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FROM FUZZY CLUSTERING TO A FUZZY RULE-BASED FAULT CLASSIFICATION MODEL

Enrico ZIO*, Piero BARALDI, Irina Crenguta POPESCU

Energy Department, Polytechnic of Milan
Via Ponzio 34/3, 20133 Milan, Italy

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The applicability in practice of a diagnostic tool is strongly related to the physical transparency of the underlying models, for the interpretation of the relationships between the involved variables and for direct model inspection and validation. In this work, a methodology is developed for transforming an opaque, fuzzy clustering-based classification model into a fuzzy logic model based on transparent linguistic rules. These are obtained by cluster projection with appropriate coverage and distinguishability constraints onto the fuzzy input partitioning interface. The methodological approach is applied to a diagnostic task concerning the classification of simulated faults in the feedwater system of a nuclear Boiling Water Reactor.

Keywords: Fault Diagnosis, Fault Classification, Fuzzy Clustering, Cluster Projection, Transparent Knowledge Base, Nuclear Transients, Boiling Water Reactor.

1. Introduction

Fault diagnosis, i.e. the detection and classification of anomalies and faults, is a particularly important task in hazardous components such as those employed in the nuclear technology, for its implications on the safety and efficiency of operation. Conceptually, the basis for performing these tasks is that different faults initiate different patterns of evolution of the interested variables, measured by properly located sensors. The diagnostic problem then becomes one of pattern classification, i.e. association of the different patterns of evolution to the different classes of system faults.

In this regard, extensive research has been carried out with respect to the investigation of fuzzy clustering techniques for classification [1,2,3,4]. These techniques have proven very effective but often remain "black boxes" as to the interpretation of the physical relationships underpinning the pattern classification [5,6,7,8,9].

In this paper, a fuzzy logic (FL) model of pattern classification for fault identification is developed. One of the main strengths offered by the proposed modeling approach is that the underlying Knowledge Base (KB) is in a rule format, easy to maintain, update, examine and understand. In general, to achieve this, one must first perform the fuzzy partitioning of the input space by an adequate choice of representative fuzzy sets (FSs) and then establish the fuzzy rules underpinning the relationships of the involved variables. In the case of FL models for pattern classification, automatic partitioning and rule construction processes are often adopted, on the basis of available pre-classified, labeled data [10,11].

In the approach proposed in the present paper, the first stage of the development of the rule-based classification model amounts to finding clusters corresponding to different types of fault. This is done by processing pre-classified, labelled 'training' data by means of a supervised evolutionary possibilistic clustering scheme [1]. Then, the fuzzy rule-

*enrico.zio@polimi.it

based model is obtained by optimally partitioning the range of each input (the so-called universe of discourse, UOD) to reflect the previously obtained clusters, with each fuzzy cluster inducing a fuzzy classification rule. More specifically, the FSs making up the rule corresponding to a given fuzzy cluster are obtained by projecting such cluster onto the individual one-dimensional coordinate axes of the involved variables [10,12]. In synthesis, the idea behind the approach is to first fuzzy-cluster the data and then derive the FSs and linguistic rules underpinning the classification model from proper projection of the fuzzy clusters found.

Section 2 sets the terminology and framework of fuzzy reasoning [13]. Section 3 sketches the possibilistic procedure for creating a cluster from available pre-classified data [1]. Section 4 illustrates the membership functions (MFs) properties and semantic constraints which are introduced to achieve a transparent model and the pruning process performed to "clean" the model. Section 5 reports the application of the approach to the classification of simulated faults in the feedwater system of a nuclear Boiling Water Reactor (BWR). A discussion concerning the advantages and limitations of the proposed approach is provided in the last Section.

2. Fuzzy reasoning

In this Section, a short description is provided with regards to how fuzzy reasoning proceeds [13,14]. The content is limited to the general basic concepts, the terminology and the notation necessary for completeness and self-consistency of the paper. Given the aim of providing a transparent fuzzy rule-based classification model, the Mamdani Fuzzy Inference scheme is chosen over the Takagi–Sugeno one due to the flexibility offered by expressing linguistic outputs.

