Optimism and Pessimism in Decision Making Based on Intuitionistic Fuzzy Sets

Ting-Yu Chen[1] Che-Wei Tsui[2]

[1]Department of Business Administration, Chang Gung University
[2]Graduate Institute of Industrial Engineering and Engineering Management, National Tsing Hua University

Abstract

This paper presents a method of relating optimism and pessimism to multiple criteria decision analysis based on intuitionistic fuzzy sets. We develop the concepts of optimistic and pessimistic point operators to measure optimism and pessimism, respectively. Furthermore, we provide an approach to effectively capture the influences of optimism and pessimism on multiple criteria decision making. The suitability function is assessed according to the influence upon overall judgments or upon individual outcomes. Finally, we establish two algorithms to solve a multi-criteria decision making problem.

Keywords: Optimism, pessimism, multiple criteria decision analysis, intuitionistic fuzzy set, point operator

1. Introduction

Intuitionistic fuzzy set (IFS for short), introduced by Atanassov (1986, 1999), is characterized by three functions expressing the degree of belongingness, the degree of nonbelongingness, and the degree of hesitation. IFSs have been found to be highly useful to deal with uncertainty and vagueness (Xu and Yager, 2008) and have become a popular topic of investigation in the fuzzy set community (Dubois et al., 2005). Although a considerable number of studies have been made on intuitionistic fuzzy multiple criteria decision making, little attention has been given to the issue of optimism and pessimism. Optimism and pessimism are fundamental inner psychological characteristics that both determine and reflect how a person responds to his or her perceived environment. The evidence in psychology suggests that one main channel through which dispositional optimism works is through developing coping habits or behavior that is more likely to lead to desired outcomes (Friedman et al., 1995; Puri and Robinson, 2007).

In view of multi-criteria decision making problems, optimism and pessimism can reflect individual differences of different decision makers. In addition, they tend to be both consistent and enduring. It follows that optimism and pessimism consistently influence how the decision maker responds to the decision environment. Therefore, the identification of their influence on the decision making process is highly valuable and useful in multiple criteria decision analysis. In this research, we will develop optimistic and pessimistic estimations based on IFSs to multi-criteria decision analysis, where optimism and pessimism are measured by using optimistic and pessimistic point operators, respectively. In addition, several important properties will be examined and discussed. For decision aiding, we provide an approach relating optimism and pessimism to multi-criteria decision analysis under the intuitionistic fuzzy decision environment.

Proceedings of the 11th Joint Conference on Information Sciences (2008)
Published by Atlantis Press

2. Preliminaries

The concept of IFSs is a generalization ordinary fuzzy sets. We briefly review some relevant definitions, relations, and operations of IFSs.

Definition 2.1. Let *X* be an ordinary finite non-empty set. An IFS *A* in *X* is an expression given by:

$$A=\{\langle x,\mu_A(x),\nu_A(x)\rangle \mid x\in X\}, \quad (1)$$

where $\mu_A(x)$: $X\to[0,1]$, $\nu_A(x)$: $X\to[0,1]$ with $0\le\mu_A(x)+\nu_A(x)\le 1$. The numbers $\mu_A(x)$ and $\nu_A(x)$ denote the membership degree and the non-membership degree of the element *x* in *A*, respectively.

Definition 2.2. For each IFS *A* in *X*, the value of

$$\pi_A(x)=1-\mu_A(x)-\nu_A(x) \quad (2)$$

represents the the intuitionistic index.

Definition 2.3. For every IFS *A* and for every $\alpha,\beta\in[0,1]$:

$$J_{\alpha,\beta}(A)=\{\langle x,\mu_A(x)+\alpha\cdot\pi_A(x),\beta\cdot\nu_A(x)\rangle \mid x\in X\} \quad (3)$$

$$J^*_{\alpha,\beta}(A)=\{\langle x,\mu_A(x)+\alpha\cdot(1-\mu_A(x)-\beta\cdot\nu_A(x)),\beta\cdot\nu_A(x)\rangle \mid x\in X\} \quad (4)$$

$$H_{\alpha,\beta}(A)=\{\langle x,\alpha\cdot\mu_A(x),\nu_A(x)+\beta\cdot\pi_A(x)\rangle \mid x\in X\} \quad (5)$$

$$H^*_{\alpha,\beta}(A)=\{\langle x,\alpha\cdot\mu_A(x),\nu_A(x)+\beta\cdot(1-\alpha\cdot\mu_A(x)-\nu_A(x))\rangle \mid x\in X\} \quad (6)$$

$$P_{\alpha,\beta}(A)=\{\langle x,\max(\mu_A(x),\alpha),\min(\nu_A(x),\beta)\rangle \mid x\in X\} \quad (7)$$

$$Q_{\alpha,\beta}(A)=\{\langle x,\min(\mu_A(x),\alpha),\max(\nu_A(x),\beta)\rangle \mid x\in X\} \quad (8)$$

3. Optimistic and Pessimistic Estimations on IFSs

This paper relates optimism and pessimism to multi-criteria decision making behavior and establishes appropriate assessment tools for measuring them in decision analysis. Optimists construe their lives and future states of the world positively, whereas pessimists construe their lives and future states of the world negatively. In addition, optimists expect greater overall utility or favorable outcomes, but pessimists expect less overall utility or unfavorable outcomes. The above rationale coincides with several Atanassov's operators, including $J_{\alpha,\beta}$, $J^*_{\alpha,\beta}$, $H_{\alpha,\beta}$, $H^*_{\alpha,\beta}$, $P_{\alpha,\beta}$ and $Q_{\alpha,\beta}$. By extension we will develop new point operators denoted by J_{α_x,β_x} , $J^*_{\alpha_x,\beta_x}$, H_{α_x,β_x} , $H^*_{\alpha_x,\beta_x}$, P_{α_x,β_x} and Q_{α_x,β_x} for each point $x\in X$ and $\alpha_x,\beta_x\in[0,1]$. These estimations on IFSs are called as optimistic or pessimistic point operators.

