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Evolution of Kink Network in Inhomogenous Systems

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Abstract

The purpose of this report is to show the influence of imperfections on creation and evolution of a kink network. Our main finding is a mechanism for reduction of the kinetic energy of kinks which works in both the overdamped and underdamped regimes. This mechanism reduces mobility of kinks and therefore prevents the kink-antikink network from annihilation.

1 Context

In recent years, topological defects have attracted the attention of many researchers. The motivation for these studies comes from the fact that they can be seen as macroscopic manifestations of underlying physical processes. On the other hand they can help to study the nature of critical dynamics.

The theory describing the dynamics of the second order phase transition was proposed by Kibble and Zurek [1]. The key point of the Kibble-Zurek mechanism is an observation that the order parameter evolves adiabatically through a sequence of nearly equilibrium configurations up to the time of freeze-in. At that instant the system loses the capacity to respond to the change in the external parameters. From that time to the time of freeze-out the field configuration remains almost unchanged. The dynamical evolution restarts below the critical temperature at the time of freeze-out. At that instant the system regains the ability to respond to the changes in the external parameters but it is too late to undo non-trivial arrangements of the order parameter from above the critical point. This paradigm works for overdamped systems and underdamped systems as well. The main prediction of this scenario is the dependence of the number density of produced defects on correlation length $n \sim \xi^{-d}$ at the time of freeze-out or its dependence on a quench time $n \sim \tau^{-d/4}$, where d denotes the number of space dimensions. This scenario was well verified in a series of numerical experiments [2].

The density of a defect network obtained at the time of freeze-out is an initial condition for dynamics which is determined by the defect-antidefect interactions. Due to annihilation of defects and antidefects the initial density of the defect network is first quickly reduced in time and then is stabilized at the level determined by the Boltzmann factor which describes the probability of thermal nucleation of the kink-antikink pairs.

In real life experiments researchers concentrate on study of creation and evolution of topological defects in helium-3, liquid crystals and in superconducting films [3]. There are also attempts to study the creation of vortices in optically cooled alkali atom clouds during the formation of Bose-Einsten condensates. There were also more controversial experiments made in superfluid helium-4 where almost no vortices of topological origin were observed [4].

In this report we would like to concentrate on the influence of impurities on creation and evolution of topological defects. It is difficult to imagine a liquid crystal or even superconductor, that is free of imperfections. The population of the superconductors and liquid crystals by the impurities and admixtures seems to be an inevitable outcome of their preparation. Although the solubility of foreign materials in liquid helium is small there are some artificial techniques, like the aerogel technique [5], which allow one to introduce impurities even into a quantum liquid. Most of the results obtained hitherto concern an homogeneous medium. On the other hand, the presence of impurities can significantly change the properties of the system.

Analytical and numerical studies of the kink distribution show that kinks are created mainly in the vicinity of knots of the force distribution which corresponds to extremes of the impurity potential, and therefore the distribution of the kinks is determined not only by the quench time but also by some length scale which characterizes the average distance between impurities [7]. Due to the existence of impurities and admixtures the topological defects are created mainly in the knots of the impurity force distribution. This observation suggests that kinks produced in systems of this kind are confined to impurity centers. To confirm this conjecture the exact solutions which describe squeezed kinks trapped by the impurity were constructed [8].

The existence of squeezed kink solutions confirms the statistical prediction concerning the distribution of kinks at the time of freeze-out. Moreover, stability of this solution against small perturbations is crucial for the behavior of the kink-antikink network at later times. It seems that, in contrary to pure systems, the disappearance of the kinks from the system via kink-antikink annihilation is substantially reduced or even stopped by fixing the positions of kinks and antikinks. This observation is additionally confirmed by the existence of a static squeezed kink-squeezed antikink solution which is also stable against small perturbations [8].

2 Interaction of the kink with the impurity

The stability against small perturbations allows one to draw conclusions concerning the behaviour of the kink network, at least at low temperature when the thermal fluctuations of the field ϕ are small. If the temperature is higher then we also need to analyze the behavior of the system for larger changes of the field. This analysis can be made numerically.