The two key elements of fuzzy reasoning are the Fuzzy Rule Base (FRB) (or Knowledge Base, KB) and the fuzzy inference engine. The former consists of a set of R *if-then* rules. The generic j-th fuzzy rule, $j = 1,2,\ldots,R$, is made up of a number of antecedent and consequent linguistic statements, suitably related by fuzzy connections:

$$\begin{aligned} R_j: \quad & \textit{if } (x_1 \textit{ is } X_{1j}) \textit{ and } \ldots \textit{ and } (x_n \textit{ is } X_{nj}) \\ & \textit{then } (y_1 \textit{ is } Y_{1j}) \textit{ and } \ldots \textit{ and } (y_m \textit{ is } Y_{mj}) \end{aligned} \quad (1)$$

The linguistic variables x_p, $p = 1,\ldots,n$, are the antecedents, represented in terms of the FSs X_{pj} of the UOD U_{x_p}, with MFs $\mu_{X_{pj}}(x_p)$. The linguistic variables y_q, $q = 1,\ldots,m$, are the consequents, represented by the FSs Y_{qj} of the UOD U_{y_q}, with MFs $\mu_{Y_{qj}}(y_q)$. The connective operator *and* links two fuzzy concepts and it is generally implemented by means of a *t-norm*, typically the minimum operator or the algebraic product. The rules of the FRB are joined by the connective *else* and are generally implemented by means of an *s-norm*, typically the maximum operator [13].

The fuzzy inference engine receives the (linguistic) variables which constitute the Fact, viz.,

$$\textit{Fact: } x_1 \textit{ is } X'_1 \textit{ and } \ldots \textit{ and } x_n \textit{ is } X'_n$$

where X'_p is a FS on the UOD U_{x_p} of the p-th linguistic input variable x_p, and compares it with the antecedents of the rules in the FRB to arrive at the Conclusion, viz.,

$$\textit{Conclusion: } y_1 \textit{ is } Y'_1 \textit{ and } \ldots \textit{ and } y_m \textit{ is } Y'_m$$

where Y'_q is a FS on the UOD U_{y_q} of the q-th output variable y_q.

In the case of fault classification, the fuzzy inference engine i) receives as Fact the n values of the monitored variables, possibly fuzzyfied to account for measurement imprecision, ii) computes the 'strength' with which each of the R rules in the FRB is activated by the incoming input Fact, i.e. the degree to which the rule matches the Fact, and iii) properly combines the consequents of the rules, weighed by their respective strengths, to determine the output memberships to the different fault classes [13,14].

3. A supervised evolutionary possibilistic clustering classifier

Fuzzy clustering algorithms have been widely studied and applied in various substantive areas such as taxonomy, medicine, geology, business, engineering, image processing and others. A general classification of these algorithms is offered in [2] in terms of three categories: fuzzy clustering based on fuzzy relations, fuzzy clustering based on the minimization of an objective function and the class of non-parametric classifiers based on the fuzzy generalized k-nearest neighbors rule. The interested reader is referred to [2] for a detailed discussion of the three categories and an extensive literature review of works in the field.

The clustering scheme adopted in the present work belongs to the second category. A set of N, n-dimensional patterns $\vec{x}_k$, $k = 1,\ldots,N$, pre-classified to c a priori known classes (in our case, corresponding to the c categories of faults whose recognition is of interest), is assumed available. The information regarding this known, physical class-membership partition $\Gamma^t \equiv (\Gamma_1^t, \Gamma_2^t, \ldots, \Gamma_c^t)$ is used to supervise an evolutionary algorithm for finding c optimal Mahalanobis metrics which define c geometric clusters as close as possible to the a priori known physical classes [1,5,15,16]. The Mahalanobis metrics are defined by the matrices $\underline{\underline{M_i}}$, $i = 1,\ldots,c$, whose elements are identified by the supervised evolutionary algorithm so as to minimize the distances $s_{ik} = (\vec{x}_k^i - \vec{v}_i^*)^{\mathrm{T}} \underline{\underline{M_i}} (\vec{x}_k^i - \vec{v}_i^*)$ between the patterns $\vec{x}_k^i$ belonging to class i and the class prototype, i.e. the cluster center $\vec{v}_i^*$.