Definition 3.1. For each $x\in X$, taking $\alpha_x,\beta_x\in[0,1]$ we define optimistic point operators J_{α_x,β_x} , $J^*_{\alpha_x,\beta_x}$, P_{α_x,β_x} : *IFS*(*X*)→*IFS*(*X*) as follows, for $A\in IFS(X)$:

$$J_{\alpha_x,\beta_x}(A)=\{\langle x,\mu_A(x)+\alpha_x\cdot\pi_A(x),\beta_x\cdot\nu_A(x)\rangle \mid x\in X\} \quad (9)$$

$$J^*_{\alpha_x,\beta_x}(A)=\{\langle x,\mu_A(x)+\alpha_x\cdot(1-\mu_A(x)-\beta_x\cdot\nu_A(x)),\beta_x\cdot\nu_A(x)\rangle \mid x\in X\} \quad (10)$$

$$P_{\alpha_x,\beta_x}(A)=\{\langle x,\max(\mu_A(x),\alpha_x),\min(\nu_A(x),\beta_x)\rangle \mid x\in X\} \text{ for } \alpha_x+\beta_x\le 1 \quad (11)$$

Definition 3.2. For each $x\in X$, taking $\alpha_x,\beta_x\in[0,1]$ we define pessimistic point operators H_{α_x,β_x} , $H^*_{\alpha_x,\beta_x}$, Q_{α_x,β_x} : *IFS*(*X*)→*IFS*(*X*) as follows, for $A\in IFS(X)$:

$$H_{\alpha_x,\beta_x}(A)=\{\langle x,\alpha_x\cdot\mu_A(x),\nu_A(x)+\beta_x\cdot\pi_A(x)\rangle \mid x\in X\} \quad (12)$$

$$H^*_{\alpha_x,\beta_x}(A)=\{\langle x,\alpha_x\cdot\mu_A(x),\nu_A(x)+\beta_x\cdot(1-\alpha_x\cdot\mu_A(x)-\nu_A(x))\rangle \mid x\in X\} \quad (13)$$

$$Q_{\alpha_x,\beta_x}(A)=\{\langle x,\min(\mu_A(x),\alpha_x),\max(\nu_A(x),\beta_x)\rangle \mid x\in X\} \quad (14)$$

Definition 3.3 Let κ, κ_1, κ_2 be real numbers and let $a_1,a_2,\cdots,a_n\in[0,1]$. For *n* IFSs $A_1,A_2,\cdots,A_n$, we define the averaging operations based on generalized means:

$$A_1 \underset{\kappa_1,\kappa_2}{\otimes} A_2=\overset{2}{\underset{i=1}{\otimes}} A_i \quad (15)$$

and

$$\overset{n}{\underset{i=1}{\otimes}} A_i=\{\langle x,f(\kappa_1,\mu_{A_1}(x),\mu_{A_2}(x),\cdots,\mu_{A_n}(x)), f(\kappa_2,\nu_{A_1}(x),\nu_{A_2}(x),\cdots,\nu_{A_n}(x)) \mid x\in X\rangle\}, \quad (16)$$

where

$$f(\kappa, a_1, a_2, \cdots, a_n) = \begin{cases} \left(\dfrac{a_1^{\kappa} + a_2^{\kappa} + \cdots + a_n^{\kappa}}{n} \right)^{1/\kappa}, & \text{if } \kappa > 0 \text{ or } (\kappa < 0 \text{ and } a_1 \cdot a_2 \cdots a_n > 0); \\ (a_1 \cdot a_2 \cdots a_n)^{1/n}, & \text{if } \kappa = 0; \\ 0, & \text{otherwise.} \end{cases} \quad (17)$$

4. Application in Multi-Criteria Decision Making

This section presents an approach for handling optimism and pessimism in multi-criteria decision making problems under the intuitionistic fuzzy environment. By similar definitions of Li (2005) and Lin et al. (2007), the evaluations of each alternative with respect to each criterion on a fuzzy concept "excellence" are given using IFSs. Suppose that there exists a non-inferior alternative set $X = \{x_1, x_2, ..., x_m\}$. Each alternative is assessed on *n* criteria, denoted by $A = \{A_1, A_2, ..., A_n\}$. Assume that μ_{ij} and ν_{ij} are the degree of membership and the degree of non-membership of the alternative $x_j \in X$ with respect to the criterion $A_i \in A$ to the fuzzy concept "excellence", respectively, where $0 \le \mu_{ij} \le 1$, $0 \le \nu_{ij} \le 1$ and $0 \le \mu_{ij} + \nu_{ij} \le 1$. Denote that $X_{ij} = \{\langle x_j, \mu_{ij}, \nu_{ij} \rangle\}$. The intuitionistic index of the alternative x_j in the set X_{ij} is defined by c.

4.1. An approach utilizing optimistic and pessimistic point operators

Assume that the decision maker consider evaluative criteria all equally important for simplicity. The suitability function to determine the degrees to which the alternative x_j satisfies and does not satisfy the decision maker's requirement can be measured as follows:

$$\overset{n}{\underset{i=1}{\otimes}} A_i = \left\{ \langle x_j, f(\kappa_1, \mu_{1j}, \mu_{2j}, \cdots, \mu_{nj}), f(\kappa_2, \nu_{1j}, \nu_{2j}, \cdots, \nu_{nj}) \rangle \middle| x_j \in X \right\} \quad (18)$$

If an optimistic decision maker construe overall judgment positively and expect favorable synthetic evaluation, the suitability function to determine the degrees to which the alternative x_j satisfies and does not satisfy the decision maker's requirement becomes $J_{\alpha_{x_j}, \beta_{x_j}}(\overset{n}{\underset{i=1}{\otimes}} A_i)$, $J^*_{\alpha_{x_j}, \beta_{x_j}}(\overset{n}{\underset{i=1}{\otimes}} A_i)$ or $P_{\alpha_{x_j}, \beta_{x_j}}(\overset{n}{\underset{i=1}{\otimes}} A_i)$. If an optimistic decision maker reconstructs the decision matrix with more desirable outcomes, the suitability function will be $\overset{n}{\underset{i=1}{\otimes}} J_{\alpha_{x_j}, \beta_{x_j}}(A_i)$, c or $\overset{n}{\underset{i=1}{\otimes}} P_{\alpha_{x_j}, \beta_{x_j}}(A_i)$.

