Attribute Reduction Algorithm Based on Discernibility Matrix with Algebraic Method

GAO Jing^{1,a}, Ma Hui¹, Han Zhidong^{2,b}

¹Information School, Capital University of Economics and Business, Beijing 100070

²Beijing Information Center of China UnionPay Co., Ltd, Beijing 100193

^aemail:gaojinggaoj@126.com, ^bemail:hanzhidong@sina.com

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Abstract. There are many algorithms of attribute reduction based on discernibility matrix, where the element of the discernibility matrix is used as heuristic information. But there are few algebra methods about attribute reduction based on discernibility matrix. In this paper, we proposed an algorithm of attribute reduction with algebra method. First of all, we should design an algorithm to compute the heuristic information. Then we used this heuristic information to design an new algorithm of attribute reduction based on discernibility matrix, whose time complexity is $O(|C|^2|U|)$, and whose space complexity is $O(|U|)$. At last, we used an example to illustrate the new algorithm's validity and high efficiency.

Introduction

Attribute Reduction is one of the researches in rough set theory ^[1], and which based on discernibility matrix has attracted a lots of attention as its simplify and intuitive ^[2]. On one hand, the research on Attribute Reduction Algorithm Based on Discernibility Matrix is more inclined to be designed by the heuristic information, which is composed by elements of the discernibility matrix. On the other hand, Professor Pawlak has put forward many design methods of Attribute Reduction Algorithm of Attribute Reduction model. Namely, they are respectively algebra method ^[3], discernibility matrix method ^[4-6] and matrix ^[7] etc.

So is attribute reduction algorithm of attribute reduction model based on information entropy proposed by Professor Wang, however, But the design of attribute reduction algorithm of discernibility matrix in algebraic method is less used, this is mainly because the model has no direct algebraic definition

Reference [10-11] illustrates a new attribute reduction model based on information. Reference [12] has proved that the model is equal to the Attribute Reduction Algorithm Based on Discernibility Matrix. Of which Reference [13] has put forward a matrix method. So, the paper designs an algebra method of Attribute Reduction based on discernibility. The Specific steps are as follows: Firstly, we design a algorithm to calculate a heuristic information with rapid response, then based on the heuristic information we design Attribute Reduction algorithm based on discernibility matrix. Whose time complexity is $O(|C|^2|U|)$, and space complexity is $O(|U|)$. At last, an example is provided to illustrate the new algorithm.

Basic Theory

Definition 1: Decision Table is defined as follows: $S = (U, C, D, V, f, d)$. In this formula, $U = \{x_1, x_2, \dots, x_n\}$ is the domain, $C = \{c_1, c_2, \dots, c_r\}$ is the conditions attribute sets. D is the decision attribute sets. $f: U \times C \rightarrow V$ and $d: U \times D \rightarrow V$ is information functions, in the functions, $F = C \cup D, C \cap D = \emptyset, V = \cup V_a, a \in F, V_a$ means range of a.

Definition 2:

Define discernibility matrix as $M = (m_{ij})$ in $S = (U, C, D, V, f, d)$. The element is as follows:

$$m_{ij} = \begin{cases} \{c_k \mid c_k \in C, f(x_i, c_k) \neq f(x_j, c_k), d(x_i, D) \neq d(x_j, D)\} \\ \emptyset \quad \text{else} \end{cases} \quad (1)$$

In this formula, $k = 1, 2, \dots, r; i, j = 1, 2, \dots, n$.

Definition 3: In decision table, $S = (U, C, D, V, f, d)$, we hypothesize that $M = (m_{ij})$ is HU discernibility matrix, $\forall B \subseteq C$, if B satisfies following conditions: (1) $\forall \emptyset \neq m_{ij} \in M$, $B \cap m_{ij} \neq \emptyset$; (2) $\forall b \in B$, $B - \{b\}$ doesn't satisfy (1), B is considered as attribute reduction based on discernibility matrix. all attribute reductions is short for $HRED_C(D)$.

