Simulation of Torricelli Effluent Flow by Using Visual Basic for Application (VBA) on Microsoft Excel

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Abstract - Simulation of Torricelli effluent flow was created by using Visual Basic for Application (VBA) on Microsoft Excel. It describes a container with a cross sectional area A_1 filled by water with a certain level h_1. At the container's wall, there is a small leakage with a leakage height h_2 and a cross sectional area A_2. If the quantity of water in the container is very large ($A_1 \gg A_2$), the speed of water is constant, and the effluent distance of water out from leakage (x) can be simply determined by applying the Bernoulli principle, the equation of continuity, and the formula of free-fall motion. However, if the cross sectional area of container is proportional to the leakage's cross sectional area, then the speed of effluent water decreases with time and the formula of effluent distance is revisited. The simulation program was generated by the Euler method and analytical calculation. The manipulated variables are water level, leakage height, buffer height, container cross sectional area, and leakage cross sectional area. The simulation presents idealized condition by ignoring the air resistance, adhesion force, and surface tension of water so that it deviates from experimental results.

Index Terms – Torricelli Principle, Bernoulli, VBA Excel

1. Introduction

Learning physics through computer simulation is common nowadays. It has been created many educational software or applications that contain physics simulation making learning materials more interesting. Learning physics through simulation is known able to enhance students' motivation and creativity. Students are also able to obtain new information from this media by constructive manner [1]. One application potentially used to create physics simulation is Microsoft Excel. This application, beside known broadly as number crunchers and mathematical operations, can present moving view. Microsoft Excel utilization in physics simulation for learning physics has some advantages. It is easy because almost all teachers are accustomed to use it and animation only based on number, graph, and object by using certain cells in the spreadsheet. Moreover, it does not consume a lot of memory [2]. On the other hand, Microsoft Excel is completed with the programming language of Visual Basic for Application (VBA) that enables users to control the running. The VBA is utilized to manipulate variables and objects movement of physical phenomena [3].

In this paper, simulation of physics concept of Torricelli effluent flow will be reported. The topic was chosen because it potentially brings out students misconceptions regarding to the width of container used [4]. The form of Torricelli principle is described as a cylindrical container filled by water and it has a small leakage in its wall. The Torricelli principle states that if the quantity of water filled is plentiful, then the quantity of water flow through the small leakage does not make the water level decreases. The speed of water out from it is constant and so does the horizontal position of water fall [5].

If the quantity of water is not plentiful, or the cross sectional area of the container is proportional to that of the leakage, then the Torricelli principle is no more valid and it should be revised. The volume of water in the container, water speed, and horizontal position of water fall will change with time. Unfortunately, the Torricelli principle taught in Indonesian Senior High School (Grade XI) does not explain the revision [6]. The program was created to show the phenomena.

In designing physics simulation, we should combine physics concepts, relevant applied mathematics, and computer programming. The applied mathematics needed to run the program is numerical methods [7]. Since real results are difficult to obtain by analytical methods, numerical methods are then employed to approach the real results. An accurate simulation is acquired when the numerical and analytical methods produce very close results. The simulation results are also compared to experimental results to give a consideration whether the simulation is feasible or not.

2. Method

The Torricelli effluent flow is described in Fig. 1. The equation of continuity states that, the product of the cross sectional area A and the fluid speed v at all points along a pipe is constant for a steady flow of incompressible fluid as expressed by Eq. (1) [5,8].

$$A_1 v_1 = A_2 v_2, \tag{1}$$

where the subscripts 1 and 2 refers to the container and leakage area, respectively.

The Bernoulli equation states that the kinetic energy density in all points along a pipe is constant for incompressible fluid [5,8]. For the Torricelli effluent flow, it is therefore written down as given in Eq. (2)

$$P_1 + \frac{1}{2}\rho_1 v_1^2 + \rho_1 g h_1 = P_2 + \frac{1}{2}\rho_2 v_2^2 + \rho_2 g h_2, \tag{2}$$

where P is the atmospheric pressure, ρ is the fluid density, and g is the gravitational acceleration. By considering that atmospheric pressure between both points is same ($P_1=P_2$), v_1 in Eq. (1) is substituted into Eq. (2) to yield the discharge water speed (v_2) as given in Eq. (3)