On the contrary, if a pessimistic decision maker reflects negative attitude on overall judgments, the suitability function must adjust to $H_{\alpha_{x_j}, \beta_{x_j}}(\overset{n}{\underset{i=1}{\otimes}} A_i)$, $H^*_{\alpha_{x_j}, \beta_{x_j}}(\overset{n}{\underset{i=1}{\otimes}} A_i)$ or $Q_{\alpha_{x_j}, \beta_{x_j}}(\overset{n}{\underset{i=1}{\otimes}} A_i)$. If a pessimistic decision maker modifies the decision matrix with more adverse outcomes, the corresponding suitability function is $\overset{n}{\underset{i=1}{\otimes}} H_{\alpha_{x_j}, \beta_{x_j}}(A_i)$, $\overset{n}{\underset{i=1}{\otimes}} H^*_{\alpha_{x_j}, \beta_{x_j}}(A_i)$ or $\overset{n}{\underset{i=1}{\otimes}} Q_{\alpha_{x_j}, \beta_{x_j}}(A_i)$. Then, measure the degree of suitability to which the alternative x_j satisfies the decision maker's requirement on the basis of the score function.

Definition 4.1. Let $X = \{x_1, x_2, ..., x_m\}$ be the set of the considered alternatives, and let $A = \{A_1, A_2, ..., A_n\}$ be the set of the criteria used for evaluating the alternatives. Let $\alpha_{x_j}, \beta_{x_j} \in [0,1]$ for all $x_j \in X$. Assume that individual differences in virtue of optimism or pessimism influence overall judgments. The suitability function to determine the degrees to which the alternative x_j satisfies and does not satisfy the decision maker's requirement can be measured as follows:

(i) For the optimistic condition:

$$\begin{aligned}
&J_{\alpha_{x_j},\beta_{x_j}}(\bigotimes_{i=1}^{n} A_i) = J_{\alpha_{x_j},\beta_{x_j}}\left(\left\{\left\langle x_j, f(\kappa_1,\mu_{1j},\mu_{2j},\cdots,\mu_{nj}), f(\kappa_2,\nu_{1j},\nu_{2j},\cdots,\nu_{nj})\right\rangle \middle| x_j \in X\right\}\right)\\
&= \left\{\left\langle x_j, f(\kappa_1,\mu_{1j},\mu_{2j},\cdots,\mu_{nj}) + \alpha_{x_j}\cdot\left(1 - f(\kappa_1,\mu_{1j},\mu_{2j},\cdots,\mu_{nj}) -\right.\right.\right.\\
&\quad \left.\left.\left. f(\kappa_2,\nu_{1j},\nu_{2j},\cdots,\nu_{nj})\right), \beta_{x_j}\cdot f(\kappa_2,\nu_{1j},\nu_{2j},\cdots,\nu_{nj})\right\rangle \middle| x_j \in X\right\},
\end{aligned} \tag{19}$$

or

$$\begin{aligned}
&J^*_{\alpha_{x_j},\beta_{x_j}}(\bigotimes_{i=1}^{n} A_i) = J^*_{\alpha_{x_j},\beta_{x_j}}\left(\left\{\left\langle x_j, f(\kappa_1,\mu_{1j},\mu_{2j},\cdots,\mu_{nj}), f(\kappa_2,\nu_{1j},\nu_{2j},\cdots,\nu_{nj})\right\rangle \middle| x_j \in X\right\}\right)\\
&= \left\{\left\langle x_j, f(\kappa_1,\mu_{1j},\mu_{2j},\cdots,\mu_{nj}) + \alpha_{x_j}\cdot\left(1 - f(\kappa_1,\mu_{1j},\mu_{2j},\cdots,\mu_{nj}) -\right.\right.\right.\\
&\quad \left.\left.\left. \beta_{x_j}\cdot f(\kappa_2,\nu_{1j},\nu_{2j},\cdots,\nu_{nj})\right), \beta_{x_j}\cdot f(\kappa_2,\nu_{1j},\nu_{2j},\cdots,\nu_{nj})\right\rangle \middle| x_j \in X\right\}.
\end{aligned} \tag{20}$$

(ii) For the optimistic condition with restrictions:

$$\begin{aligned}
&P_{\alpha_{x_j},\beta_{x_j}}(\bigotimes_{i=1}^{n} A_i)\\
&= P_{\alpha_{x_j},\beta_{x_j}}\left(\left\{\left\langle x_j, f(\kappa_1,\mu_{1j},\mu_{2j},\cdots,\mu_{nj}), f(\kappa_2,\nu_{1j},\nu_{2j},\cdots,\nu_{nj})\right\rangle \middle| x_j \in X\right\}\right)\\
&= \left\{\left\langle x_j, \max\left(f(\kappa_1,\mu_{1j},\mu_{2j},\cdots,\mu_{nj}),\alpha_{x_j}\right),\right.\right.\\
&\quad \left.\left. \min\left(f(\kappa_2,\nu_{1j},\nu_{2j},\cdots,\nu_{nj}),\beta_{x_j}\right)\right\rangle \middle| x_j \in X\right\} \text{ for } \alpha_{x_j} + \beta_{x_j} \le 1.
\end{aligned} \tag{21}$$

