Research on Operational Decision Scheme Evaluation Method

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Abstract. Operational decision scheme evaluation is an important means of inspection that operations are determined accurately or not. This article explores the evaluation methods of combat determination method, and proposes a decision scheme evaluation method based on interval numbers grey relation projection. This article introduces the basic concept and basic principle of the method, established a decision-making model which justify the decision plan's coordinate with positive ideal solution or the negative one.

Introduction

Program evaluation on operational resolution, that is to make comprehensive assessment of the feasibility, works as an important means of inspection on the accuracy of operational resolution and plays an important role in decision-making and decision-making efficiency [1, 2]. The research on Operational decision-making scheme evaluation and sorting [3], essentially a multi-objective decision problem has not yet formed a system with a broad range of instructional theory [4], nor a universally recognized ideal evaluation method.

To evaluate operational determination scheme, the probability of pure analytic calculation is very difficult to cover the uncertain factors affecting operational decisions. Interval number and fuzzy number can well reflect the uncertainty of parameters, the parameter values in a variety of types, such as precision number, interval number and fuzzy number) in the form of the decision matrix which constitutes the mixed problem of the multi-target appraisal. This article discusses evaluation problem from the uncertainty of type of index weight and fuzzy number of hybrid multi-index respect to do numerical, interval number. This paper proposes a scheme evaluation method based on interval number grey relation projection for the scheme evaluation problem provides a new solution.

The principle and the evaluation model of interval number grey relation projection based on Effectiveness

Scheme evaluation and sorting problem is the selection of decision makers in the face of multiple solutions, the use of two or more than two standard (i.e. indicators or attributes) on the comprehensive evaluation of each scheme and the priorities of the scheme and choose satisfactory solution.

Assumed the solution sets as $S = \{S_1, S_2, \dots, S_m\}$ and the attribute sets (cited as index set such as "cost type" and "benefit type") as $P = \{P_1, P_2, \dots, P_n\}$. weight vector ($w = \{w_1, w_2, \dots, w_n\}$) donates the status of properties, in which the weighting equal to $w_j = [w_j^L, w_j^R]$, $\sum_{j=1}^n w_j^L \leq 1$, $\sum_{j=1}^n w_j^R \geq 1$, $0 \leq w_j^L \leq w_j^R \leq 1$, $j = 1, 2, \dots, n$. The following are other variates.

x_{ij} denotes the value of set S_i in the attribute sets P_j which can be a variety of types (such as precision number, interval number and fuzzy number) of the form, thus come to constitute a hybrid original evaluation matrix say $X = (x_{ij})_{m \times n}$, $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$, which formulated as

$$x_{ij} = \begin{cases} x_{ij}, & \text{values are precision number} \\ [x_{ij}^L, x_{ij}^R], & \text{values are interval number} \\ [x_{ij}^L, x_{ij}^M, x_{ij}^U], & \text{values are fuzzy number} \end{cases} \quad (1)$$

Initial evaluation matrix forms the interval number. The precise number and triangular fuzzy Numbers is transformed into interval number to form the interval number initial evaluation matrix $Y = ([y_{ij}^L, y_{ij}^R])_{m \times n}$:

$$[y_{ij}^L, y_{ij}^R] = \begin{cases} [x_{ij}, x_{ij}], & \text{values are precision number} \\ [x_{ij}^L, x_{ij}^R], & \text{values are interval number} \\ [(x_{ij}^L + x_{ij}^M)/2, (x_{ij}^M + x_{ij}^U)/2], & \text{values are fuzzy number} \end{cases} \quad (2)$$

Construct the positive and negative ideal interval number of initial evaluation matrix. Assume the index of an ideal solution s_0^+ as $[y_{0j}^L, y_{0j}^R]$ ($j=1,2,\dots,n$), and satisfy the following circumstance:

$$[y_{0j}^L, y_{0j}^R] = [\max(y_{1j}^L, \dots, y_{mj}^L), \max(y_{1j}^R, \dots, y_{mj}^R)] \quad (3)$$

P_j as cost type

$$[y_{0j}^L, y_{0j}^R] = [\min(y_{1j}^L, \dots, y_{mj}^L), \min(y_{1j}^R, \dots, y_{mj}^R)] \quad (4)$$