$$v_2 = \sqrt{2g(h_1 - h_2)\big/(1-(A_2^2\big/A_1^2))}\,. \tag{3}$$

If $A_1 >> A_2$, then Eq. (3) approaches to $v_2 = \sqrt{2g(h_1 - h_2)}$. The time for water to reach ground, which is formulated as a free-fall motion, is obtained as $t = \sqrt{2h_2/g}$. Thus the horizontal distance of discharge water is $x = 2\sqrt{h_2(h_1 - h_2)}$.
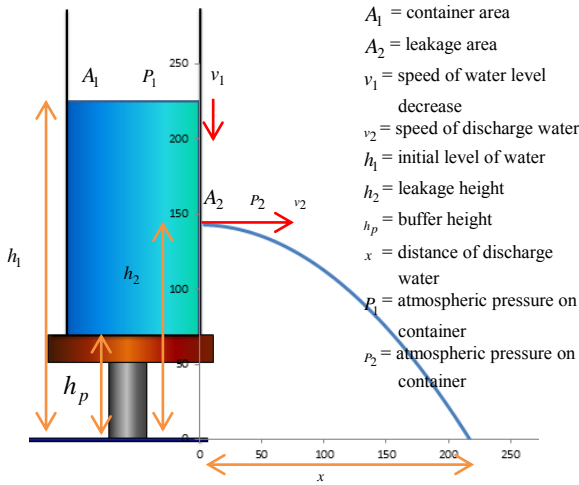

Figure 1. Torricelli Principle

If A_1 is not considerably larger than A_2, then h_1 will decreases with time. By considering that h_1 is a function of time, Eq. (3) into Eq. (1) yields the speed of water level decrease v_1 that depends on time as given in Eq. (4).

$$v_1 = dh_1(t)\big/dt$$
$$= (A_2\big/A_1)\sqrt{2g(h_1(t) - h_2)\big/(1-(A_2^2\big/A_1^2))}\,. \tag{4}$$

The water level h_1 as a function of time can be written down as in Eq. (5)

$$h_1(t) = h_{1initial} - \int_0^t v_1(t)dt\,. \tag{5}$$

The horizontal and vertical position of water discharge each time is seen as parabolic motion started from maximum height. The position with time is formulated as shown in Eqs. (6) and (7).

$$x(t) = \int_0^t v_2(t)dt \tag{6}$$

$$y(t) = h_2 - \int_0^t gtdt \tag{7}$$

Eqs (5), (6), and (7) are solved using a simple numerical technique called Euler method. The Euler method is written as in Eq. (8) [7].

$$f_{i+1} = f_i + \Delta t \cdot f'_i\,, \tag{8}$$

where Δt is time step size and f'_i is the first derivation of an arbitrary function of time f_i.

The program coding is used by combining formulas in the spread sheet and VBA. The flowchart of the program is described in Fig. 2.
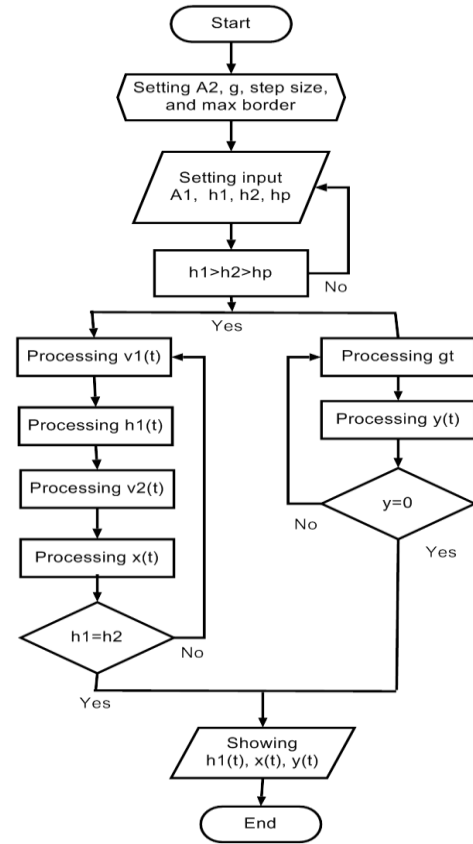

Figure 2. Flowchart of the program.

The basic of Torricelli effluent flow animation was arranged by a creative combination of three simple shapes as shown in Fig. 3 [9]. The program uses four main inputs as manipulated variables: (1) water level in container (h_1); (2) width of container (A_1); (3) buffer height; and (4) leakage height. Inputs (1), (2), and (4) are represented by a water-blue-colored cubicle in Fig. 3.(a), Fig. 3.(b), and a blue line graph in Fig. 3.(c), respectively. All inputs are set by a scrollbar to see the effect of little change of input toward output. The movement of the shapes is generated by the numerical calculation.
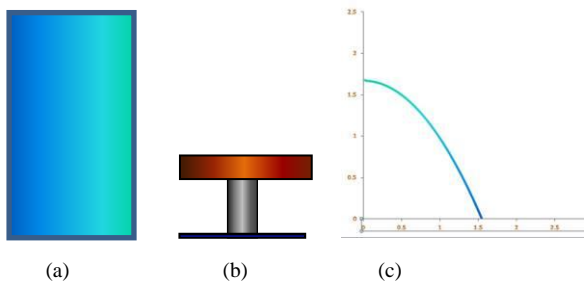
Figure 3. Basic animation: (a) cubicle, (b) buffer, and (c) graph.

