The synchronous algorithm of FTN signaling on independent pilot symbols

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***Abstract*—Mazo discovered that the performance of system symbol error would not degrade with manual Inter-Symbol Interference(ISI) when symbol rate is faster at some extend. This paper which took FTN as background mainly explained the carrier synchronization performance of FTN signaling under additive white gaussian noise (AWGN) channel. With the characteristics of FTN signaling introduced, this paper analyzed how the carrier asynchronization affected the performance of FTN signaling. The noise compound in the receiver signal through match filter was time-domain correlated for the sampling rate was faster than Nyquist rate. The traditional carrier synchronization method is not proper for FTN , which is applied under white noise background. Based on the above, we put forward the independent pilot symbols carrier synchronization method. The result shows that independent pilot symbols can estimate the carrier frequency efficiently to recover the performance of FTN signaling.**

Keywords—FTN, carrier synchronization, SNR

I. INTRODUCTION

4G has been applied into all domains of communication by mobile business in China adopted LTE standards ,which improved the data rate. The transmission system has to insert pilot symbols for the result of OFDM modulation model. As consequence, the data rate would be hindered by the pilot symbols. Now the 5G[1] wireless communication transmission system standards is under intense discussion. FTN [2]is one of the scheme of 5G. In 1975, J. Mazo discovered that the minimum distance didn't change, even the data rates was about 25 percent faster than the Nyquist rate under AWGN channel. We know that FTN can break traditional transmission scheme satisfying the Nyquist rate[3][4].

FTN brings the ISI into transmission system by compressing the time slot, which increases the complexity of decoding. With the time-slot compressing coefficient declining, the ISI of FTN become more serious. The noise at best sampling points is time-domain correlated after matched filter.

The transmission channel is not ideal channel in real communication process, which would affect the time-frequency characteristics of signals. In order to recover the ideal reception performance, need to elimilate the adverse effect of channel deterioration. Specially, the residue frequency offset will degrade the performance of FTN signaling seriously for carrier asynchronization. This passage mainly analyzed the synchronization of FTN.

The estimation of residue frequency offset plays important role in the FTN signaling and traditional transmission scheme. It's significant for satellite communication system to drop down the threshold SNR of receiver for receive for received power is limited by apparatus. With the development of Turbo code、LDPC code and Polar code, communication can still work under low SNR for these coding method have high coding gain. But these encoding must be completely in carrier and bit timing synchronization to obtain high gain. Traditional synchronization method is divided into DA(Data Aid) and NDA(None Data Aid).

In terms of traditional transmission system, the adjacent symbols' wave overlaps with each other for time-slot compressing in FTN signals, which changes signal envelope and PAPR. We can't estimate the residue frequency offset with traditional time and frequency domain estimation method. The exact frequency offset estimation can remove the harmful effect of carrier asynchronization on performance of FTN signaling.

Under background of colored noise, the performance of frequency-domain frequency offset estimation is not good. Based on frequency-domain estimation , the main idea is to search the peak of frequency-domain of single tone signal corresponding to the crude valuation and accurately estimate the frequency with the spectrum amplitude and phase information in a similar interpolation method. Low SNR will degrade the synchronization performance.

Recovering sine signal from colored noise. Pisarenko harmonic decomposition[5][6] can determine the frequency component of sine signal from colored noise with feature vector and pseudo spectrum calculation. So the Pisarenko harmonic decomposition can effectively estimate frequency only when the sine signal autocorrelation function is accurately known. So the frequency component can't be determined with small frequency and short sampling data. For the noise component at the sampling points is time-domain correlated, the passage choose two pieces pilot sequences of independent noise sequence to obtain the exact frequency offset.

Firstly, the part 2 of the passage introduced the model of FTN signal. Part 3 analyzed the performance of FTN with residue frequency offset. Part 4 shows the result of simulation to prove that the independent pilot symbols can estimate the frequency offset efficiently.

II. FTN SYSTEM MODEL

A. Transmitting model

Presume $q(t)$ is the symbol shaping wave. Expression in its spectrum: $Q(f)=0, |f|>\frac{1}{2T}$.

Traditional transmission system for sending cycle T . the transmitted waveform is expressed as:

$$s(t)=\sum_{i=0}^{N-1} u_i q(t-iT) \tag{1}$$

FTN with sending cycle $\tau T(\tau<1)$:

$$s(t)=\sum_{i=0}^{N-1} u_i q(t-i\tau T) \tag{2}$$

The following figure show the traditional transmission scheme how to compress time-slot to create the FTN signal.