We consider the ϕ^4 model with dissipation,

$$\frac{1}{c^2}\partial_t^2\phi(t,x)+\Gamma\partial_t\phi(t,x)=\partial_x^2\phi(t,x)+a\phi(t,x)-\lambda\phi^3(t,x)+\mathscr{D}(x), \tag{2.1}$$

where c is the sound speed (or the speed of light), Γ is the dissipation constant, a is a chemical potential and λ describes the self-interaction of the scalar field. In the limit $c\to\infty$ this model reduces to the Landau-Ginzburg model, as extensively studied in condensed matter physics. On the other hand in the dissipationless limit $(\Gamma=0)$ it is similar to the Higgs sector of the Standard Model of particles.

The equation (1), in the absence of dissipation $(\Gamma=0)$ and in a system free of imperfections $(\mathscr{D}(x)=0)$, possesses a stationary kink solution.

In the further investigations we choose the following form of the external force distribution

$$\mathscr{D}(x) = \mathscr{A}\left(\frac{a}{\lambda}\right)^{\frac{3}{2}} \frac{\sinh \beta_A (x - x_{imp})}{\cosh^3 \beta_A (x - x_{imp})}, \tag{2.2}$$

The force distribution of this kind is generic for the description of imperfections in this sense that the normalized potential

$$V(x) = \sqrt{\frac{a}{2}}\frac{\gamma_A}{2}\,\mathrm{sech}^2\,(\sqrt{\frac{a}{2}}\,\gamma_A(x - x_{imp}))$$

for this force, in the $\gamma_A \to \infty$ limit, has properties of the delta function i.e. $V(x = x_{imp}) \to \infty$ and $V(x \neq x_{imp}) \to 0$. This kind of potential is typically assumed in the description of impurities and defects. This form of the force distribution has an additional advantage, namely that the static kink solution with this choice of $\mathscr{D}(x)$ can be easily found by direct integration and is given by a squeezed kink profile [8]. It suggests the possible existence of a solution continuously interpolating between squeezed and free kinks,

$$\phi_{K\ ans}(x,t) = \sqrt{\frac{a}{\lambda}}\tanh\left(\sqrt{\frac{a}{2}}\gamma(t)(x - x_K(t))\right). \tag{2.3}$$

We adopt this function as an initial configuration of the field. At the initial instant of time t_{in} we assume that $\gamma(t_{in}) = 1/\sqrt{1-\frac{v^2}{c^2}}$ where $\dot{x}_K(t_{in}) = v$. We also assume that $\dot{\gamma}(t_{in}) = 0$ and initially the kink is located at $x_K(t_{in}) = x_0$. On the other hand we expect that if we wait a sufficiently long time, then at some final instant of time $\gamma(t_{fin}) = \sqrt{1+\frac{\mathscr{A}}{\lambda}}$ and $x_K(t_{fin}) = x_{imp}$. It means that at the beginning we have a stationary kink moving with speed v, and we expect that at least for some range of initial speeds the final configuration is a squeezed kink resting at the position of the impurity.

In numerical investigations we use equation (1) with rescaled variables and parameters $(ct \to t, \sqrt{\frac{a}{2}}x \to x, \phi \to \sqrt{\frac{a}{\lambda}}\phi, \sqrt{\frac{\lambda}{a}}\mathscr{D} \to \mathscr{D}, c\Gamma \to \Gamma)$.

First we studied formation of the bound state. For a better visualization of the squeezing effect we plot the first derivative of the field configuration in the spatial direction. The simulations show that the initially slowly moving kink is attracted and squeezed by the impurity (fig.1). After this period of movement the kink stops at the position of the impurity. The final state of our simulations is a squeezed kink, described in the paper [8]. The squeezing factor in the figure confirms the analytical prediction. For small values of the damping constant we also observe some damped oscillations of the kink about the position of the impurity. An example of half of the period of such oscillations is presented in figures 1.b-d. The final state of this evolution is a configuration which looks similar to the one presented in figure 1.c.

In the numerical experiments we also controlled the speed of the kink, this being identified with the speed of the zero of the field $\phi(t,x)$. Typical time dependence of the kink velocity (see fig.2) shows a dramatic increase in speed during the interaction of the kink with the imperfection (see the time interval $20 < t < 30$ in the figure). After formation of the squeezed kink we observe oscillations of this new state ($t > 30$).