The program uses two main buttons: RESET that functions to clear the process; and RUN that functions to run the program. The button RUN is driven by the following VBA coding [10]:

```
Private Sub CommandButton1_Click()
y = h2
tfall = Sqr(2 * h2 / g)
For i = 1 To 5000
v = Sqr(2 * g * (h1 - h2) / (1 - A2 ^ 2 / A1 ^ 2))
dhperdt = (A2 / A1) * v
h1 = h1 - 0.01 * dhperdt
y = y + (h / 2) * (vy(h * i) + vy(h * (i - 1)))

ScrollBar1.Value = h1 / 2.5 * 308
Sheet1.Shapes("Rectangle 1").Height = h1 / 2.5 *
308 (Coding for decreasing water level)

Cells(18, 2) = v
Cells(28 + i, 6) = y (Coding for discharge water)

If tfall - Cells(28 + i, 4).Value > 0 Then
Activate
Else
Exit For
End If
Next i
```

3. Results and Discussion

The simulation of Torricelli effluent flow used the following physical quantities: (1) water level (h_1) = 0 - 2.50 m; (2) buffer height (h_P) = 0 - 1.84 m; (3) lower limit of container area (A_1) = 0.01 m^2; (4) leakage area (A_2) = 0.0025 m^2; (5) time iteration/step size (Δt) = 0.01 s; and (6) acceleration due to gravity (g) = 9,8 m/s^2. By considering the standard, the program will run only if inputs fill criteria of $h_1 > h_2 > h_P$. The input of buffer height (h_P) functions to show that it does not contribute to the time (t) and the horizontal distance (x) of discharge water on the ground. The display of the program, which is a graphical interface unit, is shown in Fig. 4.

The source code of the program presents four outputs as shown in Fig. 4. The OUTPUT 1 consists of the decrease of water level (h_1), discharge water speed (v_2), and number of iterations. The OUTPUT 2 is data table that presents simulation display in the OUTPUT 3. The horizontal distance (x) in the OUTPUT 2 is influenced by the discharge water speed (v_2) in the OUTPUT 1. The source code for horizontal distance (x) is written in the spreadsheet, not in VBA. Comparison between x obtained by the numerical and analytical methods is presented in the OUTPUT 4.

In the following, we will discuss the comparison between the numerical and analytical methods presented in the

OUTPUT 4. We begin with first input: h_1= 2.50 m, h_2= 1.65 m, and A_1= 10 m^2 ($A_1 > A_2$). It yields the OUTPUT 2 as given in Table 1.
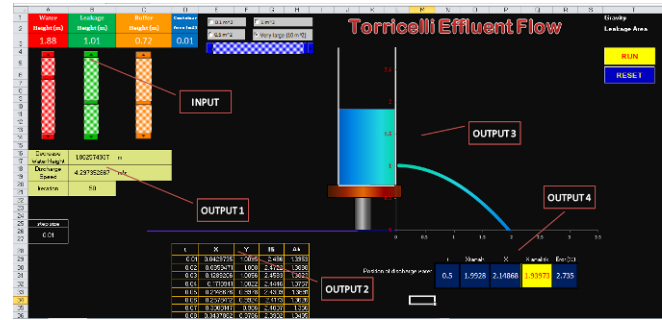

Figure 4. The display of the program.

Table 1. The OUTPUT 2 for the first input.

t	x	y	h_1	Δh
0.01	0.04080	1.64951	2.49999	0.0010
0.02	0.08160	1.64804	2.49998	0.0010
0.03	0.12240	1.64559	2.49996	0.0010
…	…	…	…	…
0.58	2.36654	0.00164	2.49940	0.0010

It is also shown in Table 1 that when y goes to 0, no significant change of h_1. The values of t and x compared to those obtained by the analytical method (Eqs. (5) and (6)) have no significant differences as shown in Table 2.

Table 2. Comparison between the OUTPUT 2 obtained by the numerical and analytical methods for the first input.

Comparison	Numerical	Analytical	Error (%)
t	0.58000	0.58028	0.049
x	2.36654	2.36854	0.084

Then, we put the second input: h_1= 2.50 m, h_2= 1.65 m, and A_1= 0.01 m^2 (A_1 is slightly larger than A_2). It yields the OUTPUT 2 as shown in Table 3.

Table 3. The OUTPUT 2 for the second input.

t	x	y	h_1	h_2
0.01	0.02720	1.64951	2.48946	1.05388
0.02	0.05440	1.64804	2.47898	1.04732
0.03	0.08160	1.64559	2.46858	1.04077
…	…	…	…	…
0.58	1.57772	0.00164	1.99713	0.68005

Tabel 3 shows that when water discharge reaches ground (y = 0), the water level has decreased until 1.99713 m height. Its decreasing speed is getting smaller. These results show that Torricelli principle has revisited when A_1 is slightly larger than A_2. The discharge speed of water (v_2) decreases gradually and reaches zero when $h_1 = h_2$. Consequently, x also decreases.

