A Comparative Study of Intuitionistic Fuzzy Entropy on Attribute Importance

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Abstract

We propose a new objective weight method by using intuitionistic fuzzy (IF) entropy measures for multiple attribute decision making (MADM). We utilize the nature of IF entropy to assess the attribute weight based on the credibility of data, and the concept is totally different with the traditional one. Moreover, there were many IF entropy measures which were originated with different theories. We choose a geometric concept which was introduced by Szmidt and Kacprzyk in 2001, and we combine the concept with several distance measures to form several IF entropy measures. We investigate the differences among those varied measures through the experiment simulation. According to the experiment results, the differences undoubtedly exist among those measures. Besides, we also understand the number of attributes and alternatives would influence the degree of difference among those measures.

Keywords: Objective weight, intuitionistic fuzzy entropy, multiple attribute decision making, intuitionistic fuzzy set

1. Introduction

Many methods for solving MADM problems require definitions of quantitative weights for the attributes. It is very important to measure attribute weights properly, because they could influence the results of the analysis such as rankings of alternatives. The weight in MADM can be classified into subjective weight and objective weight. Subjective weights can reflect the subjective judgment or intuition of the DM, and they can be obtained based on preference information of the attributes given by the DM through interviews, questionnaires or trade-off interrogation directly. The most representative method is Analytical Hierarchy Process. Objective weight can be obtained from the objective information such as decision matrix through mathematic models. The most popular method to obtain objective weights is entropy method (Hwang and Yoon, 1981).

If the data of the decision matrix are fuzzy, qualitative, incomplete or hard to quantify, it would cause misestimate. Therefore, since intuitionistic fuzzy set (IFS) was proposed by Atanassov in 1986, scholars began to apply it in MADM, because IFS can much appropriately measure human being decision making progress and can also properly solve the incomplete information. IFS contains an new element π which can measure the hesitation degree. Thereupon this causes the decision matrix and data have much more uncertainty when we apply IFS in MADM.

We proposed a new objective weight method by using the IF entropy measures for solving the problems we mentioned above. The traditional entropy method focuses on using the discrimination of data to determine the weights of attributes. If the attribute can discriminate the data more significantly, we give a higher weight to the attribute. Dissimilarly, we focus on using the credibility of data to determine the attribute weights through IF entropy measures. This concept is totally different with the traditional entropy method, but our method can combine with the traditional method. Besides, Szmidt and Kacprzyk (2001) proposed a different concept for assessing the IF entropy.

We combine Szmidt and Kacprzyk's concept with several distance measures of IFS, and then we have several IF entropy measures. We investigate the difference among those measures.

2. Entropy Measures for Intuitionisitc Fuzzy Sets

Atanassov (1986) introduced the notion of intuitionistic fuzzy sets (IFSs). IFSs can present the degrees of membership, non-membership and hesitancy. An IFS A in X has the form: Let a set X be fixed. $A=\{(x,\mu_A(x),\nu_A(x))|x\in X\}$ where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$, respectively, the numbers of $\mu_A(x)$ and $\nu_A(x)$ is the degree of membership and non-membership of $x \in X$ in A. For each $x \in X$, $0 \le \mu_A(x)+\nu_A(x) \le 1$. For each IFS A in X, we will call $\pi_A(x)=1-\mu_A(x)-\nu_A(x)$ is the intuitionistic index of x in A. It is a hesitancy degree of x to A.

The traditional entropy is based on the concepts of probability, and it could measure the discrimination of attributes while we apply it in MADM. Nevertheless, the meaning of IF entropy is different from the traditional entropy, because the IF entropy represents the credibility of the data while we apply it in MADM. Since Atanassov introduced the basic concepts of IFS, many scholars began to conduct researches for entropy measures on IFS from many kinds of viewpoints. Burillo and Bustince (1996) defined the distance measure between IFSs, and they first gave the axiom definition of IF entropy to characterize it. Szmidt and Kacprzyk (2001) proposed a new entropy method for IFS. The method is a non-probabilistic type and a geometric interpretation of IFSs. Zeng and Li (2006) discussed the relationship between similarity measure and entropy on IVFSs. They not only prove three theorems that entropy of IVFS and similarity measure can be transformed by each other but also introduced the concepts of entropy of interval-valued fuzzy sets (IVFSs). Hung and Yang (2006) exploited the concept of probability to define the IF entropy, and they proposed two families of entropy measures for IFSs and also constructed the axiom definition and properties. In the above literatures, we know many scholars used different viewpoints to assess the IF entropy, such as probability viewpoint, non-probability viewpoint and geometric viewpoint. However, in our research, we use Szmidt and Kacprzyk's concept to measure the IF entropy, because this concept could measure the whole missing information which might be required to certainly have. It is also a totally different viewpoint with others, and we use this concept to combine with several different distance measures on IFS and to analyze the difference among the measures.

