Research and Application of An improved Harmony Search Algorithm

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Abstract: An improved Harmony Search Algorithm applied to the Route Optimization has been proposed in this paper. At the same time, this paper compared this improved algorithm with the traditional Harmony Search Algorithm and the Genetic Algorithm through experiments and simulation. The experimental results show that the improved Harmony Search Algorithm has better performance in both convergence and stability in solving Route Optimization problem.

Key words: improved; Harmony Search Algorithm; Route Optimization;

1. Introduction

The route optimization belongs to the Non-deterministic polynomial hard problem in the field of combinatorial optimization. It means that the maximum or the minimum of the objective function is gained with considering the condition of the given constraints. In other words, the most optimal group or order or screening of the discrete problem is gained through the research on mathematical method[1]. The traditional Genetic Algorithm(GA), the Ant Colony Algorithm (ACA) and the Simulated Annealing Algorithm(SAA) have been applied to combinatorial optimization. Meanwhile, many satisfactory results have been obtained but falling into local optimum easily[2].

The Harmony Search (HS) Algorithm as a kind of heuristic global search algorithm has been used to the optimization design of structure and traffic path planning and the analysis of slope stability successfully and so on[3]. The HS has showed better performance than the GA, the ACA and the SAA in dealing with discrete and multi-dimensional extremum optimization problems. In spite of this, there are some defects of this algorithm. The convergence of the traditional HSA is slow at the early stage of the search, and it needs much more iterations. In addition, the values of the three main parameters (HMCR, PAR, bw) will affect the search results directly. In order to avoid the algorithm falling into local optimum as much as possible with group of species diversity and accelerate convergence speed, an improved Harmony Search(IHS) Algorithm with dynamic parameters has been put forward in this paper. Experimental results show that the improved algorithm has better effectiveness in dealing with such route optimization problems.

2. The traditional Harmony Search Algorithm

The Harmony Search Algorithm was proposed by South Korean Geem Z W in 2001 as a new intelligent algorithm[4]. There are three parameters: HMCR(Harmony memory considering rate), PAR(Pitch adjusting rate), bw(band width). The basic steps of this algorithm are as follows:

Step 1: Set the basic parameters.
①The number of variables: N; ②The value range of each variable; ③The total number of iterations: Tmax; ④The size of the Harmony memory:HMS; ⑤ The harmony memory considering rate:HMCR; ⑥ The pitch adjusting rate: PAR; ⑦ The band width : bw

Step 2: Initialize the harmony memory.
It means that generate HMS solution vectors randomly according to the value range of each variable.

$$HM = \begin{bmatrix}
    x_1 \quad f(x_1) \\
    x_1 \quad f(x_1) \\
    \vdots \quad \vdots \\
    x_i \quad f(x_i) \\
    x_i \quad f(x_i) \\
\end{bmatrix} = \begin{bmatrix}
    x_1 \quad x_1 \quad \ldots \quad x_1 \quad f(x_1) \\
    x_2 \quad x_2 \quad \ldots \quad x_2 \quad f(x_2) \\
    \vdots \quad \vdots \\
    x_N \quad x_N \quad \ldots \quad x_N \quad f(x_N) \\
\end{bmatrix}$$

Step 3: Generate a new solution vector
Generate a new harmony vector, $x' = (x'_1, x'_2, \ldots, x'_N)$, each tone $x'_i \ (i=1,2,\ldots,N)$ is gained through the following three kinds of mechanism: ① Taken from the harmony memory; ② Tone fine-tuning; ③ Selected randomly; As shown following:

$$x_i = \begin{cases}
    x_i \in (x_i^1, x_i^2, \ldots, x_i^{HMS}) & \text{if rand}<\text{HMCR} \\
    x_i \in X_i, \text{otherwise}; & i=1,2,\ldots,N
\end{cases} \quad (1)$$

In the above formula, the $X_i$ denotes the value range of the $i$th variable, and the rand is the uniformly distributed random number in the interval of $[0,1]$. The $x'_i \ (i=1,2,\ldots,N)$ taken from the harmony memory is adjusted by tone fine-tuning according to the following formula:

$$x'_i = \begin{cases}
    x_i + \text{rand1} \cdot \text{bw} & \text{if rand1}<\text{PAR (continuous)} \quad (2) \\
    x_i (k + m), m \in \{-1,1\} & \text{if rand1}<\text{PAR (discrete)} \\
    x_i, \text{otherwise}; & \text{if rand1}<\text{PAR (discrete)}
\end{cases}$$

In the above equation, rand1 is the uniformly distributed random number in the interval of $[0,1]$. PAR is the pitch adjusting rate, and the bw is band width of the tone fine-tuning.