(iii) For the pessimistic condition:

$$\begin{aligned}
&H_{\alpha_{x_j},\beta_{x_j}}(\bigotimes_{i=1}^{n} A_i) = H_{\alpha_{x_j},\beta_{x_j}}\left(\left\{\left\langle x_j, f(\kappa_1,\mu_{1j},\mu_{2j},\cdots,\mu_{nj}), f(\kappa_2,\nu_{1j},\nu_{2j},\cdots,\nu_{nj})\right\rangle \middle| x_j \in X\right\}\right)\\
&= \left\{\left\langle x_j, \alpha_{x_j}\cdot f(\kappa_1,\mu_{1j},\mu_{2j},\cdots,\mu_{nj}), f(\kappa_2,\nu_{1j},\nu_{2j},\cdots,\nu_{nj}) +\right.\right.\\
&\quad \left.\left. \beta_{x_j}\cdot\left(1 - f(\kappa_1,\mu_{1j},\mu_{2j},\cdots,\mu_{nj}) - f(\kappa_2,\nu_{1j},\nu_{2j},\cdots,\nu_{nj})\right)\right\rangle \middle| x_j \in X\right\},
\end{aligned} \tag{22}$$

or

$$\begin{aligned}
&H^*_{\alpha_{x_j},\beta_{x_j}}(\bigotimes_{i=1}^{n} A_i) = H^*_{\alpha_{x_j},\beta_{x_j}}\left(\left\{\left\langle x_j, f(\kappa_1,\mu_{1j},\mu_{2j},\cdots,\mu_{nj}), f(\kappa_2,\nu_{1j},\nu_{2j},\cdots,\nu_{nj})\right\rangle \middle| x_j \in X\right\}\right)\\
&= \left\{\left\langle x_j, \alpha_{x_j}\cdot f(\kappa_1,\mu_{1j},\mu_{2j},\cdots,\mu_{nj}), f(\kappa_2,\nu_{1j},\nu_{2j},\cdots,\nu_{nj}) +\right.\right.\\
&\quad \left.\left. \beta_{x_j}\cdot\left(1 - \alpha_{x_j}\cdot f(\kappa_1,\mu_{1j},\mu_{2j},\cdots,\mu_{nj}) - f(\kappa_2,\nu_{1j},\nu_{2j},\cdots,\nu_{nj})\right)\right\rangle \middle| x_j \in X\right\}.
\end{aligned} \tag{23}$$

(iv) For the pessimistic condition with restrictions:

$$\begin{aligned}
&Q_{\alpha_{x_j},\beta_{x_j}}(\bigotimes_{i=1}^{n} A_i) = Q_{\alpha_{x_j},\beta_{x_j}}\left(\left\{\left\langle x_j, f(\kappa_1,\mu_{1j},\mu_{2j},\cdots,\mu_{nj}), f(\kappa_2,\nu_{1j},\nu_{2j},\cdots,\nu_{nj})\right\rangle \middle| x_j \in X\right\}\right)\\
&= \left\{\left\langle x_j, \min\left(f(\kappa_1,\mu_{1j},\mu_{2j},\cdots,\mu_{nj}),\alpha_{x_j}\right), \max\left(f(\kappa_2,\nu_{1j},\nu_{2j},\cdots,\nu_{nj}),\beta_{x_j}\right)\right\rangle \middle| x_j \in X\right\} \text{for } \alpha_{x_j} + \beta_{x_j} \le 1.
\end{aligned} \tag{24}$$

Definition 4.2. Let $X = \{x_1, x_2, \ldots, x_m\}$ be the set of the considered alternatives, and let $A = \{A_1, A_2, \ldots, A_n\}$ be the set of the criteria used for evaluating the alternatives. Let $\alpha_{x_j}, \beta_{x_j} \in [0,1]$ for all $x_j \in X$. Assume that individual differences in virtue of optimism or pessimism influence outcomes in the decision matrix. The suitability function to determine the degrees to which the alternative x_j satisfies and does not satisfy the decision maker's requirement can be measured as follows:

(i) For the optimistic condition:

$$\begin{aligned}
&\bigotimes_{i=1}^{n} J_{\alpha_{x_j},\beta_{x_j}}(A_i) = \bigotimes_{i=1}^{n}\left\{\left\langle x_j, \mu_{ij} + \alpha_{x_j}\cdot\pi_{ij}, \beta_{x_j}\cdot\nu_{ij}\right\rangle \middle| x_j \in X\right\}\\
&= \left\{\left\langle x_j, f(\kappa_1, \mu_{1j} + \alpha_{x_j}\cdot\pi_{1j}, \mu_{2j} + \alpha_{x_j}\cdot\pi_{2j}, \cdots, \mu_{nj} + \alpha_{x_j}\cdot\pi_{nj}),\right.\right.\\
&\quad \left.\left. f(\kappa_2, \beta_{x_j}\cdot\nu_{1j}, \beta_{x_j}\cdot\nu_{2j}, \cdots, \beta_{x_j}\cdot\nu_{nj}\right\rangle \middle| x_j \in X\right\},
\end{aligned} \tag{25}$$

or

$$\begin{aligned}
&\bigotimes_{i=1}^{n} J^*_{\alpha_{x_j},\beta_{x_j}}(A_i) = \bigotimes_{i=1}^{n}\left\{\left\langle x_j, \mu_{ij} + \alpha_{x_j}\cdot(1 - \mu_{ij} - \beta_{x_j}\cdot\nu_{ij}), \beta_{x_j}\cdot\nu_{ij}\right\rangle \middle| x_j \in X\right\}\\
&= \left\{\left\langle x_j, f(\kappa_1, \mu_{1j} + \alpha_{x_j}\cdot(1 - \mu_{1j} - \beta_{x_j}\cdot\nu_{1j}), \mu_{2j} + \alpha_{x_j}\cdot(1 - \mu_{2j} - \beta_{x_j}\cdot\nu_{2j}), \cdots,\right.\right.\\
&\quad \left.\left. \mu_{nj} + \alpha_{x_j}\cdot(1 - \mu_{nj} - \beta_{x_j}\cdot\nu_{nj}), f(\kappa_2, \beta_{x_j}\cdot\nu_{1j}, \beta_{x_j}\cdot\nu_{2j}, \cdots, \beta_{x_j}\cdot\nu_{nj}\right\rangle \middle| x_j \in X\right\}.
\end{aligned} \tag{26}$$