Definition 4: in decision table, $S = (U, C, D, V, f, d)$, any attribute in U $B \subseteq C \cup D$ (knowledge, Equivalence relation family, $U/B = \{B_1, B_2, \dots, B_t\}$) is a random variable consisted in subset of U in algebra., the Probability distribution is confirmed by the following methods:

$$[B: p] = \begin{bmatrix} B_1 & B_2 & \dots & B_t \\ p(B_1) & p(B_2) & \dots & p(B_t) \end{bmatrix} \quad (2)$$

In the formula, $p(B_j) = |B_j| / |U|$, $j = 1, 2, \dots, t$

Definition 5:

In decision table, $S = (U, C, D, V, f, d)$, the knowledge information of decision attributes sets $U/D = \{D_1, D_2, \dots, D_k\}$ respecting to condition attributes sets $U/C = \{C_1, C_2, \dots, C_m\}$ is defined as follows:

$$I(D|C) = \sum_{i=1}^m \sum_{j=1}^k \left| \frac{X_i \cap D_j}{|U|} - \left| \frac{D_j}{|U|} - \frac{X_i \cap D_j}{|U|} \right| \right| \quad (3)$$

$$X_i^j = X_i \cap D_j \in X_i/D = \{X_i^1, X_i^2, \dots, X_i^k\} \quad (4)$$

Definition 6:

In decision table, $S = (U, C, D, V, f, d)$, for $\forall b \in B \subseteq C$, if $I(D|B) = I(D|(B - \{b\}))$, b is Omitted for B with respect to D . or, b is cannot be omitted for B with respect to D . for $\forall B \subseteq C$, if any element is necessary for D , then B is independent for D .

Definition 7:

In decision table, $S = (U, C, D, V, f, d)$, if $\forall B \subseteq C$, $I(D|B) = I(D|C)$ and B is independent for D , then B is attribute reduction based on knowledge for C with respect to D . mark all the attribute reduction as $NewRED_C(D)$.

Theorem 1, in the decision table, $S = (U, C, D, V, f, d)$, Attribute reduction based on discernibility matrix is equivalent to attribute reduction based on amount of knowledge.

We will design attribute reduction based on discernibility matrix with algebraic method under the condition of Theorem 1;

Theorem 2, in the decision table, $S = (U, C, D, V, f, d)$, $\forall C \supseteq A \supseteq B$, $I(D|A) \leq I(D|B)$ is deduced.

Definition 8: in the decision table, $\forall a \notin C - B$, define the heuristic information as $sig_B(a) = I(D|B) - I(D|B - \{a\})$.

Obviously, $sig_B(a) \geq 0$, and $sig_B(a)$ get bigger, the incremental amount of information of the a property is greater

Definition 9:

In the decision table, $S = (U, C, D, V, f, d)$, $\forall X \subseteq U$, $X/\{a\} = \{X_1, X_2, \dots, X_r\}$,

$$g(X, a) = \sum_{i=1}^r \sum_{j=1}^k \left| \frac{X_i \cap D_j}{|U|} - \left| \frac{X_i \cap D_j}{|U|} - \frac{X \cap D_j}{|U|} \right| \right| \text{ is the function we define.}$$

Theorem 3, in the decision table, $S = (U, C, D, V, f, d)$, $\forall U/B = \{B_1, B_2, \dots, B_t\}$, $a \notin C - B$,

$sig_B(a) = \sum_{i=1}^t g(B_i, a)$ can be deduced from definition 8 and definition 9.

High-efficient attribute reduction algorithm

In order to get the high-efficient attribute reduction algorithm, firstly we get the algorithm of quick response calculation heuristic information.

Algorithm1: calculate $g(X, a)$

Input: decision table $S = (U, C, D, V, f, d), \forall X \subseteq U, a \in C$

Output: $g(X, a)$

- 1). Calculating $X / \{a\}$ with Radix sort in the reference [3]
- 2).calculating $X / D, X_i / D$ with radix sort in the reference [3]
- 3).calculating $g(X, a)$ with the formula in definition 9.