In case of large damping all processes are much less violent. For example, for $\Gamma = 5$ we observe a gradual decrease of the velocity in the case of the kink located initially far from the imperfection ($x_0 = -25$). This kink quickly stops, on the time scale of the experiment, and never reaches the impurity. On the other hand, if it is located enough close to the centre of the impurity ($x_0 = -3$) then after a period of slowing down, connected to large friction, we observe an increase in the velocity, connected to formation of the squeezed kink.

We also studied the underdamped regime of the ϕ^4 model. In this sector we have also found that the creation of the bound state corresponds to a local increasing of the velocity $\dot{x}_K$ of the zero of field $\phi(t,x)$ (see fig.3.a-b). Particularly if the initial velocity is sufficiently large, then the speed of the zero of the scalar field may exceed the speed of sound (see fig.3.b). We think that this phenomenon is not an artefact of the numerics but has the same origin as exceeding the speed of light, as described in the literature in a different context. For example, in the papers [9] the phenomenon of exceeding the speed of light by the zeros of the scalar field has been observed numerically in the process of the interaction of two vortices. It seems that, in spite of the enormous increasing of the speeds of zeros of the Higgs field, nothing unphysical happens in the system. It has been found that at the same time the energy distribution is almost static. This observation leads to the conclusion that during the interaction process, in relativistic models, the variable $x_K(t)$ becomes unphysical. This observation explain why the collective coordinate methods fail in the description of interaction process for relativistic speeds. After interaction some oscillations around the impurity position are also observed. We also have checked that for a sufficiently large initial speed, after some interaction of the kink with the impurity, the kink leaves the interaction area without forming a bound state (see fig.3.c). The evolution of the field ϕ during this process is presented in figures 4.a-c. Figure 4.d shows that this interaction, even in the dissipation-free model ($\Gamma = 0$), is the reason for the reduction of the kink speed during its collision with the admixture. There are two reasons for the reduction of the kinetic energy of the kink during this process. The first is confinement of some part of the energy of the kink in the form of a gradient of the scalar field trapped by the impurity, and the second is the radiation. Both are visible in figures 4.b-c. The first corresponds, in the figures, to the stable structure formed at the position of the impurity ($x_{imp} = 0$) - see the area enclosed in the dashed circle. We also confirmed the formation of this structure at the position of the impurity in the system with small friction $\Gamma = 0.03$ (which reduces radiation in the system).

We also studied the interaction of the kink with a larger number of centres of impurity. For example, in figure 5 we present the effect of this interaction with two admixtures. We showed that the collision with the first imperfection reduces the speed of the kink to the level 0.3-0.4 which is sufficient for the kink to be confined by the second impurity.

3 Remarks

In this report we considered the influence of inhomogeneities on the creation and further evolution of a kink-antikink network.

We know that during the phase transition in an homogeneous system we obtain an homogeneous distribution of kinks determined by the K-Z exponent $n \sim \tau^{-1/4}$. The further evolution of this network is dominated by kink-antikink attraction and then annihilation. This process causes substantial reduction of the number density of kinks created at the time of freeze-out. On the other hand, at nonzero temperature one can observe a competitive process of thermal nucleation of kink-antikink pairs [10]. Usually thermal nucleation is substantially less efficient than annihilation and therefore at the final stage of evolution the system almost does not contain any kinks.

On the other hand, in a system populated by imperfections the situation is significantly differ-