Szmidt and Kacprzyk (2001) proposed a new entropy method for IFS. In their paper, they proposed the IF entropy is a ratio of distances between the (F,F_{near}) and (F,F_{far}). We express it as follows:

$$E_{SK}(F)=\frac{(F,F_{near})}{(F,F_{far})}, \tag{1}$$

where (F,F_{near}) is the distance from F to the nearer point F_{near} among positive ideal point and negative ideal point, and (F, F_{far}) is the distance from F to the farther point F_{far} among positive ideal point and negative ideal point.

De Luca and Termini (1972) have already proposed the axioms of entropy for FSs. Szmidt and Kacprzyk (2001) then expressed them as Definition 1 for the IF entropy as follows:

Definition 1. A real function E: $IFSs(X) \to [0,1]$ is called an entropy on IFSs, if E has the following properties:

(P1) $E(A) = 0$ if and only if $A \in 2^x$.

(P2) $E(A) = 1$ if and only if $\mu_A(x) = \nu_B(x)$.

(P3) $E(A) \le E(B)$ if A is less fuzzy than B, i.e., $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for $\mu_B(x) \le \nu_B(x)$, or $\mu_A(x) \ge \mu_B(x)$ and $\nu_A(x) \le \nu_B(x)$ for $\mu_B(x) \ge \nu_B(x)$.

(P4) $E(A) = E(A^c)$, where A^c is the complement of A.

Szmidt and Kacprzyk (2001) also proved (1) satisfied the Definition 1. Because the entropy concept of Szmidt and Kacprzyk constructs on distance, we need to concern the relative concepts of measuring distance for IFSs. Distances between IFSs should be calculated taking account three parameters describing an IFS. There have been many researches proposed the formulas to measure the distance between IFSs. We introduce some major and the newest formulas for measuring distance at present. We combine those distance formulas with (1), and then we can get various equations for measuring IF entropy based on geometric concept. Szmidt and Kacprzyk (2000) introduced two different distance measures for IFS as follows: Let $X = \{x_1, x_2, \ldots x_n\}$. The Hamming distance between IFS A, B belonging to $IFSs(X)$ is defined by

$$d_{SK}^1(A,B) = \sum_{i=1}^{n} \left(|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)| \right) \quad (2)$$

Clearly, $0 \le d_{SK}^1(A,B) \le n$. Then they also defined another Euclidean distance between IFS A, B belonging to $IFSs(X)$ as follow:

$$d_{SK}^2(A,B) = \sqrt{\sum_{i=1}^{n} [(\mu_A(x_i) - \mu_B(x_i)^2) + (\nu_A(x_i) - \nu_B(x_i)^2) + (\pi_A(x_i) - \pi_B(x_i)^2)]} \quad (3)$$

Clearly, $0 \le d_{SK}^2(A,B) \le n$. Wang and Xin (2005) introduced another two distance measure between IFS A, B belonging to $IFSs(X)$ as following (4) and (5).

$$d_{WX}^1(A,B) = \frac{1}{n} \sum_{i=1}^{n} \left[\frac{|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|}{4} + \frac{\max(|\mu_A(x_i) - \mu_B(x_i)|, |\nu_A(x_i) - \nu_B(x_i)|)}{2} \right] \quad (4)$$

$$d_{WX}^2(A,B) = \frac{1}{\sqrt[l]{n}} \sqrt[l]{\sum_{i=1}^{n} \left[\frac{|\mu_A(x_i) - \mu_B(x_i)|}{2} + \frac{|\nu_A(x_i) - \nu_B(x_i)|}{2} \right]^l} \quad (5)$$

We combine (2), (3), (4) and (5) with (1), and we get four different IF entropy measures based on the geometric concept as following (6), (7), (8) and (9). We totally have four different IF entropy measures.