Augmented matrix type $Y^+ = ([y_{ij}^{L+}, y_{ij}^{R+}])_{(m+1) \times n}$ ($i=0,1,\dots,m; j=1,2,\dots,n$) turn out be solution set of index set which is ideal interval number of initial evaluation matrix. Therefore, solution set or S is related to P as followed:

$$Y^- = ([y_{ij}^{L-}, y_{ij}^{R-}])_{(m+1) \times n} \quad (i=0,1,\dots,m; j=1,2,\dots,n) \quad (5)$$

Construct the positive and negative ideal interval evaluation matrix. To align the ideal interval number of initial value as the followed to get normalized matrix as the ideal interval number of matrix:

$$Y^+ = ([y_{ij}^{L+}, y_{ij}^{R+}])_{(m+1) \times n} \quad (i=0,1,\dots,m; j=1,2,\dots,n) \quad Z^+ = ([z_{ij}^{L+}, z_{ij}^{R+}])_{(m+1) \times n} \quad (6)$$

When P_j is benefit type:

$$[z_{ij}^{L+}, z_{ij}^{R+}] = \left[\frac{y_{ij}^{L+}}{y_{0j}^{L+}}, \frac{y_{ij}^{R+}}{y_{0j}^{R+}} \right] \quad (7)$$

When P_j is cost type:

$$[z_{ij}^{L+}, z_{ij}^{R+}] = \left[\frac{y_{0j}^{L+}}{y_{ij}^{L+}}, \frac{y_{0j}^{R+}}{y_{ij}^{R+}} \right] \quad (8)$$

Then the negative ideal interval evaluation matrix: $Z^- = ([z_{ij}^{L-}, z_{ij}^{R-}])_{(m+1) \times n}$.

According to the interval number multiplication formula, calculate the positive and negative ideal interval number of weighted evaluation matrix Z^+ and Z^- :

$$Z^+ = ([z_{ij}^{L+}, z_{ij}^{R+}])_{(m+1) \times n} = ([w_j^L, w_j^R] \otimes [z_{ij}^{L+}, z_{ij}^{R+}])_{(m+1) \times n} \quad (9)$$

$$Z^- = ([z_{ij}^{L-}, z_{ij}^{R-}])_{(m+1) \times n} = ([w_j^L, w_j^R] \otimes [z_{ij}^{L-}, z_{ij}^{R-}])_{(m+1) \times n} \quad (10)$$

To construct the positive and negative ideal grey relational evaluation matrix. The row vector in $Z^+ = ([z_{ij}^{L+}, z_{ij}^{R+}])_{(m+1) \times n}$ and that in s_0^+ work out to get ideal grey relational evaluation matrix or $G^+ = (r_{ij}^+)_{(m+1) \times n}$, $i = 0, 1, \dots, m$; $j = 1, 2, \dots, n$, in case:

$$r_{ij}^+ = \frac{\min_{0 \leq i \leq m} \min_{0 \leq j \leq n} [z_{0j}^{L+}, z_{0j}^{R+}] - [z_{ij}^{L+}, z_{ij}^{R+}] + \rho \max_{0 \leq i \leq m} \max_{0 \leq j \leq n} [z_{0j}^{L+}, z_{0j}^{R+}] - [z_{ij}^{L+}, z_{ij}^{R+}]}{[z_{0j}^{L+}, z_{0j}^{R+}] - [z_{ij}^{L+}, z_{ij}^{R+}] + \rho \max_{0 \leq i \leq m} \max_{0 \leq j \leq n} [z_{0j}^{L+}, z_{0j}^{R+}] - [z_{ij}^{L+}, z_{ij}^{R+}]} \quad (11)$$

The interval distance calculation chosen is geometric distance formula in this paper [7, 8]:

$$[z_{0j}^{L+}, z_{0j}^{R+}] - [z_{ij}^{L+}, z_{ij}^{R+}] = \left((z_{0j}^{L+} - z_{ij}^{L+})^2 + (z_{0j}^{R+} - z_{ij}^{R+})^2 \right)^{1/2} \quad (12)$$

$r_{0j}^+ = 1$, ρ as resolution ratio, $\rho \in [0, 1]$, assumed to be $\rho = 0.5$. The negative ideal grey relational evaluation matrix can be defined as $G^- = (r_{ij}^-)_{(m+1) \times n}$, $i = 0, 1, \dots, m$; $j = 1, 2, \dots, n$.