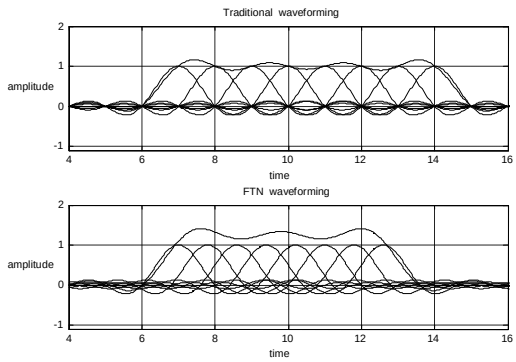

Figure 1: traditional transmission and FTN signal.

B. Matched receiving model

FTN signal goes through the matched filter under AWGN channel with carrier asynchronization. The receiving signal expression of FTN :

$$r_n(t)=s_n(t)e^{(j2\pi f_d t+\varphi)}+n_n(t) \tag{3}$$

Here, presume it's AWGN channel with noise power spectrum density $N_0/2$, obtain the best sampling sequence:

$$\begin{aligned} x[n] &= \int_{-\infty}^{\infty} r(t)q^*(t-n\tau T)dt \\ &= \int_{-\infty}^{\infty} [s_n(t)e^{(j2\pi f_d t+\varphi)}+n_n(t)]q^*(t-n\tau T)dt \\ &= \int_{-\infty}^{\infty} s_n(t)e^{(j2\pi f_d t+\varphi)}q^*(t-n\tau T)dt+\int_{-\infty}^{\infty} n_n(t)q^*(t-n\tau T)dt \end{aligned} \tag{4}$$

In FTN transmission system, the sampling value is composed of adjacent symbols for ISI. In real progress, we take its truncated waveform:

$$q(t)=0, |t|>ZT, (Z=1,2,3\cdots) \tag{5}$$

Define $K=\lceil 2Z/\tau \rceil$ as the number of related symbol in a symbol interval. $e^{(j2\pi f_d nt+\varphi)}$ in (5) is approximate to constant within the interval length less than $2\tau T$. (4) can turn into :

$$\begin{aligned} x[n] &= \sum_{i=0}^{N-1} u_i e^{(j2\pi f_d n\tau T+\varphi)} \int_{-\infty}^{\infty} q(t-i\tau T)q^*(t-n\tau T)dt \\ &= \sum_{i=n-(K-1)}^{n+K-1} u_i g_q[n-i]e^{(j2\pi f_d n\tau T+\varphi)}+\eta[n] \end{aligned} \tag{6}$$

Where :

$$\begin{aligned} g_q[n-m] &= \int_{-\infty}^{\infty} q(t-n\tau T)q^*(t-m\tau T)dt \\ \eta[n] &= \int_{-\infty}^{\infty} n(t)q^*(t-n\tau T)dt \end{aligned} \tag{7}$$

C. FTN whitening

The autocorrelation function of the noise sequence $\eta[n]$ is $E[\eta(n)\eta^*(m)]=\frac{N_0}{2}g_q[n-m]$. The receiving signal needs to go through whitening filter and be decoded with VA. According to Forney whitening method, $G(z)$ is the Z-transform of $g_q(n)$, which can turn into $F(z)F(1/z^*)$. After whitened by filter $1/F(z)$, the signal can be expressed by $y[n]$. The z-transform progress is represented by following equation:

$$Y(z)=\frac{U(z)G(z)}{F(z)}+\frac{N(z)}{F(z)} \tag{8}$$

Where $U(z), N(z)$ denotes the receiving signal and noise z-transform. What remains can be expressed as

$$y=u*h+w \tag{9}$$

D. Decoding algorithm of FTN

From (9), we can find the right decoding sequence with MLSD[7][8]. The decoding algorithm is as follow:

$$\tilde{\mathbf{U}}=\arg\min_{\tilde{\mathbf{U}}} \|Y-\mathbf{UH}\|^2 \tag{10}$$

when the FTN signal is received with accurate carrier synchronization, noise power is the main factor affecting the performance of receiving signal of FTN. Unable to decode accurately with frequency offset in real process of communication. The whitening filter used in accurate carrier synchronization.