Complexity analysis of algorithm 1:

The time complexity of the first step of algorithm is $O(|X|)$, The time complexity of second step is $O(|X|)$, so the time complexity of algorithm is $O(|X|)$, The worst-case complexity of space is $O(|X|)$.

Algorithm 2: calculate $sig_B(a)$.

Input: in decision table, $S = (U, C, D, V, f, d), \forall U / B = \{B_1, B_2, \dots, B_t\}, a \in C - B$

Output: $sig_B(a)$.

- 1) calculate $g(B_i, a)$ with algorithm 1.
- 2) calculate $sig_B(a)$ with algorithm 3.

Complexity principal analysis of algorithm 2:

The time Complexity of the first step of algorithm is $O(|B_i|)$, so the worst-case complexity of time is $\sum_{i=1}^t O(|B_i|) = O(|U|)$, the worst-case complexity of space is $O(|U|)$.

Algorithm 3: attribute reduction algorithm

Input: decision table $S = (U, C, D, V, f, d)$.

Output: attribute reduction R;

- 1) $R = \emptyset$;
- 2) for any $a \in C - R$, calculate $sig_R(a)$ with algorithm 2.
- 3) make $sig_R(b) = \max_{a \in C-R} |sig_R(a)|$, if $sig_R(b) > 0$, then $R = R \cup \{b\}$, get to the step 2, or get to the step 4.
- 4) output the attribute reduction R.

Complexity principal analysis of algorithm 3:

The worst-case time complexity of second step of algorithm 3 is $O(|C - R| |U|)$, so the worst-case time complexity of algorithm 3 is $\sum_{|R|=0}^{|C|} \sum_{i=1}^t O(|C - R| |U|) = O(|C|^2 |U|)$, the worst-case space complexity is $O(|U|)$.

Example analysis

Decision table in the literature [13] is used to illustrate the new algorithm.

Table 1 decision table

	a	b	c	d	D
X1	2	1	2	1	0
X2	2	2	2	1	1
X3	2	1	2	1	0
X4	2	2	2	1	1
X5	3	1	2	1	0
X6	1	2	3	2	2

Calculate $sig_{\emptyset}(a) = \frac{2}{6}(\frac{0}{6} + \frac{0}{6}) + \frac{2}{6}(\frac{0}{6} + \frac{0}{6}) + \frac{1}{6}(\frac{2}{6} + \frac{2}{6} + \frac{1}{6}) + \frac{1}{6}(\frac{2}{6} + \frac{2}{6} + \frac{1}{6}) = \frac{10}{36}$ with algorithm 3.

By the same token, $sig_{\emptyset}(b) = \frac{18}{36}$, $sig_{\emptyset}(c) = \frac{10}{36} = sig_{\emptyset}(d)$. We choose attribute b according the third step, so we get the equation $R = \{b\}$, and then get to the step 2. Calculate $sig_{\{b\}}(a) = \frac{8}{36}$, $sig_{\{b\}}(c) = sig_{\{b\}}(d) = \frac{4}{36}$, we choose attribute 2 according to the third step. So we get the expression- $R = \{b, a\}$, then jump to step 2, calculate $sig_{\{b,a\}}(c) = sig_{\{b,a\}}(d) = 0$ by step 4, attribute reduction $R = \{b, a\}$ is outputted.

Conclusion

In this paper, algebraic expression attribute reduction definitions based on discernibility matrix is put forward, and then starting from the attribute reduction definitions of algebraic methods, the heuristic information is defined, and the algorithm of heuristic information calculating with rapid reaction is provided, finally design an attribute reduction algorithm based on discernibility matrix, witch time complexity is $O(|C|^2|U|)$ and space complexity is $O(|U|)$. Finally, an example is used to illustrate the effectiveness and efficiency of the new algorithm.

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