(ii) For the optimistic condition with restrictions:

$$\begin{aligned}
&\bigotimes_{i=1}^{n} P_{\alpha_{x_j},\beta_{x_j}}(A_i) = \bigotimes_{i=1}^{n}\left\{\left\langle x_j, \max(\mu_{ij},\alpha_{x_j}), \min(\nu_{ij},\beta_{x_j})\right\rangle \middle| x_j \in X\right\}\\
&= \left\{\left\langle x_j, f(\kappa_1, \max(\mu_{1j},\alpha_{x_j}), \max(\mu_{2j},\alpha_{x_j}), \cdots, \max(\mu_{nj},\alpha_{x_j})), f(\kappa_2, \min(\nu_{1j},\beta_{x_j}),\right.\right.\\
&\quad \left.\left. \min(\nu_{2j},\beta_{x_j}), \cdots, \min(\nu_{nj},\beta_{x_j})\right\rangle \middle| x_j \in X\right\} \text{ for } \alpha_{x_j} + \beta_{x_j} \le 1.
\end{aligned} \tag{27}$$

(iii) For the pessimistic condition:

$$\begin{aligned}
&\bigotimes_{i=1}^{n} H_{\alpha_{x_j},\beta_{x_j}}(A_i) = \bigotimes_{i=1}^{n}\left\{\left\langle x_j, \alpha_{x_j}\cdot\mu_{ij}, \nu_{ij} + \beta_{x_j}\cdot\pi_{ij}\right\rangle \middle| x_j \in X\right\}\\
&= \left\{\left\langle x_j, f(\kappa_1, \alpha_{x_j}\cdot\mu_{1j}, \alpha_{x_j}\cdot\mu_{2j}, \cdots, \alpha_{x_j}\cdot\mu_{nj}),\right.\right.\\
&\quad \left.\left. f(\kappa_2, \nu_{1j} + \beta_{x_j}\cdot\pi_{1j}, \nu_{2j} + \beta_{x_j}\cdot\pi_{2j}, \cdots, \nu_{nj} + \beta_{x_j}\cdot\pi_{nj}\right\rangle \middle| x_j \in X\right\},
\end{aligned} \tag{28}$$

or

$$\begin{aligned}
&\bigotimes_{i=1}^{n} H^*_{\alpha_{x_j},\beta_{x_j}}(A_i) = \bigotimes_{i=1}^{n}\left\{\left\langle x_j, \alpha_{x_j}\cdot\mu_{ij}, \nu_{ij} + \beta_{x_j}\cdot(1 - \alpha_{x_j}\cdot\mu_{ij} - \nu_{ij})\right\rangle \middle| x_j \in X\right\}\\
&= \left\{\left\langle x_j, f(\kappa_1, \alpha_{x_j}\cdot\mu_{1j}, \alpha_{x_j}\cdot\mu_{2j}, \cdots, \alpha_{x_j}\cdot\mu_{nj}), f(\kappa_2, \beta_{x_j}\cdot(1 - \alpha_{x_j}\cdot\mu_{1j} - \nu_{1j}),\right.\right.\\
&\quad \left.\left. \beta_{x_j}\cdot(1 - \alpha_{x_j}\cdot\mu_{2j} - \nu_{2j}), \cdots, \beta_{x_j}\cdot(1 - \alpha_{x_j}\cdot\mu_{nj} - \nu_{nj}))\right\rangle \middle| x_j \in X\right\}.
\end{aligned} \tag{29}$$

(iv) For the pessimistic condition with restrictions:

$$\begin{aligned}
&\bigotimes_{i=1}^{n} Q_{\alpha_{x_j},\beta_{x_j}}(A_i) = \bigotimes_{i=1}^{n}\left\{\left\langle x_j, \min(\mu_{ij},\alpha_{x_j}), \max(\nu_{ij},\beta_{x_j})\right\rangle \middle| x_j \in X\right\}\\
&= \left\{\left\langle x_j, f(\kappa_1, \min(\mu_{1j},\alpha_{x_j}), \min(\mu_{2j},\alpha_{x_j}), \cdots, \min(\mu_{nj},\alpha_{x_j})), f(\kappa_2, \max(\nu_{1j},\beta_{x_j}),\right.\right.\\
&\quad \left.\left. \max(\nu_{2j},\beta_{x_j}), \cdots, \max(\nu_{nj},\beta_{x_j})\right\rangle \middle| x_j \in X\right\} \text{ for } \alpha_{x_j} + \beta_{x_j} \le 1.
\end{aligned} \tag{30}$$

Based on the simple additive model, the steps for solving a multi-criteria decision making problem can be summed up as follows.

Algorithm (I): for the effect of optimism or pessimism upon overall judgments.

Step 1: Establish evaluation criteria A_i's which have been judged relevant in a given decision situation by the decision maker. Denote $A = \{A_1, A_2, \ldots, A_n\}$.

Step 2: Develop feasible alternatives x_j's for achieving the goals or attaining the decision maker's needs and desires. Denote $X = \{x_1, x_2, \ldots, x_m\}$. Set $\alpha_{x_j}, \beta_{x_j} \in [0,1]$ for all $x_j \in X$ according to the decision maker's preference information on alternatives.

Step 3: Evaluate alternatives in terms of criteria on the fuzzy concept "excellence", where the values of criterion functions are expressed by IFSs. Denote $X_{ij} = \{\langle x_j, \mu_{ij}, \nu_{ij}\rangle\}$.

Step 4: Acquire the synthetic evaluation, unconsidering optimism or pessimism, of the alternative x_j by employing the averaging operation in (18).

Step 5: Determine the suitability function of the alternative x_j by employing the optimistic and pessimistic point operators. Use (19) or (20) for the optimistic condition, (21) for the optimistic condition with restrictions, (22) or (23) for the pessimistic condition, and (24) for the pessimistic condition with restrictions.

Step 6: Measure the degree of suitability for each alternative by means of the score function.

Step 7: Rank the preference order of all alternatives according to the degree of suitability. If there exists $j_o \in \{1, 2, \cdots, m\}$ such that the score function is largest, then the alternative x_{j_o} is the best choice.

Algorithm (II): for the effect of optimism or pessimism upon individual outcomes.

Steps 1-3: See Steps 1-3 of Algorithm (I).

