Design of the MFCS in Cutting Systems with Time-Delay

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Abstract. From the last ten years, control and analysis of systems with delays is subject to lots of studies and contributions. This paper presents about the design of cutting system for time-delay. The design of the control system is constructed using the easy algebraic algorithm of matrices whose elements are polynomials of the two kinds of operators. Bounded of the inner states for the control system is given and the utility of this control design is guaranteed. It is confirmed on basis of the example that the output signal of the control system asymptotically follows the reference model signal in the case of the existence.

Introduction

The time-delay is always existent in the practical systems. Analysis of the delay phenomenon in a continuous-time domain is sophisticated [1]. There are several reasons which motivate the continuous interest and development in the time-delay systems field research. Time delays naturally appear in many physical systems [2]. Cutting process is one of the most popular manufacturing methods; however, it generates self-excited vibrations of negative effects. Specifically, they can reduce the volumetric efficiency, increase the tool wear, and decrease the geometric accuracy and surface finish [3]. Time-delay in the cutting process may result in shortened tool life and poor surface finishes, hence it should be avoided in practice. In order to predict the cutting process stability, much effort has been devoted to modeling the dynamic cutting process [4-5].

In this paper, design of model following control system (MFCS) for cutting systems with time-delay is discussed. It was proposed by [6-8]. The method of linear MFCS will be extended to cutting control system with time delays. In this case, the effectiveness of this method has verified by a numerical example and simulations. The paper structure is as follows. First, the cutting systems with time-delay are described in Section 2. The design of the cutting systems with time-delay is detailed in Section 3, an example to show the effectiveness of the proposed method in Section 4. The conclusion is finally drawn in Section 5

The cutting systems with time-delay [9]

Turing operation, as shown in Fig. 1, is investigated in this paper. With the assumption of rigid workpiece, the corresponding equation of motion is

$$m\ddot{x} + c\dot{x} + kx = \cos(\beta)K_s b(h_m + x(t - \tau) - x(t)) \quad (1)$$

where

R_{ss} is spindle speed in r/min;

$\tau = 60 / R_{ss}$ is the time delay;

m is the modal mass;

c is the modal damping;

k is the modal stiffness;

K_s is an empirical coefficient which depends on workpiece material;

b is chip width;

β represents the force angle between F and F_n ;

h_m is a constant and can be ignored on the effect of the cutting vibration.

Fig. 1 Turning operation^[9]

Thus, standard discrete form of (8) without time delay can be obtained as^[9]

$$x(k+1) = Ax(k) + Bu(k) + d(k) \quad (2)$$

$$y(k) = Cx(k) \quad (3)$$

where $\bar{F} = [F_0 \quad F_1 \quad F_2]$ and

$$F_0 = \begin{bmatrix} F \\ 0 \\ \vdots \\ 0 \\ [1 \quad 0] \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad F_1 = \begin{bmatrix} G_1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \quad F_2 = \begin{bmatrix} G_2 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \quad \bar{G} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

Here $A \in R^{n \times n}$, $B \in R^{n \times m}$, $C \in R^{m \times n}$.

Design of cutting system with time-delay

Control law $u(t)$ can be written

$$u(k) = -Hx(k) + Kx(k) \quad (4)$$

$$x(k+1) = x(k) + [r - y(k)] \quad (5)$$

Then we take

$$\begin{bmatrix} x(k+1) \\ z(k+1) \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & I_m \end{bmatrix} \begin{bmatrix} x(k) \\ z(k) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(k) + \begin{bmatrix} d \\ r \end{bmatrix} \quad (6)$$

$$y(k) = [C \quad 0] \begin{bmatrix} x(k)^T & z(k)^T \end{bmatrix}^T \quad (7)$$

Therefore, $u(k+1)$ defined by

$$\begin{aligned} u(k+1) &= -Hx(k+1) + KZ(K+1) \\ &= -[H(A - I_m) + KC]x(k) + [I_m - HB]u(k) + Kr - Hd \end{aligned} \quad (8)$$

Example

To illustrate some of the results presented in section 4, we consider the following example with time-delay $h_m = 0.2$. The system is described by

$$x(k+1) = 2x(k) + u(k)$$

$$y(k) = x(k)$$

Then we have

$$\begin{bmatrix} x(k+1) \\ z(k+1) \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x(k) \\ z(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(k) + \begin{bmatrix} 0 \\ r \end{bmatrix}$$

$$u(k) = -(H, -K) \begin{bmatrix} x(k) & z(k) \end{bmatrix}^T$$

and

$$\begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} (H, -K) = \begin{bmatrix} 2-H & K \\ -1 & 1 \end{bmatrix}$$

Intrinsic value is described as follows

$$\begin{bmatrix} z-2+H & -K \\ 1 & z-1 \end{bmatrix} = z^2$$

We have

$$H = 3, K = 1$$

Closed-loop system is given as follows

$$\begin{bmatrix} x(k+1) \\ z(k+1) \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x(k) \\ z(k) \end{bmatrix} + \begin{bmatrix} 0 \\ r \end{bmatrix}$$

$$y(k) = x(k)$$

Let $z(0) = 0$, using $x(0) = 0$ and $r = 1$, we have

$$\begin{bmatrix} x(0) \\ z(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} x(1) \\ z(1) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} x(i) \\ z(i) \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

The output responses for the above case are shown in Fig. 2. From the results, it can be seen that the control systems are valid.

Fig. 2 Responses for the system

Conclusion

The paper discussed study of the cutting systems with time-delay. The cutting controller and corresponding control laws are designed. Based on MFCS theory, we prove the resulting closed-loop error system is uniformly ultimately bounded stable. An example is given to verify the feasibility and validity of the main results.

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