More specifically, the supervised training procedure for the optimization of the c Mahalanobis metrics is carried out via an evolutionary optimization method previously presented in the literature within a supervised fuzzy clustering scheme [16] and further extended to diagnostic applications by both fuzzy [5] and possibilistic clustering schemes [1].

The target of the supervised optimization is the minimization of the distance $D(\Gamma^t, \Gamma)$ between the a priori known physical class partition $\Gamma^t \equiv (\Gamma_1^t, \Gamma_2^t, \ldots, \Gamma_c^t)$ and the obtained geometric cluster partition $\Gamma \equiv (\Gamma_1, \Gamma_2, \ldots, \Gamma_c)$:

$$D(\Gamma^t, \Gamma) = \sum_{i=1}^{c} \frac{D(\Gamma_i^t, \Gamma_i)}{c} = \sum_{i=1}^{c} \sum_{k=1}^{N} \frac{\left| \mu_i^t(\vec{x}_k) - \mu_i(\vec{x}_k) \right|}{N \cdot c} \quad (2)$$

where $0 \leqslant \mu_i^t(\vec{x}_k) \leqslant 1$ is the a priori known membership of the k-th pattern to the i-th physical class and $0 \leqslant \mu_i(\vec{x}_k) \leqslant 1$ is the membership to the corresponding geometric cluster in the space of the monitored variables.

A sketch of the procedure is provided in Appendix A, but the interested reader should refer to [1] for a more thorough mathematical treatment.

To overcome some known limitations associated to fuzzy clustering [17], the framework of possibility theory is adopted for the patterns membership to the different clusters [17,18,19]. In this interpretation, the MF $\mu_i(\vec{x}_k)$ represents the degree of similarity of the generic incoming pattern $\vec{x}_k$ with the prototypical member $\vec{v}_i^*$ of cluster i [1]. If the classes represented by the clusters are thought of as a set of FSs defined over the UOD, then there should be no constraint on the sum of the memberships, as there is instead in fuzzy clustering [2,20]. The only constraint is that the membership values do represent degrees of similarity, or possibility, i.e. they must lie in $[0,1]$ [21]:

$$0 \leqslant \mu_i(\vec{x}_k) \leqslant 1, \quad i = 1,\ldots,c, \quad k = 1,\ldots,N \quad (3)$$

$$\max_i \mu_i(\vec{x}_k) > 0, \quad k = 1,\ldots,N \quad (4)$$

where constraint (4) simply ensures that the set of fuzzy clusters covers the entire UOD.

A possibilistic partition derived under these constraints defines a set of distinct, uncoupled possibilistic distributions (and the corresponding fuzzy subsets) over the UOD [21].

Thus, at convergence the supervised evolutionary possibilistic clustering algorithm provides the c metrics $\underline{\underline{M_i}}$, the c possibilistic cluster centers $\vec{v}_i^*$ and the $c \cdot N$ possibilistic membership values $\mu_i(\vec{x}_k)$ of the patterns $\vec{x}_k$, $k = 1,\ldots,N$, to the clusters $i = 1,\ldots,c$, optimal with respect to the classification task.

The c identified clusters are FSs in the n-dimensional space of the monitored variables, each FS being associated to a different class. These are to be translated into the antecedent part of c rules of the kind [22]:

$$\textit{if } (x_1,\ldots,x_n) \textit{ is } X_i \quad i = 1,\ldots,c \quad (5)$$

where $(x_1,\ldots,x_n)$ is the multi-dimensional linguistic variable describing the n variables monitored for performing the classification and X_i is the FS associated to the i-th multi-dimensional cluster Γ_i, $i = 1, \ldots, c$.

Once the antecedent part of the fuzzy *if-then* rule associated to class i has been constructed, the corresponding consequent part must be set up. In this work, this is accomplished consistently with a possibilistic approach by providing the degree of membership of a pattern to each class [1,23]. To achieve this, a discrete output variable y_q is associated to each class, $q = 1, \ldots, c$. Each output variable is described by two linguistic labels $\{YES, NO\}$, with corresponding singletons FSs Y_q^{NO} and Y_q^{YES} (Fig. 1). Then, in the consequent part of the fuzzy rule representing the i-th class, all the output variables y_q, $q \neq i$, appear labelled with the FS Y_q^{NO}, except the i-th output variable y_i, representing the i-th class, which is labelled with Y_q^{YES}:

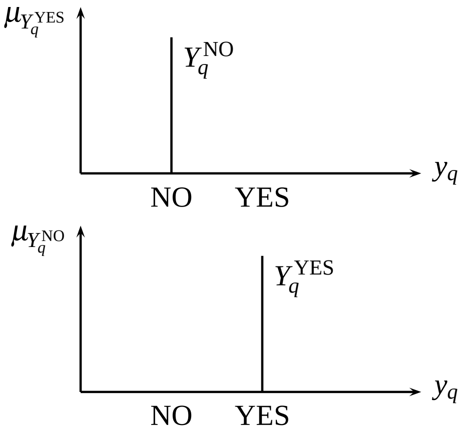

Fig. 1. The two singletons FSs Y_q^{NO} and Y_q^{YES} associated to the q-th output variable

$$\begin{aligned}
&\textit{if } (x_1, \ldots, x_n) \textit{ is } X_i \textit{ then } (y_1 \textit{ is } Y_1^{NO}) \\
&\textit{and } (y_2 \textit{ is } Y_2^{NO}) \textit{ and} \ldots \textit{ and } (y_i \textit{ is } Y_i^{YES}) \\
&\ldots \textit{ and } (y_c \textit{ is } Y_c^{NO}) \qquad (6)
\end{aligned}$$

Note that this form of the consequent part of the rule also allows an easier handling of multiple faults [23].

The c fuzzy logic rules derived from the identified clusters constitute the FRB of the classification model. On the basis of these rules, the possibilistic classification of the generic pattern $\overrightarrow{x}'$ of the values of the monitored variables is performed by a Mamdani Fuzzy Inference Engine leading to the fuzzy conclusion y_1 *is* Y_1' *and* $\ldots$ *and* y_c *is* Y_c', where Y_q', $q = 1, \ldots, c$, is the discrete output FS of the variable y_q, constituted by the two values of membership or non-membership to class q. Figure 2 shows an example of output FSs for a given input pattern to be classified into one of three possible classes: the pattern most possibly belongs to class 3 (with degree 0.95) but it could possibly belong also to class 1 (with degree 0.7) and 2 (with degree 0.3).

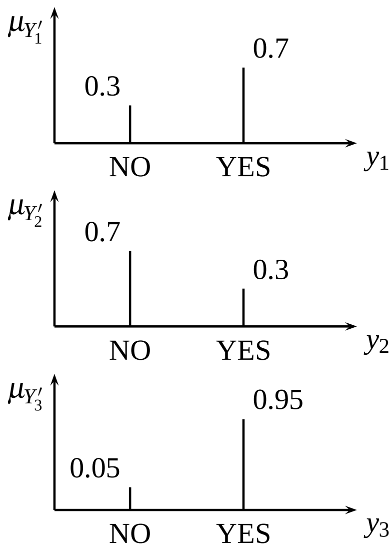

Fig. 2. Example of a possibilistic classification into 3 classes.

As a final remark, we note that the supervised evolutionary possibilistic clustering algorithm is run a priori, off-line to obtain the partition of the monitored variables space into clusters from which the fuzzy inference model is derived. Once this is achieved, the diagnostic model can be set on-line for performing the fault classification in real time.

4. Obtaining a transparent fuzzy rule-based model

The classification model derived with the approach illustrated in Section 3 is really still a ‘black box’, due to the difficulties of describing and interpreting in terms of rules antecedents the multi-dimensional FSs representing the identified clusters. In the present Section, a method is propounded to extract a transparent, rule-format KB from the previously obtained multi-dimensional FSs. To ease the presentation of the procedure, reference will be made to the two-dimensional artificial classification problem of Fig. 3. The relative data comprise 4 classes of patterns obtained by random sampling from different Gaussian distributions. Each class can be considered to correspond to a different type of system fault to be classified.