III. FREQUENCY OFFSET ESTIMATION

A. Traditional ML estimation based on DA

For improving the performance of FTN, need to estimate the frequency offset efficiently. Usually the accurate frequency estimation rely on the estimation of a single tone signal which can be formed by eliminating the modulation information. The single tone followed by:

$$x(t)=\mathrm{e}^{j2\pi f_d t+\varphi}+w(t) \tag{11}$$

There are many manners to estimate the frequency of single tone under AWGN channel. The frequency estimation

variety can close to CRML in a certain complexity. Then restore the original information sequence with the right frequency estimation value. Carrier frequency maximum likelihood estimation is expressed as:

$$\tilde{f} = \arg \max_{\tilde{f}} \{p(r \mid f)\} \tag{12}$$

The probability is as follow:

$$p(r \mid f) = \frac{1}{L} \exp[\frac{\int_0^{N_p \tau T} (r(t) - \tilde{s}_f(t))^2 dt}{2N_0}] \tag{13}$$

Where:

$$\tilde{s}_f(t) = e^{j2\pi ft} \sum_{i=0}^{N-1} u_i q(t - i\tau T) \tag{14}$$

where N_p is the number of pilot symbols, N_0 is the noise power density. (14) can be expressed as:

$$\Lambda(r \mid f) = \int_0^{N_p \tau T} \mathrm{Re}(r(t) \bullet \tilde{s}_f^{\,*}(t)) dt \tag{15}$$

Get a rough estimation of the frequency offset based on the frequency scanning mode. So it's necessary to obtain more accurate frequency offset estimation.

In traditional orthogonal conditions($\tau = 1$), (16) can be expressed as :

$$\begin{aligned} &\int_0^{N_p T} r(t) \bullet \tilde{s}_f^{\,*}(t) dt \\ &\approx \sum_{k=0}^{N_p - 1} u_k^* x(k) \end{aligned} \tag{16}$$

Where:

$$x(k) = \int_{-\infty}^{\infty} e^{j2\pi fkT} r(t) q(t - kT) dt \tag{17}$$

The likelihood function as follow:

$$\Gamma(f) = \left| \sum_{k=0}^{N_p - 1} u_k^* x(k) \right| \tag{18}$$

In FTN signal, (16) isn't suitable anymore for the orthogonal basis of $\tilde{s}_f(t)$ is not $q(t - i\tau T)$.

B. Frequency Offset Estimation in Colored Noise

The paper assumed the transmitting signal goes through AWGN with white noise which will become time-domain correlated at sample point.

Recover a single complex sinusoidal signal from FTN signal with the overlapping sampling values. Direct elimination modulation information will change the distribution of noise. The know correct overlap value is expressed as:

$$c[n] = \sum_{i=n-(K-1)}^{n+K-1} u_i g_q[n - i] \tag{19}$$

The single complex sinusoid signal:

$$\begin{aligned} z[n] &= x[n] / c[n] \\ &= e^{(j2\pi f_d n\tau T + \varphi)} + \gamma[n] \end{aligned} \tag{20}$$

Where:

$$\gamma[n] = \frac{\eta[n]}{c[n]} \tag{21}$$

The autocorrelation function of noise :

$$E[\gamma(n)\gamma^*(m)] = \frac{N_0}{2} \frac{g_q[n-m]}{c(n)c(m)} \tag{22}$$

Due to the correlation of noise samples, unbiased frequency estimation can't be obtained directly. Therefore the paper proposed the improved method of time-domain frequency estimation with two independent pilot sequences:

$$\begin{aligned} &z_1[n_1] = e^{(j2\pi f_d n_1 \tau T + \varphi)} + \gamma_1[n_1], (n_1 = 0,1,2 \cdots N_p - 1) \\ &z_2[n_2] = z_1[n_1 + N_p + K], (n_2 = 0,1,2 \cdots N_p - 1) \end{aligned} \tag{23}$$

The correlation function between the two sequences is expressed as:

$$\begin{aligned} r_{z_2 z_1}(m) &= E[z_2(n+m) z_1^*(n)] \\ &= E[(e^{(j2\pi f_d (K+N_p+n+m)\tau T + \varphi)} + \gamma_2[n+m])(e^{(j2\pi f_d n\tau T + \varphi)} + \gamma_1[n])^*] \\ &= e^{j2\pi f (m+K+N_p)\tau T}, \quad (m > 0) \end{aligned} \tag{24}$$