Step 4: Update the harmony memory
The new solution vector is assessed according to a certain formula. If the new harmony vector is better than the worst harmony vector in the harmony memory, the worst harmony will be replaced by the new harmony vector in harmony memory. For example, to seek the optimal path, then:

$$f(x') < f(x^{\text{worst}}) = \max_{j=1,2,\ldots,HMS} f(x^j)$$

then $x^{\text{worst}} = x'$

Step 5: Determine that the termination conditions are reach or not. If not, the Step 3 and Step 4 will be repeated. Otherwise, the optimal solution is outputted.

3. The improved Harmony Search Algorithm
The HMCR of harmony search algorithm can improve the ability of global search to find the optimal solution when solving the discrete problem, but it can’t complete the local search to details of HM. The parameters called PAR and bw can improve the ability of local search to make the solution more close to the optimal solution from the perspective of local to enhance the local search ability of the algorithm[5]. In the traditional Harmony Search Algorithm, the value of PAR is constant which will greatly affect the search performance of the algorithm[6]. The PAR is smaller, the ability of local search is stronger, but it is easy to fall into local optimal. The PAR is bigger, the search range of the algorithm is larger, but the convergence speed is slower at the same time. Inspired by literature[5,6], in order to search in all effective space as much as possible and improve the convergence speed, dynamic PAR is adopted to gain the optimal path according to the following formula in this paper:

$$\text{PAR} = \begin{cases}
    \text{PAR min} + \frac{\text{PAR max} - \text{PAR min}}{T_{\text{max}}/2} \cdot t & \text{when } t \leq T_{\text{max}}/2 \\
    \text{PAR max} - \frac{\text{PAR min}}{T_{\text{max}}} \cdot t & \text{when } T_{\text{max}}/2 < t \leq T_{\text{max}} \quad (4)
\end{cases}$$

In the above equation, the PARmax is the maximum of the dynamic PAR, and the PARmin is the minimum of the dynamic PAR, $t$ is the current number of iterations, and the Tmax is the total number of iterations.

4. The model simulation
4.1 Problem description
Start from a certain place, chose reasonable route to visit $N$ customers who are in the different positions, and then return the original position. Designing the optimal path means determine the access sequence of $N$ nodes with
considering the constraint of maximum mileage of vehicles. Assuming that any two nodes are interlinked and the distance of them is known. This problem mainly involves several parameters as following:

\[ N \]: The number of customers;
\[ D \]: The maximum mileage of vehicles;
\[ s \]: The visit order of a customer in the access path, when \( s=0 \), means that it is origin.
\[ d(s-1,s) \]: The directly distance between the \( s-1 \)th customer and the \( s \)th customer, \( s=1,2 \cdots N \).

Then, the constraint of maximum mileage of vehicles can be described as follow:

\[
\sum_{s=1}^{N} d(s-1,s) + d(N,0) \leq D \quad (5)
\]

Establishing the mathematical model with the shortest path as the optimization goal can be expressed:

\[
L = \min \left( \sum_{s=1}^{N} d(s-1,s) + d(N,0) \right) \quad (6)
\]

4.2 Numerical Experiments

There are 10 customers to visit, and the maximum mileage of vehicles is 80km. The positions of the 10 customers have been given randomly, and the coordinates are in the following set \{\((x,y)\)|(6,6),(11,11),(9,4),(14,9), (9,14),(13,12),(12,3),(15,6),(5,11),(8,7),(10,10)\}. The coordinate of the origin is \((10,10)\). The distribution is showed in the following figure 1 (The serial numbers of customers is 1 to 10, and the serial number of the origin is 0):

According to the coordinates, the distances of customers and origin are calculated as shown in following table 1:

<table>
<thead>
<tr>
<th>num</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
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<td>6.000</td>
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<td>0.000</td>
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<td>11.401</td>
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<td>0.000</td>
<td>9.055</td>
<td>6.324</td>
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<td>0.000</td>
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<td>6.324</td>
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<table>
<thead>
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<th>4</th>
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<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
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<tbody>
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<td>Value</td>
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<td>44.3</td>
<td>47.9</td>
<td>45.2</td>
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<td>44.3</td>
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</tbody>
</table>