(A1)

$$E_{SK}^1(A) = \frac{|\mu_{near}(x_i) - \mu_A(x_i)| + |\nu_{near}(x_i) - \nu_A(x_i)| + |\pi_{near}(x_i) - \pi_A(x_i)|}{|\mu_{far}(x_i) - \mu_A(x_i)| + |\nu_{far}(x_i) - \nu_A(x_i)| + |\pi_{far}(x_i) - \pi_A(x_i)|} \quad (6)$$

(A2)

$$E_{SK}^2(A) = \frac{\sqrt{(\mu_{near}(x_i) - \mu_A(x_i))^2 + (\nu_{near}(x_i) - \nu_A(x_i))^2 + (\pi_{near}(x_i) - \pi_A(x_i))^2}}{\sqrt{(\mu_{far}(x_i) - \mu_A(x_i))^2 + (\nu_{far}(x_i) - \nu_A(x_i))^2 + (\pi_{far}(x_i) - \pi_A(x_i))^2}} \quad (7)$$

(A3)

$$E_{SK}^3(A) = \frac{\frac{|\mu_{near}(x_i) - \mu_A(x_i)| + |\nu_{near}(x_i) - \nu_A(x_i)|}{4} + \frac{\max(|\mu_{near}(x_i) - \mu_A(x_i)|, |\nu_{near}(x_i) - \nu_A(x_i)|)}{2}}{\frac{|\mu_{far}(x_i) - \mu_A(x_i)| + |\nu_{far}(x_i) - \nu_A(x_i)|}{4} + \frac{\max(|\mu_{far}(x_i) - \mu_A(x_i)|, |\nu_{far}(x_i) - \nu_A(x_i)|)}{2}} \quad (8)$$

(A4)

$$E_{SK}^4(A) = \frac{\frac{1}{\sqrt[l]{n}} \sqrt[l]{\sum_{i=1}^{n} \left[\frac{|\mu_{near}(x_i) - \mu_A(x_i)|}{2} + \frac{|\nu_{near}(x_i) - \nu_A(x_i)|}{2} \right]^l}}{\frac{1}{\sqrt[l]{n}} \sqrt[l]{\sum_{i=1}^{n} \left[\frac{|\mu_{far}(x_i) - \mu_A(x_i)|}{2} + \frac{|\nu_{far}(x_i) - \nu_A(x_i)|}{2} \right]^l}}. \quad (9)$$

3. Research Method

Zeleny (1976) proposed the algorithm for calculating the attribute weights by using the traditional entropy. We use Zeleny's algorithm to calculate the attribute weights, but we change the procedure of calculating the traditional entropy to the IF entropy. In our algorithm, we first use IF entropy formulas to measure the IF entropy value of each IFS in the decision matrix. In the original algorithm, Zeleny used traditional entropy in the first step, but we change it to the IF entropy measures. After calculating the IF entropy of each IFS, we do the normalization for those values, and the method of normalization refers to Zeleny's method. For normalization, we have to let all IF entropy values which are in the same column divide by the maximal one in this column. The reason for normalization is for conforming to the postulate of weight. Because the postulate of weight, we make all the value of IF entropy values situate between 0 and 1, and the method of normalization could not only let all the value situate between 0 and 1 but also keep the feature of ratio scale. After the normalization, we use a weight transformation equation which was used in Zeleny's method, and we introduce it in the following context.

We give an easy model to illustrate our method. Let an IFS decision matrix D of m alternatives and n attributes be

$$D = \begin{matrix} P_1 \\ P_2 \\ \vdots \\ P_i \\ \vdots \\ P_m \end{matrix} \begin{bmatrix} (\mu_{11},\nu_{11},\pi_{11}) & (\mu_{12},\nu_{12},\pi_{12}) & \cdots & (\mu_{1j},\nu_{1j},\pi_{1j}) & \cdots & (\mu_{1n},\nu_{1n},\pi_{1n}) \\ (\mu_{21},\nu_{21},\pi_{21}) & (\mu_{22},\nu_{22},\pi_{22}) & \cdots & (\mu_{2j},\nu_{2j},\pi_{2j}) & \cdots & (\mu_{2n},\nu_{2n},\pi_{2n}) \\ \vdots & & \ddots & & & \vdots \\ (\mu_{i1},\nu_{i1},\pi_{i1}) & & & (\mu_{ij},\nu_{ij},\pi_{ij}) & & (\mu_{in},\nu_{in},\pi_{in}) \\ \vdots & & & & \ddots & \vdots \\ (\mu_{m1},\nu_{m1},\pi_{m1}) & (\mu_{m2},\nu_{m2},\pi_{m2}) & \cdots & (\mu_{mj},\nu_{mj},\pi_{mj}) & \cdots & (\mu_{mn},\nu_{mn},\pi_{mn}) \end{bmatrix}$$