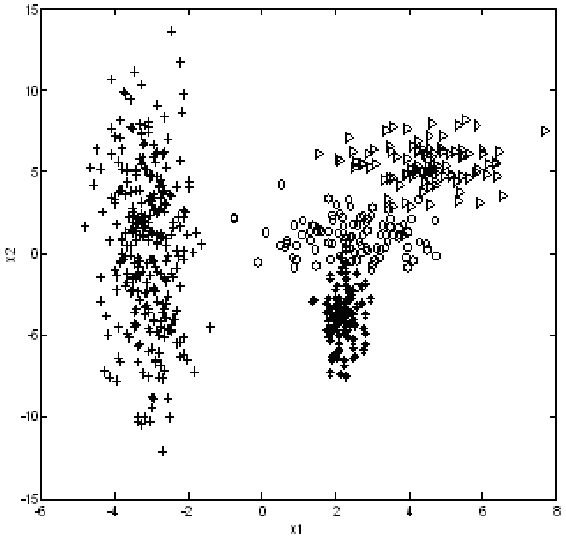

Fig. 3. Two-dimensional artificial case study comprised of 4 fault classes: '+' represents patterns of class Γ_1^t,'$*$' of class Γ_2^t, '$\circ$' of class Γ_3^t and '$\triangleright$' of class Γ_4^t

To obtain a transparent KB, the following two steps are performed:
- projection of the n-dimensional fuzzy clusters into n mono-dimensional FSs (Section 4.1);
- enforcement of appropriate semantic constraints on the obtained FSs (Section 4.2).

4.1. Projection of the n-dimensional fuzzy clusters into n mono-dimensional FSs

As a result of the clustering classification algorithm presented in Section 3, each n-dimensional training pattern $\vec{x}_k$, $k = 1,\ldots,N$, is possibilistically classified by its membership $\mu_i(\vec{x}_k)$ to each class $i = 1,\ldots,c$. The projection of the generic n-dimensional fuzzy cluster onto the n mono-dimensional UODs of the rules antecedent variables is performed as follows:

(i) the mono-dimensional MFs of the antecedents FSs are generated by pointwise projection of the membership value $\mu_i(\vec{x}_k)$ onto the antecedent variables UODs [10,12,22,23,24,25]. In the particular case of two patterns $\vec{x}_1$ and $\vec{x}_2$ of the same cluster i having the same projection onto the p-th input variable axis (i.e. $x_{1p} = x_{2p}$), the membership function on the p-th projection is taken equal to $\mu_{X_{pi}}(x_p) = \max\left(\mu_{X_{pi}}(x_{1p}), \mu_{X_{pi}}(x_{2p})\right)$, where X_{pi} is the FS resulting from the projection of cluster i onto the p-th input variable, $i = 1,\ldots,c$. This is in force of the compositional rule of inference [22]. For example, by projecting onto the ranges of the antecedents x_p, $p = 1, 2$, the cluster of Fig. 3, associated to class Γ_2^t, corresponding to the second Gaussian distribution (symbol '$*$' in Fig. 3), the FSs X_{12} and X_{22} shown in Fig. 4, are obtained.

(ii) the resulting non-convex MFs are transformed into convex MFs. To do this, starting from the smallest value of the antecedent x_p, only the membership of those values that have membership higher than the previous one are kept, until the maximum membership value is reached [12]. Then, the same procedure is applied starting from the highest value of the antecedent, until the maximum MF is reached. Fig. 5, shows the application of this procedure to the non-convex FSs of Fig. 4. The enveloping solid line represents the resulting convex MFs.

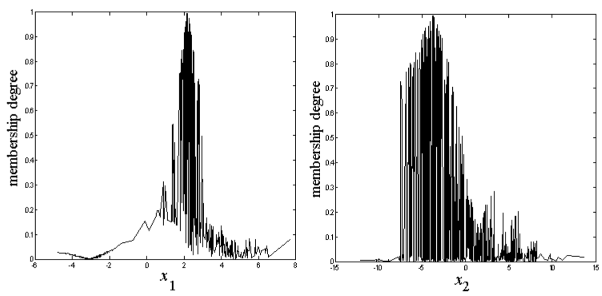

Fig. 4. Projections of the cluster of Fig. 3, corresponding to class Γ_2^t onto the UODs of the 2 antecedents x_1 and x_2 (abscissa: antecedent values; ordinate: membership value of the generic k-th pattern to the cluster projection, $k = 1,\ldots,N$

(iii) The convex FSs are approximated by linear interpolation to FSs with MFs of trapezoidal shape. The use of trapezoidal MFs allows a good identification of the projections zones. Other alternative representations, e.g. truncated Gaussian FSs, may work as well. Before performing the linear interpolation, all membership values under a threshold (chosen to be 0.1 in the present work) are rounded off to 0 and analogously all membership values above an upper threshold (chosen to be 0.9 in the present work) are rounded off to 1. Figure 6 shows the two FSs that represent the projection of the cluster of Fig. 3, corresponding to class Γ_2^t onto the 2 antecedents x_1 and x_2.