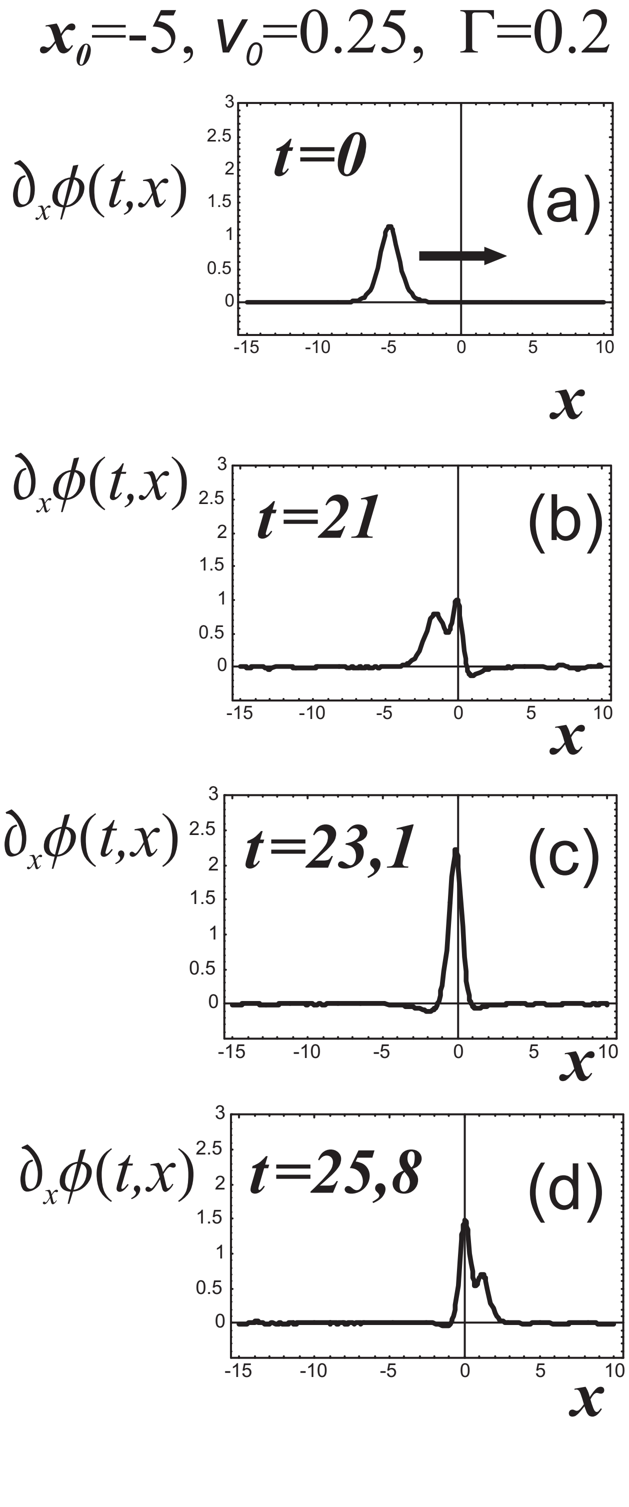

Figure 1: Formation of the bound state through a sequence of damped oscillations. a) Kink pushed with the speed $v = 0.25$. b)-d) Half period of oscilations of the kink interacting with the impurity located at $x_{imp} = 0$.