where P represents the alternative and x represents the attribute. Then, we calculate the IF entropy values of each IFS. Then the decision matrix D can be illustrated as follows:

$$D = \begin{matrix} P_1 \\ P_2 \\ \vdots \\ P_i \\ \vdots \\ P_m \end{matrix} \begin{bmatrix} E_{11} & E_{12} & \cdots & E_{1j} & \cdots & E_{1n} \\ E_{21} & E_{22} & & & & E_{2n} \\ \vdots & & \ddots & & & \vdots \\ E_{i1} & & & E_{ij} & & E_{in} \\ \vdots & & & & \ddots & \vdots \\ E_{m1} & E_{m2} & \cdots & E_{mj} & \cdots & E_{mn} \end{bmatrix},$$

where E_{ij} is the IF entropy value of each IFS in the decision matrix. Furthermore, we have to normalize the IF entropy values in the decision matrix. We use h_{ij} to represent the outcomes of normalization, and it can be defined as

$$h_{i1} = \frac{E_{i1}}{\max(E_{i1})}, h_{i2} = \frac{E_{i2}}{\max(E_{i2})}, \ldots, h_{ij} = \frac{E_{ij}}{\max(E_{ij})}.$$

Then the decision matrix can be expressed as

$$D = \begin{matrix} P_1 \\ P_2 \\ \vdots \\ P_i \\ \vdots \\ P_m \end{matrix} \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1j} & \cdots & h_{1n} \\ h_{21} & h_{22} & \cdots & h_{2j} & \cdots & h_{2n} \\ \vdots & & \ddots & & & \vdots \\ h_{i1} & & & h_{ij} & & h_{in} \\ \vdots & & & & \ddots & \vdots \\ h_{m1} & h_{m2} & \cdots & h_{mj} & \cdots & h_{mn} \end{bmatrix}.$$

Finally, we want to calculate the weight of attributes by using weight formula. We use w_j to represent the outcome of weight value of attribute j, and it can be defined as

$$w_j = \frac{1}{n-T} \times (1 - a_j), \tag{10}$$

where $a_j = \sum_{d=1}^{i} h_{dj}$ and $T = \sum_{f=1}^{j} a_f$ .The a_j represents the summation of the normalized entropy values which are corresponding to the attribute j. The T is the summation of a_j, and n is the number of attributes.

The (10) is a transformation. It gives an attribute a higher weight when the IF entropy values belonged to this attribute are lower, and vice versa. To sum up, we simplify our algorithm to three steps, and we show them as follows:

Step 1: Use IF entropy formulas to calculate the entropy value of each IFS in the decision matrix.

Step 2: Normalize the IF entropy value that we have gotten in the first step. For normalization, we have to divide all entropy values by the maximal entropy value in the same column.

Step 3: Use weight formula to calculate the attribute weights.

From the three steps above, we can assess the attribute weights by using the IF entropy measures.

4. Experiment Analysis

We assume that we have 4, 6, 8, 10, 12, 14, 16, 18, 20 and 22 alternatives/attributes. So, we can get 10 × 10 combinations, and it means we have totally 100 kinds of decision matrix. We use MATLAB to simulate 1000 times for each kind of decision matrix, and we measure the average of Spearman rank correlation coefficient, contradiction rate and inversion rate between each two different IF entropy measures. The contradiction rate is used to measure the frequency of difference of the first one attribute in two ranks. For inversion rate, we separate the rank into two parts. The first part is from the first one to the medium of the rank, and the second part is from the medium to the last one of the rank. If there is any attribute appear in the first part of the rank 1, but it also appears in the second part of the rank 2. Then we count 1 time, and vice versa.