If we see the discharge position reaching ground for the second input, numerical result has no significant different compared to analytical one as presented in Table 4.

Table 4. Comparison between the OUTPUT 2 obtained by the numerical and analytical methods for the second input.

Comparison	Numeric	Analytic	Error (%)
t	0.58000	0.58028	0.048
x	1.57772	1.56324	0.979

It is proved that the Euler numerical method is accurate to simulate the phenomena of Torricelli effluent flow. The error is small (<2%) compared to the analytical calculation.
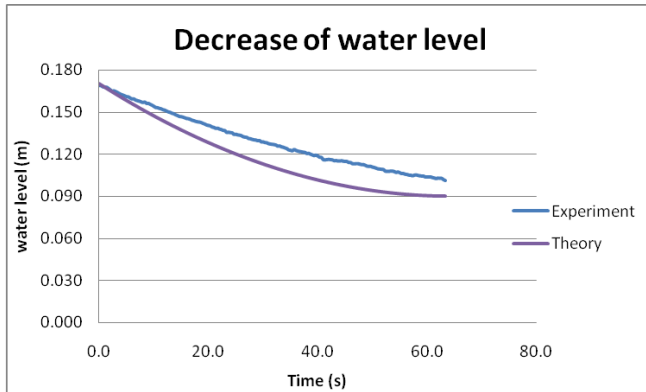

Figure 5. The decrease of water level as a function of time.

The simulation above presents a description of Torricelli effluent flow in an idealized condition, thus some physical aspects does not include in calculation. If we conduct a real experiment, we will find some differences compared to the model presented here.

For instance, we set a trial by using a container whose diameter of 10 cm, leakage diameter of 4 mm, water level of 17 cm, and leakage height of 9 cm. So we have quantity $A_1 =$ 63.6 cm^2, $A_2 = 0.126$ cm^2, $h_1 = 17$ cm, and $h_2 = 9$ cm. We compare the decrease of water level (h_1) and the distance of discharge water (x) between experiment and simulation. The difference is presented in Figs. 5 and 6, respectively.
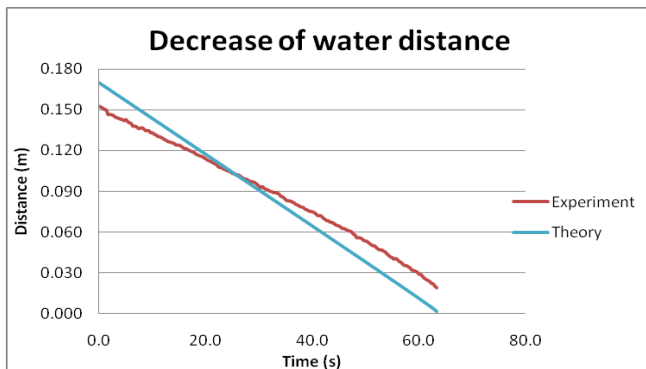

Figure 6. The decrease of water distance as a function of time.

Theoretically, the water level will decrease gradually as purple line in Fig. 5. The water decrease will stop, when the height reaches the leakage height (9 cm), at 63.3 second. However experimentally, it gets longer and never reaches the height leakage. This is because when the speed of discharge water (v_2) approaches zero, the water out from container will perks through container's wall. Low discharge speed will cause surface tension of water in the leakage area

so relatively large that preclude water squirt out. That is why the experiment graph (blue line) does not stop on the leakage height; we cannot measure how long it will be.

The same difference also occurs on distance of water (x) falling out from the container (Fig. 6). Both theory and experiment results show that the water distance decreases linearly. However, the longest distance reached by water is 16.9 cm in theory, while it is only 15.2 cm in experiment. This is because the existence of air resistance influences of water discharge. The air resistance causes the water distance not as long as the result from the simulation [10].

If we manipulate the cross sectional area of container, either larger or smaller, we will obtain similar results as presented above. Here we find that air resistance, adhesion force, and surface tension of water influence the experiment results; and those do not include in simulation calculation.

5. Conclusion

Simulation of Torricelli effluent flow can be created by utilizing VBA codes and formulas on spreadsheet in Microsoft Excel. The basic of animation is operating shapes and graph creatively and generated by numerical calculation. Result of numerical method has no significant error (< 2%) compared to that of analytical method.

However, an error is found in describing the effluent distance of discharge water (x) compared with experimental results since the simulation presents idealized condition, i.e. ignoring the existence of air resistance, adhesion force, and surface tension that influence the experimental result.

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