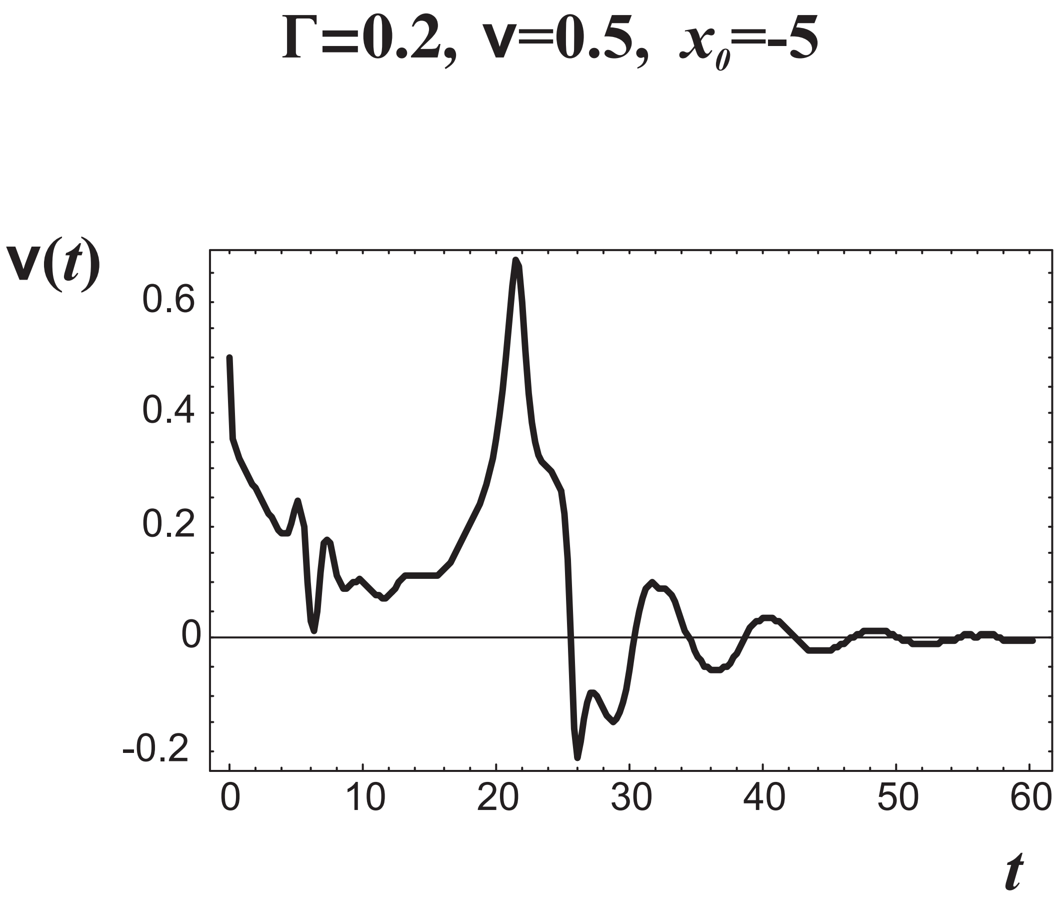

Figure 2: Time dependence of the kink speed during the formation of the bound state.