First, we analyze the results of average ρ-values, and we show them in the Figures 1-6. All the figures are in the same form. The number of attributes increases from 4 to 22, but the number of alternatives decreases from 22 to 4. The shapes in these six figures are very similar, so we take (a) to represent them for discussion. The effect of attributes is quite clear in (a), and the ρ-value increases when the value of attributes goes down from 22 to 4. Extremely, the highest point of ρ-values is about 0.85 and locates on the line when the value of attributes is 22, and contrary to highest point, the lowest one is about 0.4 and locates on the line while the value of attributes is 4. Relatively, the value of alternatives does not have great effect to the ρ-value, and it could only cause little undulation on the slope.

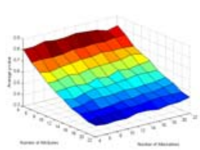

Fig. 1: A1 vs A2.

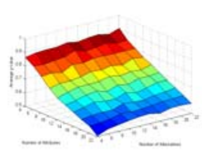

Fig. 2: A1 vs A3.

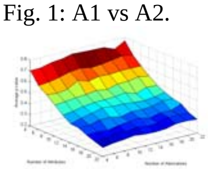

Fig. 3: A1 vs A4.

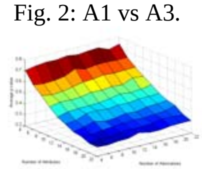

Fig. 4: A2 vs A3.

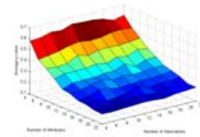

Fig. 5: A2 vs A4.

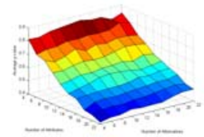

Fig. 6: A3 vs A4.

We further discuss about which IF measure could be more suitably used while we apply it in reality. We do the total average of contradiction rates and inversion rates, and we show them in the Table 1. In the Table 1, we see that A2 performs the highest average contradiction rate and inversion rate, but A3 performs the lowest average contradiction rate and inversion rate. Therefore, while we apply those measures in reality, if the first attribute is particularly important in the decision, we should avoid choosing A2 as the tool for measuring the attribute weight, because A2 performs the highest average contradiction rate and would cause different results easier. Oppositely, we should choose A3. In other realistic condition, such as our attributes might equally important, we should avoid choosing A2, because A2 performs the lowest average inversion rate. Oppositely, we should choose A3 as the measure for assessing the attribute weights.

Tab. 1: The total average of contradiction rate and inversion rate.

Measure	Average contradiction rate	Average Inversion rate
A1	0.150030	0.353940
A2	0.233223	0.495540
A3	0.147263	0.345683
A4	0.217837	0.473510

We also do the total average of average ρ-values of each comparison, and we show them in the Table 2. There are three different values of m and n in each comparison which are m and n = 4 to 22, m and n = 4 to 8 and m and n = 18 to 22. In the Table 2, while n = 4 to 22, A1 vs A3 performs 0.7043, and it means highly positive. Therefore, we understand that A1 is similar with A3. When *m* and *n* = 4 to 8, A1 vs A2, A1 vs A3 and A3 vs A4 perform higher than 0.7, so A1 is similar with A2, A1 is similar with A3 and A3 is similar with A4. When *m* and *n* = 18 to 22, there are no results perform higher than 0.7. We especially mention that those similarities between each two measures do not contain transitivity, even A1 is similar with A3 and A3 is similar with A4, but we still can not conclude that A1 is similar with A4.

Tab. 2: The total average of average ρ-values of each comparison.

Comparison	*m* and *n* = 4 to 22	*m* and *n* = 4 to 8	*m* and *n* = 18 to 22
A1 vs A2	0.5800	0.7307	0.4633
A1 vs A3	0.7043	0.8212	0.6049
A1 vs A4	0.4763	0.6235	0.3559
A2 vs A3	0.4627	0.6171	0.3419
A2 vs A4	0.3425	0.4814	0.2346
A3 vs A4	0.6173	0.7506	0.5105

5. Conclusions

In our research, we propose a new objective weight method by using the IF entropy measures, and the new method can help us measuring the objective weight of attributes based on the credibility of data. According to our experimental results, we have already known that different IF entropy measures would cause a totally different weight result. The differences undoubtedly exist among those IF entropy measures. Even so, we still can not certainly know which IF entropy measure can be most suitably used in reality because our data is obtained from simulation. Hence the follow-up researches can devote to understand which IF entropy measure is more proper in actual example or in some occasions by using the evidence-based data.

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