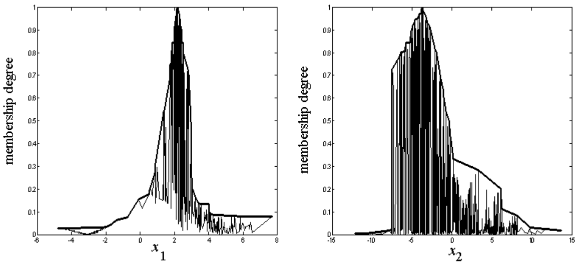

Fig. 5. Approximation of the projections of the cluster of Fig. 4, associated to the class Γ_2^t into convex non typical FSs

By so doing, the original premise of the i-th rule for cluster i, "*if* $(x_1,\ldots,x_n)$ *is* X_i", $i=1,\ldots,c$, of Eq. (4) is transformed into a new fuzzy proposition of the kind:

$$\textit{if } (x_1 \textit{ is } X_{1i}) \textit{ and } \ldots \textit{ and } (x_n \textit{ is } X_{ni}) \qquad (7)$$

Obviously, the method is approximate and some information on the cluster is inevitably lost in the projection, due to the decomposition error arising from projecting the multi-dimensional FS into its monodimensional constituents [22]. On the other hand, it enables expressing the KB in a form with a clear and interpretable semantic meaning.

4.2. *Enforcement of appropriate semantic constraints on the obtained FSs*

To achieve the physical interpretability of the model, semantic constraints are imposed to the FSs obtained in the previous step in an attempt to obtain an "optimal" interface [26,27,28,29]. This is sought through the procedure described below in Sections 4.2.1–4.2.4; note that at each step of the procedure, the corresponding FSs modification required to achieve an improved physical interpretability is actually carried out only if the classification performance on the training data is not significantly decreased.

4.2.1. *Pruning of FSs covering a large portion of the UOD*

Some FSs projections can turn out to be covering great portions of the variables UODs, adding little specific information to the model and overshadowing more focused FSs. Such sets can be removed from the antecedents of the rules [24]. For example, in Fig. 7, the projection of the clusters of Fig. 3, associated to the classes Γ_1^t, Γ_2^t, Γ_3^t, and Γ_1^t onto the antecedent variable x_2 results in FS X_{21} covering a wide portion of the UOD U_{x_2}.

The pruning of a FS modifies only the rules in which the FS appears as antecedent. The modification amounts to canceling from the antecedents the one corresponding to the eliminated FS.

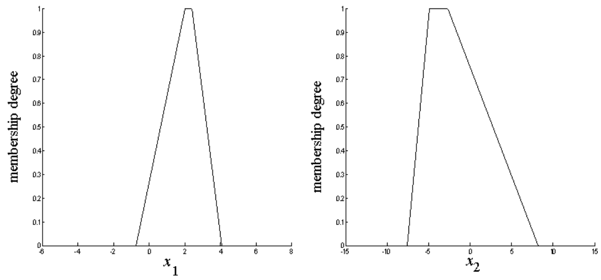

Fig. 6. The trapezoidal FSs corresponding to the cluster of Fig. 3, associated to the class Γ_2^t

The criterion for elimination of a FS X_{pi} widely covering the UOD U_{x_p} is [26]:

$$\beta_o l_{X_{pi}} \geqslant U_{x_p}; \;\; p=1,\ldots,n; \;\; i=1,\ldots,c \qquad (8)$$

where $l_{X_{pi}}$ is the width at half height of the i-th FS X_{pi} of variable x_p and $\beta_o \geqslant 1$ is the so-called overlap parameter which quantifies the portion of UOD U_{x_p} that can be covered by the support of the FS X_{pi}. The larger is the value of β_o, the more severe is the pruning criterion. A value $\beta_o = 1$ implies that a FS is eliminated only if its support covers the whole UOD; a value of $\beta_o > 1$ is such that the elimination criterion of Eq. (8) is satisfied by FSs with supports smaller than the entire UOD. A value of $\beta_o = 1.5$ was found to be optimal by trial-and-error for the application which follows in Section 5.