Where rate is $R = \frac{1}{\tau T}$ and relative frequency offset is $f_r = f / R$,(24) can be expressed as:

$$r_{z_1 z_2}(m) = e^{j2\pi f_r (m+K+N_p)}, \quad (m \geq 0) \tag{25}$$

Two pilot sequences have independent colored noise which will affect the frequency estimation. Obtain the phase difference of the two pilot symbols by calculating the correlation function of them. The accumulation of frequency difference turns into phase difference, which can be used to estimate the frequency offset. The expectation of phase difference $E[angle(r_{z_1 z_2}(m))] = 2\pi f_r (m + K + N_p)$ can be used to obtain unbiased frequency estimation:

$$f = -\frac{angle(r_{z_1 z_2}(m))}{2(m + K + N_p)\pi} \tag{26}$$

Every correlation function values involve the frequency information, so

$$\tilde{f} = \frac{1}{\pi N_p (3N_P + 2K + 1)} \sum_{m=0}^{N_p - 1} angle(r_{z_1 z_2}(m)) \tag{27}$$

With the increasing of pilot symbol number, the performance of frequency estimation can be improved .

C. Improved frequency estimation method

Because pilot symbols reduce the transmission efficiency, communication system needs to decline the length of pilot symbols. Based on above, it's efficient to choose two pilot

sequences with independent noise not independent information, which can still obtain unbiased frequency estimation:

$$\begin{aligned} z_1[n_1] &= e^{(j2\pi f_d n_1 \tau T+\varphi)} + \gamma_1[n_1], (n_1 = 0,1,2\cdots N_p - 1) \\ z_2[n_2] &= z_1[n1+K_p] \end{aligned} \tag{28}$$

Where $K_p = \begin{cases} N_p & N_p \geq K \\ K & N_p < K \end{cases}$.

The estimation of frequency offset can be expressed as :

$$\tilde{f} = \frac{1}{\pi N_p (N_P + 2K_p - 1)} \sum_{m=0}^{N_p - 1} angle(r_{z_1 z_2}(m)) \tag{29}$$

IV. SIMULATION AND COMPARISONS

[6] proposed the method of estimation of sinusoidal frequency in colored noise by Extended-Order Hankle Matrix SVD. In the process of simulation the length of pilot symbols is limited, frequency can't be estimated efficiently.

In order to verify the efficiency of frequency offset estimation with independent pilot symbols, assume ideal bit timing synchronization with QPSK modulation. Simulate for 10000 times with 32 pilot symbols and normalized frequency offset component 1×10^{-4}. Respectively 0.5,0.7 and 0.9 in the compression coefficient, the frequency offset estimation simulation as shown TABLE I.

TABLE I. FREQUENCY OFFSET ESTIMATION PERFORMANCE

		Independent pilot symbol		**Frequency domain[9]**	
Coefficient	**SNR**	***Average***	***Variance***	***Average***	***Variance***
0.5	-10	0.000108	2.64E-06	-0.001342	0.004161
	-5	7.98E-05	2.86E-06	0.002796	1.94E-06
	0	7.35E-05	7.10E-07	8.71E-05	6.20E-07
0.7	-10	5.35E-05	2.43E-06	-0.0001094	0.009274
	-5	0.000182	1.88E-06	1.77E-05	2.14E-06
	0	8.45E-05	6.80E-07	3.35E-04	5.80E-07
0.9	-10	7.58E-05	2.09E-06	-2.90E-03	0.015623
	-5	8.74E-05	1.55E-06	-3.22E-04	3.41E-05
	0	9.72E-05	7.10E-07	-2.16E-04	6.40E-07

We can observe that the frequency-domain of noise has peak for the noise is time-domain correlated. In low SNR ,the sine signal's frequency peak interfere with the noise, which cause wrong frequency estimation.

The noise component $\gamma[n] = \frac{\eta[n]}{c[n]}$ will degrade the performance for sampling points in small energy can be easily affected by same noise. [2] proved that the BER performance of FTN can keep same when symbol rate is faster than Nyquist rate 1/3 with 0.3 forming waveform rolling coefficient. The frequency offset would degrade the performance of FTN with ISI. After carrier synchronization, we can recover the BER performance of FTN signal.

V. CONCLUSION

FTN signal can improve the symbol rate without degrading BER performance. As a new thinking of 5G, there are many problems to be solved. Firstly, the decoding complexity[10] is very high, then it's difficult to obtain the traditional synchronization precision. This paper proposed the new method of frequency synchronization with independent pilot symbols which can avoid the noise's time-domain correlation to acquire right frequency estimation.

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