4.2.1 The results of traditional Harmony Search Algorithm

The parameters were setted as following: HMS=50, HMCR=0.75, PAR=0.3, Tmax=1000. The program was written and run in Matlab. After running 10 times, the results were recorded as shown in table 2, and the path of the optimal solution(0\(\rightarrow\)9\(\rightarrow\)5\(\rightarrow\)6\(\rightarrow\)2\(\rightarrow\)4\(\rightarrow\)8\(\rightarrow\)7\(\rightarrow\)3\(\rightarrow\)1\(\rightarrow\)10\(\rightarrow\)0) and it’s iterative process are showed in figure 2 and figure 3:

<table>
<thead>
<tr>
<th>Time</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
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<th>9</th>
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</tr>
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<tbody>
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<td>Value</td>
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<td>47.9</td>
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<td>40.4</td>
<td>42.7</td>
<td>43.4</td>
<td>45.1</td>
<td>44.3</td>
</tr>
<tr>
<td>Average</td>
<td>44.5</td>
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</tbody>
</table>
4.2.2 The results of improved Harmony Search Algorithm

In this algorithm, the parameters were set as following: HMS=50, HMCR=0.75, PARmax=0.9, PARmin=0.0001, Tmax=1000. The program was written and run in Matlab. After running 10 times, the results were recorded as shown in Table 3, and the path of the optimal solution (0 2 6 4 8 7 3 1 10 9 5 0) and its iterative process are showed in Figure 4 and Figure 5:

Table 3 The results of improved harmony search algorithm

<table>
<thead>
<tr>
<th>Time</th>
<th>Value</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>39.6</td>
<td>40.5</td>
<td>43.7</td>
<td>40.5</td>
<td>40.0</td>
<td>37.3</td>
<td>41.3</td>
<td>42.9</td>
<td>43.1</td>
<td>38.9</td>
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<tr>
<td></td>
<td>Average</td>
<td>40.8</td>
<td></td>
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</tbody>
</table>

4.2.3 The results of the Genetic Algorithm

The parameter of the Genetic algorithm are as follows: popsize (the population size) = 50, PC (the probability of crossover) = 0.95, PM (the probability of mutation) = 0.001, the number of iterations Tmax = 1000. In the same way, the program was written and run in Matlab. After running 10 times, the results were recorded as shown in Table 4, and one of the optimal solution (0 5 2 6 4 8 7 3 9 1 10 0) and its iterative process are showed in Figure 6 and Figure 7:

Table 4 The results of Genetic algorithm

<table>
<thead>
<tr>
<th>Time</th>
<th>Value</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<th>10</th>
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</thead>
<tbody>
<tr>
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<td></td>
<td>45.1</td>
<td>49.0</td>
<td>42.7</td>
<td>48.2</td>
<td>45.3</td>
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<td>46.6</td>
<td>50.4</td>
<td>43.1</td>
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<tr>
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<td>Average</td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>
4.3 Comparison and Analysis

In Intel G645, 2.9 GHZ, 1 GB of memory, using the three algorithms respectively, with the same number of iterations, the optimal solutions or the near optimal solutions (the length of path less than 48) were recorded, the and the are shown in table 5:

Table 5 The comparison of the three algorithms

<table>
<thead>
<tr>
<th>Item</th>
<th>Iterations</th>
<th>Run time</th>
<th>Excellent rate</th>
<th>Average Run time</th>
<th>Excellent rate</th>
<th>Average Run time</th>
<th>Excellent rate</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>HS</td>
<td>1000</td>
<td>&lt;2</td>
<td>53%</td>
<td>46.06</td>
<td>&lt;4</td>
<td>81%</td>
<td>44.31</td>
<td>&lt;5</td>
</tr>
<tr>
<td>IHS</td>
<td>2000</td>
<td>≈2</td>
<td>67%</td>
<td>42.21</td>
<td>≈3</td>
<td>89%</td>
<td>39.47</td>
<td>≈5</td>
</tr>
<tr>
<td>GA</td>
<td>5000</td>
<td>≈8</td>
<td>49%</td>
<td>46.98</td>
<td>≈15</td>
<td>61%</td>
<td>43.39</td>
<td>≈22</td>
</tr>
</tbody>
</table>

5. Conclusion and Discussion

From the experimental results, the improved harmony search algorithm showed better performance compared to the GA and the HS in dealing with discrete optimization problems, and it has better stability and approximation than the other two kinds of algorithm.

The values of the parameters in harmony search algorithm will directly affect the search results. This algorithm has the same advantage whether or not to solve continuous optimization problems. It’s worth to research further.

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References


