3 r_{n_2^2} q_{n_2^2}
\]

\[
J_1, Y_1, K_4 \text{ stand for Bessel function of the first kind, Bessel function of the second kind and modified Bessel function of the second kind. If the surrounding medium is air } (n_4 = 1.0), \text{ the type of the bare fiber is identified, only } n_1, a_3, \lambda \text{ will affect the coupling process.}
\]

Numerical analysis and results

The type of the fiber is SMF-28 from Coning Company, where \( n_1 = 1.4681 \), \( n_2 = 1.4628 \), \( a_1 = 4.15um \), \( a_2 = 62.5um \). The type of the liquid crystal is MDA-98-3699 from Merck Germany. When temperature changes from 20°C to 65°C, \( n_3 \), the refractive index of the LC layer changes from 1.477 to 1.515 [6].

(1)Relationship of \( n_{\text{eff}}' \) and the order of the cladding mode \( \nu \)

In the simulation, the incident wavelength \( \lambda = 1550nm \), the layer thickness \( a_3 - a_2 = 500nm \), relationship curve of the effective refractive index of the cladding mode is obtained in Figure 2. As can be seen, when the order of the cladding mode increase, the effective refractive index of the cladding mode decrease monotonically and have a mutation at \( \nu = 30 \) and \( \nu = 40 \).
Fig. 2. Curve of the effective refractive index of the cladding mode

(2) Relationship of $n_{\text{eff}}^{cl}$ and the layer thickness $d_3 - d_2$

In the simulation, the incident wavelength $\lambda = 1550\,\text{nm}$, the temperature of the surrounding medium is $20\,\text{°C}$ ($\mathcal{G} \approx 1.477$). When the layer thickness range from 200nm–700nm, relationship curve of $n_{\text{eff}}^{cl}$ and $d_3 - d_2$ is obtained in Figure 2. The curve from top to bottom represents the order of the cladding mode ranged from 1 to 10 respectively. As can be seen, when the thickness of the layer increase, the effective refractive index of the cladding mode increase monotonically. While comparing with other parameters, this unapparent effect can be ignored. The only point to pay attention to is that $d_3 - d_2$ should be smaller than the penetration depth to let the electromagnetic field go through.

(3) Relationship of $n_{\text{eff}}^{cl}$ and the incident wavelength $\lambda$

In the simulation, the layer thickness $d_3 - d_2 = 500\,\text{nm}$, the temperature of the surrounding medium is $20\,\text{°C}$ ($\mathcal{G} \approx 1.477$). When the incident wavelength $\lambda$ range from 1350nm to 1750nm, relationship curve of $n_{\text{eff}}^{cl}$ and $\lambda$ is obtained in Figure 3. As can be seen from the top to the bottom, when the incident wavelength increase, the effective refractive index of the cladding mode decrease monotonically. The higher the order is, the higher the rate of change is, which means $n_{\text{eff}}^{cl}$ will be more sensitive to $\lambda$. To conclude, as for $\lambda$, choosing higher order of the cladding mode can improve the sensitivity.

(4) Relationship of $n_{\text{eff}}^{cl}$ and the refractive index of the layer $n_3$

In the simulation, the layer thickness $d_3 - d_2 = 450\,\text{nm}$, the incident wavelength $\lambda = 1550\,\text{nm}$, the temperature of the surrounding medium ranges from 20°C to 65°C ($n_3$ ranges from 1.477 to 1.515). Relationship curve of $n_{\text{eff}}^{cl}$ and $n_3$ with the order ranged from 7 to 8 is obtained in Figure 4 (a), (b). When the refractive index of the LC layer increase, the effective refractive index of the cladding mode increase almost linearly. The slope of the higher order curve is greater than the lower order, which means $n_{\text{eff}}^{cl}$ of the higher order is more sensitive to temperature. To conclude, as for $n_3$,
choosing higher order of the cladding mode can improve the sensitivity.

(a) Curve of \( \nu = 7 \)  
(b) Curve of \( \nu = 8 \)

Fig.5. Curve of \( n_{cl}^{eff} \) and LC refractive index

Conclusion

A four-layer structure model was proposed in this paper. Simulation results showed that by changing parameters, such as \( n_3, a_i, \lambda \), the effective refractive index of the cladding mode would shift monotonically and then affect the coupling process. In addition, choosing higher order would enhance the sensitivity, which is of great significance to the temperature sensor.

References