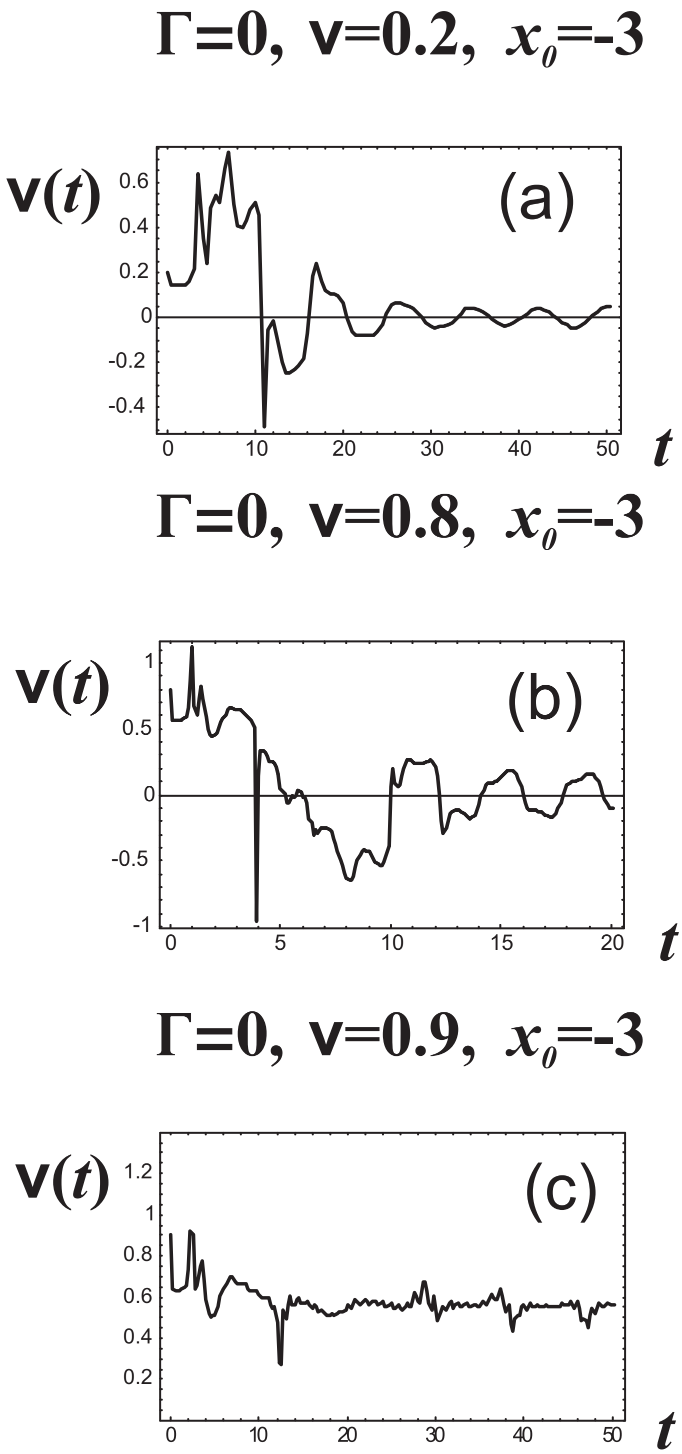

Figure 3: Interaction of the kink with the impurity in underdamped system. a)-b) Formation of the bound state. c) Reduction of the velocity during the interaction of the kink with the impurity without formation of the bound state.

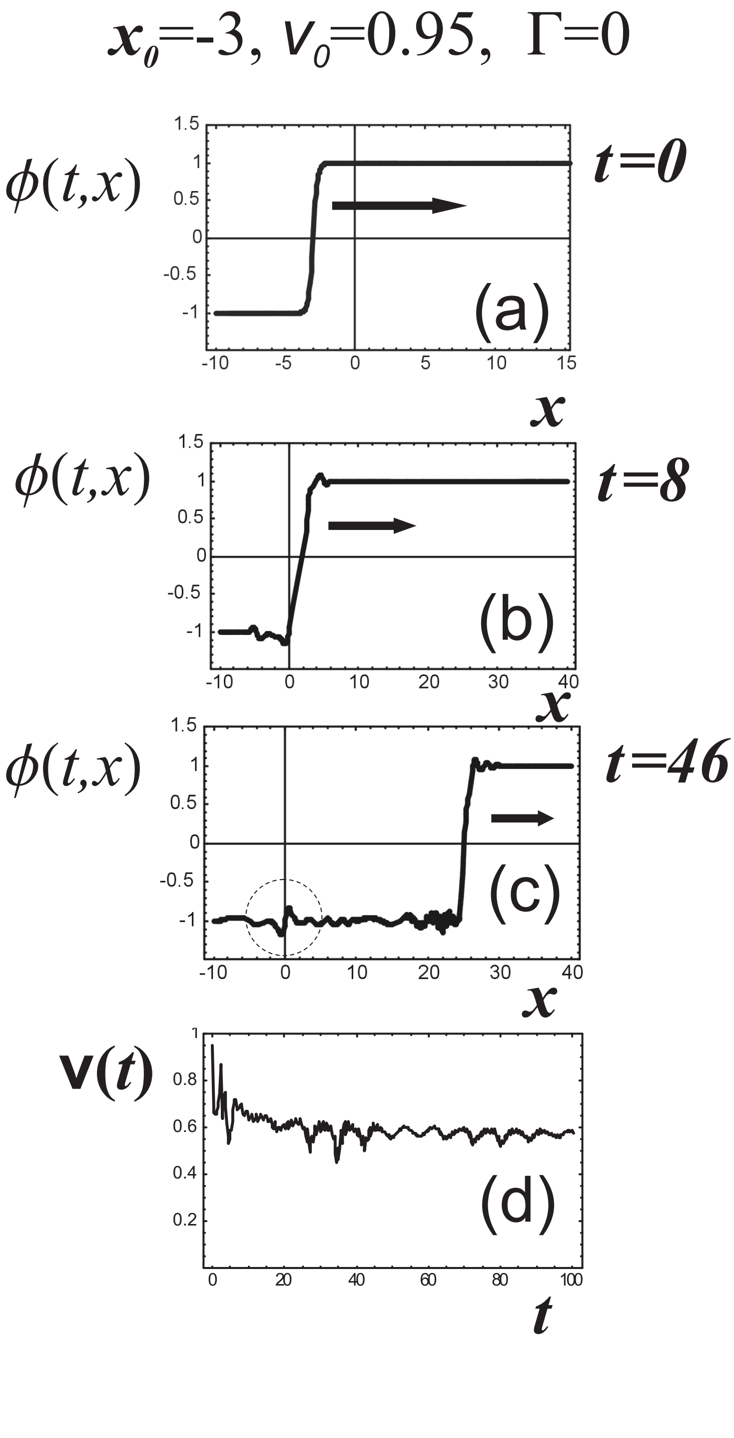

Figure 4: a)-c)Fast kink leaves interaction area without formation of the bound state. d) Reduction of the kink velocity.