Step 4: Construct the intuitionistic fuzzy decision matrix that is composed of all X_{ij}'s.

Step 5: Utilize the optimistic point operators, (9), (10), or (11), and pessimistic point operators, (12), (13), or (14), to reconstruct the intuitionistic fuzzy decision matrix.

Step 6: Determine the suitability function of the alternative x_j by employing the averaging operation. Use (25) or (26) for the optimistic condition, (27) for the optimistic condition with restrictions, (28) or (29) for the pessimistic condition, and (30) for the pessimistic condition with restrictions.

Steps 7-8: See Steps 6-7 of Algorithm (I).

4.2. Illustrative example

Assume that the degrees μ_{ij} of membership and the degrees ν_{ij} of non-membership for alternative $x_j \in X$ with respect to criterion $A_i \in A$ to the fuzzy concept "excellence" are give below:

$$((\mu_{ij}, \nu_{ij}))_{5\times 3} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{array} \begin{array}{c} \begin{array}{ccc} A_1 & A_2 & A_3 \end{array} \\ \begin{bmatrix} (0.39, 0.19) & (0.83, 0.12) & (0.27, 0.25) \\ (0.27, 0.55) & (0.73, 0.17) & (0.54, 0.19) \\ (0.34, 0.51) & (0.68, 0.01) & (0.55, 0.27) \\ (0.63, 0.16) & (0.48, 0.20) & (0.30, 0.41) \\ (0.55, 0.23) & (0.54, 0.25) & (0.38, 0.56) \end{bmatrix} \end{array}$$

We measure the degree of suitability of each alternative by employing all of optimistic and pessimistic point operators. Different settings of parameters are adopted, including (i) $\alpha_{x_j} = 0.4$, $\beta_{x_j} = 0.3$; (ii) $\alpha_{x_j} = 0.6$, $\beta_{x_j} = 0.4$; (iii) $\alpha_{x_j} = 0.2$, $\beta_{x_j} = 0.7$, and (iv) $\alpha_{x_j} = \mu_{ij} / (\mu_{ij} + \nu_{ij})$, $\beta_{x_j} = \nu_{ij} / (\mu_{ij} + \nu_{ij})$ for all $x_j \in X$. The resulting Algorithms (I) and (II) analyses are summarized in Tables 1 and 2, respectively.

Tab. 1: The results of Algorithm (I) in the case of $\kappa_1 = \kappa_2 = 1$ in the numerical example.

	Fixed point operator		Variant point operator	
Suitability function	α_{x_j} =0.4 β_{x_j} =0.3	α_{x_j} =0.6 β_{x_j} =0.4	α_{x_j} =0.2 β_{x_j} =0.7	$\alpha_{x_j} = \mu_{ij} / (\mu_{ij} + \nu_{ij})$ $\beta_{x_j} = \nu_{ij} / (\mu_{ij} + \nu_{ij})$
$J_{\alpha_{x_j}, \beta_{x_j}}(\bigotimes_{i=1}^{3} A_i)$	x_1: 0.5673 x_2: 0.4957 x_3: 0.5297 x_4: 0.5023 x_5: 0.4513 $x_1 \succ x_3 \succ x_4 \succ x_2 \succ x_5$	x_1: 0.6120 x_2: 0.5020 x_3: 0.5460 x_4: 0.5313 x_5: 0.4493 $x_1 \succ x_3 \succ x_4 \succ x_2 \succ x_5$	x_1: 0.4293 x_2: 0.3377 x_3: 0.3817 x_4: 0.3450 x_5: 0.2800 $x_1 \succ x_3 \succ x_4 \succ x_2 \succ x_5$	x_1: 0.6758 x_2: 0.5159 x_3: 0.5771 x_4: 0.5561 x_5: 0.4420 $x_1 \succ x_3 \succ x_4 \succ x_2 \succ x_5$
$J^*_{\alpha_{x_j}, \beta_{x_j}}(\bigotimes_{i=1}^{3} A_i)$	x_1: 0.6196 x_2: 0.5806 x_3: 0.6034 x_4: 0.5742 x_5: 0.5484 $x_1 \succ x_3 \succ x_2 \succ x_4 \succ x_5$	x_1: 0.6792 x_2: 0.6112 x_3: 0.6408 x_4: 0.6237 x_5: 0.5741 $x_1 \succ x_3 \succ x_4 \succ x_2 \succ x_5$	x_1: 0.4405 x_2: 0.3559 x_3: 0.3975 x_4: 0.3604 x_5: 0.3008 $x_1 \succ x_3 \succ x_4 \succ x_2 \succ x_5$	x_1: 0.7744 x_2: 0.6358 x_3: 0.6936 x_4: 0.6635 x_5: 0.5609 $x_1 \succ x_3 \succ x_4 \succ x_2 \succ x_5$
$P_{\alpha_{x_j}, \beta_{x_j}}(\bigotimes_{i=1}^{3} A_i)$	x_1: 0.3100 x_2: 0.2133 x_3: 0.2600 x_4: 0.2133 x_5: 0.1900 $x_1 \succ x_3 \succ x_2 \sim x_4 \succ x_5$	x_1: 0.4133 x_2: 0.2967 x_3: 0.3367 x_4: 0.3433 x_5: 0.2533 $x_1 \succ x_4 \succ x_3 \succ x_2 \succ x_5$	x_1: 0.3100 x_2: 0.2100 x_3: 0.2600 x_4: 0.2133 x_5: 0.1433 $x_1 \succ x_3 \succ x_4 \succ x_2 \succ x_5$	x_1: 0.5402 x_2: 0.3252 x_3: 0.4019 x_4: 0.3901 x_5: 0.2390 $x_1 \succ x_3 \succ x_4 \succ x_2 \succ x_5$
$H_{\alpha_{x_j}, \beta_{x_j}}(\bigotimes_{i=1}^{3} A_i)$	x_1: -0.0830 x_2: -0.1530 x_3: -0.1180 x_4: -0.1507 x_5: -0.1997 $x_1 \succ x_3 \succ x_4 \succ x_2 \succ x_5$	x_1: -0.0153 x_2: -0.0687 x_3: -0.0347 x_4: -0.0840 x_5: -0.1180 $x_1 \succ x_3 \succ x_2 \succ x_4 \succ x_5$	x_1: -0.3090 x_2: -0.3290 x_3: -0.3080 x_4: -0.3540 x_5: -0.3630 $x_3 \succ x_1 \succ x_2 \succ x_4 \succ x_5$	x_1: 0.0878 x_2: -0.0488 x_3: 0.0134 x_4: -0.0492 x_5: -0.1274 $x_1 \succ x_3 \succ x_2 \succ x_4 \succ x_5$
$H^*_{\alpha_{x_j}, \beta_{x_j}}(\bigotimes_{i=1}^{3} A_i)$	x_1: -0.1724 x_2: -0.2454 x_3: -0.2122 x_4: -0.2353 x_5: -0.2879 $x_1 \succ x_3 \succ x_4 \succ x_2 \succ x_5$	x_1: -0.0948 x_2: -0.1508 x_3: -0.1184 x_4: -0.1592 x_5: -0.1964 $x_1 \succ x_3 \succ x_2 \succ x_4 \succ x_5$	x_1: -0.5871 x_2: -0.6165 x_3: -0.6011 x_4: -0.6172 x_5: -0.6374 $x_1 \succ x_3 \succ x_2 \succ x_4 \succ x_5$	x_1: 0.0508 x_2: -0.1196 x_3: -0.0452 x_4: -0.1079 x_5: -0.2115 $x_1 \succ x_3 \succ x_4 \succ x_2 \succ x_5$
$Q_{\alpha_{x_j}, \beta_{x_j}}(\bigotimes_{i=1}^{3} A_i)$	x_1: 0.1000 x_2: 0.0967 x_3: 0.1000 x_4: 0.1000 x_5: 0.0533 $x_1 \sim x_3 \sim x_4 \succ x_2 \succ x_5$	x_1: 0.0967 x_2: 0.1133 x_3: 0.1233 x_4: 0.0700 x_5: 0.0900 $x_3 \succ x_2 \succ x_1 \succ x_5 \succ x_4$	x_1: -0.5000 x_2: -0.5000 x_3: -0.5000 x_4: -0.5000 x_5: -0.5000 $x_1 \sim x_2 \sim x_3 \sim x_4 \sim x_5$	x_1: 0.2235 x_2: 0.1419 x_3: 0.1886 x_4: 0.1168 x_5: 0.0757 $x_1 \succ x_3 \succ x_2 \succ x_4 \succ x_5$

