Performance Analysis and Simulation of Look-up Table Predistortion Algorithm

Qu Xiaoxu\textsuperscript{a}, Lou Jingyi\textsuperscript{b}

Naval University of Engineering, Wuhan, Hubei, China
\textsuperscript{a}const.qq@163.com, \textsuperscript{b}jingyi.lou@163.com

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Abstract. The performance of look-up table (LUT) predistortion system is analyzed and simulated, and three conclusions are gotten. The first is the system SNR increases with the increasing of the LUT length, and if the length of LUT increases doublely the SNR increases at least 6 dB. And the second is the system with weaker nonlinearity characteristic PA has better system SNR. And the third is if the input signal with the characteristic of higher occurrence probability of little signal, the system SNR is finer. These conclusions are helpful to design the suitable parameter of the LUT predistortion system.

Introduction

Power amplifier (PA) is an integral part of the wireless communication system, but the inherent nonlinearity of PA will lead to signal distortion. Predistortion technology is an important branch of PA linearization technology, and the baseband predistortion technology has gained widely attention with the advantages of wide working bandwidth, low cost and outstanding performance [1-3].

In the existing researches, there are usually two kinds of predistorter (PD) model: polynomial model and lookup table (LUT) model. The polynomial model works well with those PAs which have weak nonlinearity characteristic [4-6], and its advantages include less system parameters, simple system structure, good system stability. But with the strongly nonlinear PA, polynomial model will bring about considerable modeling error. In this case, we need to select the LUT model as the PD model, namely use line segment fitting the PD characteristic approximatively [3,7-8].

The performance of the LUT predistortion algorithm is affected by many factors, and the relationship between them is complex. In this paper, the performance of the PD LUT algorithm has been analysed and emulated, and the research focuses on the effect on the system performance with the length of LUT, the nonlinearity characteristic of PA, and different input signal distribution.

System model

The transmitter with baseband predistortion function is shown in Fig. 1. There is a loop couplings signal from the output of PA, and after downconvertor and analog to digital converter, the digital baseband signal $v_a$ is gotten. We define the RF circuit, including upper and down converters and PA, as the function $PA(\bullet)$, and basing on the transmitted signal $v_m$ and feedback signal $v_a$, the function $PD(\bullet)$ is gotten, and with the joint system combing PD and PA, there is a linear relationship between the system input and output:

$$v_a = PA(PD(v_m)) = k \cdot v_m, \quad k \text{ is a constant}$$

(1)
Performance analysis

Defining the input signal as \( v_m = r_m e^{i\theta_m} \), where \( r_m \) is the amplitude of input signal, and \( \theta_m \) is the phase. The output signal of PA is

\[
v_a = r_a e^{i\theta_a} = PA\left(PD\left(v_m\right)\right) = r_m f_a\left(r_m\right) g_a\left(r_m f_a\left(r_m\right)\right) e^{i(\theta_a + f_p\left(r_m\right) + g_p\left(r_m f_a\left(r_m\right)\right))}
\]

(2)

Fig. 2 The characteristic of PD and PA

Where \( f_a(\bullet) \) is the amplitude gain function of PD, and \( f_p(\bullet) \) is the additive phase shift function of PD, \( g_a(\bullet) \) is the amplitude gain function of PA, and \( g_p(\bullet) \) is the additive phase shift function of PA.

In theory, normalize the system gain \( k \), we can get

\[
f_a\left(r_m\right) g_a\left(r_m f_a\left(r_m\right)\right) = 1
\]

(3)

\[
f_p\left(r_m\right) + g_p\left(r_m f_a\left(r_m\right)\right) = 0
\]

(4)

In the LUT PD system, the characteristic of PD is modeled by LUT. For the limited length of LUT, quantization process is unavoidable. Fig. 2 is the characteristic diagram of PA and PD, the PD amplitude gain of the input signal with amplitude \( r_m \) is \( f_a\left(r_0\right) = f_a\left(r_m\right) + \delta f_a \), and the PD additive phase is \( f_p\left(r_0\right) = f_p\left(r_m\right) + \delta f_p \). So the amplitude and phase of output signal of PA can be written as

\[
r_a = g_a\left(r_m \left(f_a + \delta f_a\right)\right)\left(f_a + \delta f_a\right) r_m
\]

(5)

\[
\theta_a = g_p\left(r_m \left(f_a + \delta f_a\right)\right) + f_p\left(r_m\right) + \delta f_p + \theta_m
\]

(6)
Where \( f_a \) is short for \( f_a(r_m) \), \( f_p \) is short for \( f_p(r_m) \), expand (5) and neglect the high order term of \( \delta f_a \), we can get
\[
 r_a = r_m \left( g_a f_a + r_m g'_a f_a \delta f_a + g_a \delta f_a \right) \tag{7}
\]

Where \( g_a \) is short for \( g_a \left( r_m, f_a(r_m) \right) \). And from Fig. 2 we know \( \delta f_a = -\left( f_a(r_m) - f_a(r_0) \right) = -f'_a \delta r_m \), then (7) can be written as
\[
 r_a = r_m \left( g_a f_a - r_m g'_a f_a \delta r_m - g_a f'_a \delta r_m \right) \tag{8}
\]