Grey relation projection principle. Assuming each decision scheme as a row vector, θ_i (the grey relation projection Angle) is formed between s_i (Decision Solutions) and s^* (ideal one) and the cos is calculated as:

$$\cos \theta_i = \frac{S_i \cdot S^*}{\|S_i\| \|S^*\|} = \frac{\sum_{j=1}^n r_{ij} r_{0j}}{\sqrt{\sum_{j=1}^n r_{ij}^2} \sqrt{\sum_{j=1}^n r_{0j}^2}} \quad (13)$$

In Eq.13, $i = 1, 2, \dots, m$, with the θ_i reduce, $\cos \theta_k$ grows as θ_i declines, indicating the consistent change direction between s_i and s^* .

Assuming the modulus of s_i is $d_i = \sqrt{\sum_{j=1}^n r_{ij}^2}$, close degree of each scheme and ideal scheme can be reflected taking the variation of modulus (d_i) and $\cos \theta_i$ into consideration.

The grey relation projection $D_i = d_i \cos \theta_i$ is the projection of decision scheme s_i at ideal scheme s^* . The decision schemes values s_i on grey relation projection for positive ideal solution D_i^+ and negative ideal solution on the grey relation projection values D_i^- are as following:

$$D_i^+ = \frac{\sum_{j=1}^n r_{ij}^+ r_{0j}^+}{\sqrt{\sum_{j=1}^n (r_{0j}^+)^2}}, \quad i = 1, 2, \dots, m \quad (14)$$

$$D_i^- = \frac{\sum_{j=1}^n r_{ij}^- r_{0j}^-}{\sqrt{\sum_{j=1}^n (r_{0j}^-)^2}}, \quad i = 1, 2, \dots, m \quad (15)$$

For decision scheme s_i , D_i^+ is the bigger the better, and D_i^- is as small as better.

Grey relation projection coefficient. In sort of scheme s_i , hoping for the best plan is closest to the ideal solution, and also the most far away from the negative ideal solution. E_i is defined as grey relation projection coefficient to comprehensive measure scheme s_i close to the positive ideal solution and away from negative ideal solution.

Theorem 1[9]: For scheme s_i , assuming that the grey relation projection values of positive and negative ideal solution are D_i^+ and D_i^- , the grey relation projection coefficient as

$$E_i = \frac{D_i^{+2}}{D_i^{+2} + D_i^{-2}} \quad (16)$$

Sort grey relation projection coefficient value E_i from big to small, the maximum value E_i is the

optimal evaluation schemes.

Evaluation algorithms

To sum up the theory and model, based on the interval grey relation projection method of effectiveness evaluation steps can be summarized as follows:

Step 1 According to the known solution set S and index set P , construct mixed initial evaluation matrix X .

Step 2 Evaluation index into precise Numbers and triangle fuzzy Numbers is transformed into interval number, the formation of interval number Y initial evaluation matrix Y .

Step 3 To construct ideal interval number of initial evaluation matrix Y^+ and the negative ideal interval number of initial evaluation matrix Y^- .

Step 4 Initial treatment of the positive, negative ideal interval number evaluation matrix Y^+ and Y^- , and obtained positive, negative ideal interval number evaluation matrix Z^+ and Z^- ; According to the interval numbers multiplication formula[10], calculation of the positive, negative ideal interval number evaluation matrix and weight.

Step 5 All the row vector of the positive (negative) ideal interval number weighted evaluation matrix Z^+ and Z^- respectively compared to the index row vector of positive (negative) scheme s_0^+ (s_0^-) calculation of grey relation coefficient, thus forming positive, negative ideal grey relational evaluation matrix G^+ and G^- .

Step 6 Calculation the value D_i^+ and D_i^- of grey relation projection for every scheme s_i for the positive and negative ideal solution.

Step 7 Calculation coefficient of grey correlation projection of each scheme s_i .

Step 8 Accordance with the grey relation projection coefficient E_i value from big to small order; make a comprehensive and objective assessment of each program.

Summary

This paper combines the concept of grey relational degree, the concept of TOPSIS and the vector projection method, according to the operation rules of interval number, interval number is proposed of grey correlation projection method for assessment of efficacy. This method is suitable for evaluation of multi index of mixed type in weight is not completely determined (such as precision number, interval number, fuzzy number), which has broad applicability.

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