5. Conclusions

In this study, we consider the influences of optimism and pessimism to develop different decision models and methods for multiple criteria decision making problems in an intuitionistic fuzzy environment. On the basis of the measurement tool estimations defined on IFSs, we utilize optimistic and pessimistic point operators to determine the influences of optimism and pessimism, respectively, and apply to multi-criteria decision analysis. For each point operator, we investigate and discuss several important properties to acquire a thoughtful understanding in concrete meanings of the relevant operations and relations. In addition, we develop an approach to deal with the effects caused by optimism and pessimism, consisting of changes in overall judgments and in separate evaluations of each alternative with respect to each criterion. The corresponding algorithms are provided to relate optimism and pessimism to multi-criteria decision analysis under the intuitionistic fuzzy decision setting.

Tab. 2: The results of Algorithm (II) in the case of $\kappa_1=\kappa_2=1$ in the numerical example.

Suitability function	Fixed point operator		Variant point operator	
	$\alpha_{x_j}=0.4$ $\beta_{x_j}=0.3$	$\alpha_{x_j}=0.6$ $\beta_{x_j}=0.4$	$\alpha_{x_j}=0.2$ $\beta_{x_j}=0.7$	$\alpha_{x_j}=\mu_{ij}/(\mu_{ij}+\nu_{ij})$ $\beta_{x_j}=\nu_{ij}/(\mu_{ij}+\nu_{ij})$
$\bigotimes_{i=1}^{3} J_{\alpha_{x_j},\beta_{x_j}}(A_i)$	x_1: 0.5673 x_2: 0.4957 x_3: 0.5297 x_4: 0.5023 x_5: 0c.4513 $x_1 \succ x_3 \succ x_4 \succ x_2 \succ x_5$	x_1: 0.6120 x_2: 0.5020 x_3: 0.5460 x_4: 0.5313 x_5: 0.4493 $x_1 \succ x_3 \succ x_4 \succ x_2 \succ x_5$	x_1: 0.4293 x_2: 0.3377 x_3: 0.3817 x_4: 0.3450 x_5: 0.2800 $x_1 \succ x_3 \succ x_4 \succ x_2 \succ x_5$	x_1: 0.6226 x_2: 0.4765 x_3: 0.5537 x_4: 0.5326 x_5: 0.4375 $x_1 \succ x_3 \succ x_4 \succ x_2 \succ x_5$
$\bigotimes_{i=1}^{3} J^{*}_{\alpha_{x_j},\beta_{x_j}}(A_i)$	x_1: 0.6196 x_2: 0.5806 x_3: 0.6034 x_4: 0.5742 x_5: 0.5484 $x_1 \succ x_3 \succ x_2 \succ x_4 \succ x_5$	x_1: 0.6792 x_2: 0.6112 x_3: 0.6408 x_4: 0.6237 x_5: 0.5741 $x_1 \succ x_3 \succ x_4 \succ x_2 \succ x_5$	x_1: 0.4405 x_2: 0.3559 x_3: 0.3975 x_4: 0.3604 x_5: 0.3008 $x_1 \succ x_3 \succ x_4 \succ x_2 \succ x_5$	x_1: 0.7042 x_2: 0.5684 x_3: 0.6247 x_4: 0.6242 x_5: 0.5450 $x_1 \succ x_3 \succ x_4 \succ x_2 \succ x_5$
$\bigotimes_{i=1}^{3} P_{\alpha_{x_j},\beta_{x_j}}(A_i)$	x_1: 0.3567 x_2: 0.3367 x_3: 0.3500 x_4: 0.2833 x_5: 0.2367 $x_1 \succ x_3 \succ x_2 \succ x_4 \succ x_5$	x_1: 0.4900 x_2: 0.3900 x_3: 0.4000 x_4: 0.3567 x_5: 0.3067 $x_1 \succ x_3 \succ x_2 \sim x_4 \succ x_5$	x_1: 0.3100 x_2: 0.2100 x_3: 0.2600 x_4: 0.2133 x_5: 0.1433 $x_1 \succ x_3 \succ x_4 \succ x_2 \succ x_5$	x_1: 0.5018 x_2: 0.3234 x_3: 0.4221 x_4: 0.3853 x_5: 0.2510 $x_1 \succ x_3 \succ x_4 \succ x_2 \succ x_5$
$\bigotimes_{i=1}^{3} H_{\alpha_{x_j},\beta_{x_j}}(A_i)$	x_1: -0.0830 x_2: -0.1530 x_3: -0.1180 x_4: -0.1507 x_5: -0.1997 $x_1 \succ x_3 \succ x_4 \succ x_2 \succ x_5$	x_1: -0.0153 x_2: -0.0687 x_3: -0.0347 x_4: -0.0840 x_5: -0.1180 $x_1 \succ x_3 \succ x_2 \succ x_4 \succ x_5$	x_1: -0.3090 x_2: -0.3290 x_3: -0.3080 x_4: -0.3540 x_5: -0.3630 $x_3 \succ x_1 \succ x_2 \succ x_4 \succ x_5$	x_1: 0.0643 x_2: -0.0131 x_3: 0.0771 x_4: -0.0354 x_5: -0.0988 $x_3 \succ x_1 \succ x_2 \succ x_4 \succ x_5$