The derivative of (3) is
\[
 r_m g'_a f'_a f_a + g'_a f_a^2 + g_a f'_a = 0 \tag{9}
\]

Put the (3) and (9) in (8) we can get
\[
 r_a = r_m \left( 1 + \frac{g'_a}{g_a} \delta r_m \right) \tag{10}
\]

The same operation is done to the addative phase \( \theta_a \), we can get
\[
 \theta_a = k \frac{g'_p}{g_a} \delta r_m + \theta_m \tag{11}
\]

Where \( g_p \) is short for \( g_p \left( r_m \left( f_a + \delta f_a \right) \right) \). The the out put signal can be written as
\[
 v_a = r_m \left( 1 + \frac{g'_a}{g_a} \delta r_m \right) e^{\left( \frac{g'_p}{g_a} \delta r_m + \theta_m \right)} \tag{12}
\]

Expanding (12) and neglect the high order term we can get
\[
 v_a = r_m \left( 1 + \frac{g'_a}{g_a} \delta r_m + \frac{j g'_p}{g_a} \delta r_m \right) \tag{13}
\]

From (13) we know that there is system error causing by the quantization of PD LUT, that is \( v_v = v_a - v_m \). Deifine \( \delta r_m = r_m - r_0 \), and the signal to quantization noise ration (SNR) is
\[
 SNR = \frac{P_{ac}}{P_{v_c}} = \frac{E \left[ |v_m|^2 \right]}{E \left[ |v_c|^2 \right]} = \int_0^1 r_m^2 p(r_m) dr_m / \int_0^1 r_m^2 \left[ \frac{g_a^2}{g_a^2} + \frac{g_p^2}{g_a^2} \right] |\delta r_m|^2 p(r_m) dr_m \tag{14}
\]

In the quantization process, \( \delta r_m = 2^{-N} \), where \( 2^N \) is the length of LUT with the coder bit length \( N \). And from the mean value theorem for integrals, we can get
\[ SNR \geq 2^{2N} \left( \frac{g_a^2}{g_u^4} + \frac{g_p^2}{g_u^2} \right)_{\text{max}}^{-1} \] (15)

From (14) and (15) the conclusions can be gotten that the SNR of the transmitter with LUT PD is determined by the nonlinearity character of PA, including the amplitude nonlinearity \( g_a \) and phase nonlinearity \( g_p \), and the distribution of the input signal \( p(r_m) \), and the length of LUT. The increasing of the SNR with the increasing of length of LUT following the rule that the length of coder bit \( N \) increases to \( N+1 \), the SNR increases at least 6dB.

### Simulations

In following simulations, the PA model is Saleh model [9], and the performance of PD system is scaled by system SNR defined as

\[
SNR(dB) = 10 \log_{10} \left[ \frac{\sum_{n=0}^{N-1} |v_m|^2}{\sum_{n=0}^{N-1} |v_e|^2} \right]
\]

Fig. 3 is the relationship diagram of length of LUT and the system SNR. With the uniform distribution input signal, the SNR increases with the increasing of length of LUT, and the trend as our expectation is SNR increases at least 6 dB when the LUT length increases doubly.

Fig. 4 is the learning curves of the PD systems with different PAs. Using the saleh model as PA model as follows

\[
g_a(r) = \frac{(1+a_1)r}{1+a_1\cdot r^2}, \quad g_p(r) = \frac{\pi \left(1+a_2\right)r}{12 + a_2 \cdot r^2}
\]

And three groups of values as \( a_1 = a_2 = 0.2 \), \( a_1 = a_2 = 0.5 \) and \( a_1 = a_2 = 1 \) define three different nonlinearity characteristic PAs, those are PA1, PA2 and PA3. And with Nagata arithmetic [10], one of the adaptive PD LUT arithmetic, and the LUT length is 64, the conclusion is that the PA with weak nonlinearity characteristic has good system performance, the reason is that the weak nonlinearity PA comes to weak nonlinearity PD, which has slow variety nonlinearity characteristic line due to little PD LUT quantization noise.

Fig. 5 shows the relationship of system SNR with different input signal distribution, including uniform distribution and OFDM signal. With same input backoff value, the OFDM signal has good
performance, and the reason is that OFDM signal has higher occurrence probability of little signal than the uniform distribution, and the little signal has weaker nonlinearity and smaller distortion than the large one.

![Graph](image)

**Fig. 4 The system SNR with different PAs**

**Fig. 5 The system SNR with different input signals**

**Conclusion**

Theoretical investigation and simulation have been done on the LUT PD system, the results show that ignoring the effect of measurement noise, the system SNR is decided by the length of LUT, the nonlinearity characteristic of PA and the distribution of input signal together. Fristly, the SNR increases with the increasing of the LUT length, and if the length of LUT increases doublely the SNR increases at least 6 dB. Secondly, the weaker nonlinearity characteristic PA comes to good system performance. And lastly, the input signal with higher exiting probability of little signal has finer SNR. These conclusions are helpful to make correct trade-off between system implementation complexities with system performance, and design the suitable parameter of the LUT predistortion system.

**References**