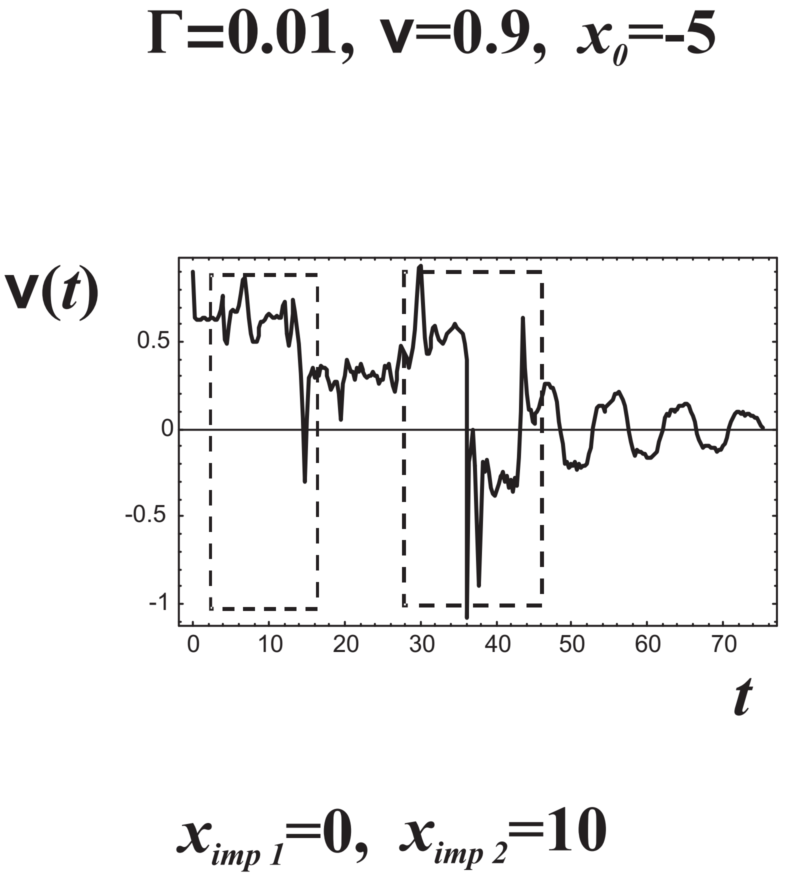

Figure 5: Interaction of the kink with two impurities. Collision with the first impurity reduces the kinetic energy of the kink but the kink still remains unbounded. As a result of interaction of the kink with the second impurity the squeezed kink is formed. Interaction areas are denoted by dashed boxes.

ent. First, the number density of kinks created at the time of freeze-out is determined not only by the quench time but also by the length scale which describes the distribution of impurities in the system. If one considers the system with a single imperfection then it becomes clear that interfaces are created mainly in the vicinity of the imperfection. In a system populated by strong imperfections, the kinks are created mainly at the positions of the imperfections.

The next question raised in this report is the further evolution of the kink network created at the time of freeze-out. We showed that the presence of impurities substantially reduces mobility of kinks in the system and therefore reduces or even stops kink-antikink annihilation. According to our studies, each interaction of the kink with the impurity reduces the kinetic energy of the kink, or if this energy is sufficiently small causes confinement of the kink by the imperfection. The reason for this reduction is independent on friction and is connected to confinement of some part of the kink energy in the form of the gradient of the scalar field located in the vicinity of the imperfection. What is important is that this mechanism of reduction of kink velocity works even in an underdamped system. The other reason for reduction of the kinetic energy is radiation of the energy during collision of the kink with the imperfection. We have shown that subsequent collisions reduce the energy of the kink until its energy is so small that it can be confined by the impurity. From our research it follows that, in a system that is densely populated by imperfections, most kinks created during the phase transition are confined by imperfections and therefore the number density of kinks remains almost unchanged during the evolution. On the other hand, in a system with friction, the kinetic energy is reduced due to dissipation (instead of radiation), but the mechanism works in the same way as previously described. Additionally, depending on the temperature of the system, we expect thermal nucleation of a small number of kink-antikink pairs [10].

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