$\bigotimes_{i=1}^{3} H^{*}_{\alpha_{x_j},\beta_{x_j}}(A_i)$	x_1: -0.1724 x_2: -0.2454 x_3: -0.2122 x_4: -0.2353 x_5: -0.2879 $x_1 \succ x_3 \succ x_4 \succ x_2 \succ x_5$	x_1: -0.1345 x_2: -0.1919 x_3: -0.1603 x_4: -0.1968 x_5: -0.2356 $x_1 \succ x_3 \succ x_2 \succ x_4 \succ x_5$	x_1: -0.5176 x_2: -0.5446 x_3: -0.5278 x_4: -0.5514 x_5: -0.5688 $x_1 \succ x_3 \succ x_2 \succ x_4 \succ x_5$	x_1: 0.0251 x_2: -0.0745 x_3: 0.0164 x_4: -0.0912 x_5: -0.1778 $x_1 \succ x_3 \succ x_2 \succ x_4 \succ x_5$
$\bigotimes_{i=1}^{3} Q_{\alpha_{x_j},\beta_{x_j}}(A_i)$	x_1: 0.0533 x_2: -0.0267 x_3: 0.0100 x_4: 0.0300 x_5: 0.0067 $x_1 \succ x_4 \succ x_3 \succ x_5 \succ x_2$	x_1: 0.0200 x_2: 0.0200 x_3: 0.0600 x_4: 0.0567 x_5: 0.0367 $x_3 \succ x_4 \succ x_5 \succ x_1 \sim x_2$	x_1: -0.5000 x_2: -0.5000 x_3: -0.5000 x_4: -0.5000 x_5: -0.5000 $x_1 \sim x_2 \sim x_3 \sim x_4 \sim x_5$	x_1: 0.1851 x_2: 0.1400 x_3: 0.2087 x_4: 0.1120 x_5: 0.0876 $x_3 \succ x_1 \succ x_2 \succ x_4 \succ x_5$

6. References

[1] K.T. Atanassov, "Intuitionistic Fuzzy Sets," *Fuzzy Sets and Systems*, vol. 20, pp. 87-96, 1986.

[2] K.T. Atanassov, *Intuitionistic Fuzzy Sets: Theory and Applications*, New York: Physica-Verlag, Heidelberg, 1999.

[3] K. T. Atanassov, G. Pasi, and R. Yager, "Intuitionistic Fuzzy Interpretations of Multi-criteria Multi-person and Multi-measurement Tool Decision Making," *International Journal of Systems Science* , vol. 36, no. 14, pp. 859-868, 2005.

[4] D. Dubois, S. Gottwald, P. Hajek, J. Kacprzyk, and H. Prade, "Terminological Difficulties in Fuzzy Set Theory—The Case of "Intuitionistic Fuzzy Sets"," *Fuzzy Sets and Systems*, vol. 156, pp. 485-491, 2005.

[5] L. Friedman, A. Weinberg, J. Webb, H. Cooper, and S. Bruce, "Skin Cancer Prevention and Early Detection Intentions and Behavior," *American Journal of Preventive Medicine*, vol. 11, pp. 59-65, 1995.

[6] D.-F. Li, "Multiattribute Decision Making Models and Methods Using Intuitionistic Fuzzy Sets," *Journal of Computer and System Sciences*, vol. 70, no. 1, pp. 73-85, 2005.

[7] L. Lin, X.-H. Yuan, and Z.-Q. Xia, "Multicriteria Fuzzy Decision-making Methods Based on Intuitionistic Fuzzy Sets," *Journal of Computer and Systems Sciences*, vol. 73, no. 1, pp. 84-88, 2007.

[8] M. Puri, and D.T. Robinson, "Optimism and Economic Choice," *Journal of Financial Economics*, vol. 86, pp. 71-99, 2007.
[9] L.J. Sanna, and E.C. Chang, "The Past is not What It Used to be: Optimists' Use of Retroactive Pessimism to Diminish the Setting of Failure," *Journal of Research in Personality*, vol. 37, pp. 388-404, 2003.
[10] Z. Xu, and R.R. Yager, "Dynamic Intuitionistic Fuzzy Mutli-attribute Decision Making," *International Journal of Approximate Reasoning*, vol. 48, no. 1, pp. 246-262, 2008.

Proceedings of the 11th Joint Conference on Information Sciences (2008)
Published by Atlantis Press