4.2.2. Addition of FS "nearly zero"

In practical diagnostic applications it is important also to be able to distinguish that the system is in a condition of no faults, as depicted by the absence of deviations of the measured signals from their normal behaviour. However, if the training data do not contain realizations from the class of no faults (stationary state), there is no cluster representing such situation and correspondingly no antecedents and no rules.

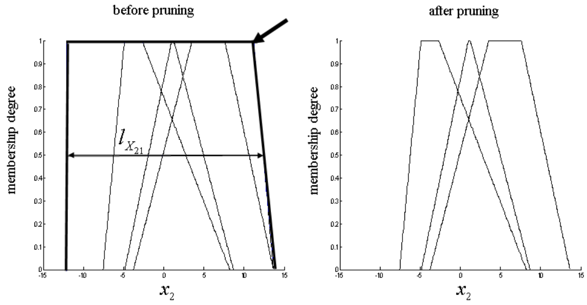

Fig. 7. Overlapping MFs obtained from the clusters projection. The thick solid line in the left Figure denotes the FS X_{21} to be pruned

To overcome this limitation, a triangular FS called "nearly zero" is forced in the partition of the UOD U_{x_p} of each variable x_p (Fig. 8). The new FS is centered in 0 and the zero-membership vertices are arbitrarily chosen equal to ± 0.1 of the minimum and maximum of the UOD U_{x_p} of the antecedent variable x_p, respectively. A rule tailored to stationary conditions can then be added to the FRB:

$$\begin{aligned} &\textit{if } (x_1 \textit{ is 'nearly zero'}) \textit{ and } (x_2 \textit{ is 'nearly zero'}) \\ &\quad \textit{then } (y_1 \textit{ is } Y_1^{NO}) \textit{ and } (y_2 \textit{ is } Y_2^{NO}) \\ &\quad \textit{and } (y_3 \textit{ is } Y_3^{NO}) \textit{ and } (y_4 \textit{ is } Y_4^{NO}) \qquad (9) \end{aligned}$$

By this addition, it is expected that when a Fact representative of the system in no-fault, stationary conditions is input to the fuzzy classification inference model, the above new rule will be the most activated so that the corresponding pattern is correctly classified.

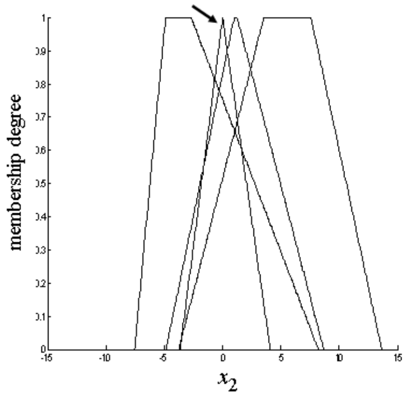

Fig. 8. FS "nearly zero" for variable x_2 (arrow)

4.2.3. Annihilation of narrow FSs

In order to avoid the overlapping among pairs of linguistic terms and the possible consequent semantic inconsistencies, it is necessary to have sufficiently distinct FSs [28]. If a FS X_{pj} is too narrow, its contribution is too specific and model transparency is somewhat lost. Annihilation of FS X_{pj} is performed if there is a FS X_{pi} for which the following criterion is satisfied (Fig. 9) [24]:

$$l_{X_{pi}} \mu_{X_{pi}} \left(\frac{z_{j,1} + z_{j,2} + z_{j,3} + z_{j,4}}{4} \right) \geqslant \beta_a l_{X_{pj}} \qquad i = 1, \ldots, c; \;\; j = 1, \ldots, c; \;\; i \neq j \qquad (10)$$

where $l_{X_{pi}}$ and $l_{X_{pj}}$ are the half-height widths of the FSs X_{pi} and X_{pj} of the same input variable x_p, $z_{j,s}$, $s = 1, 2, 3, 4$, stand for the input variable values corresponding to the four vertices of the trapezoidal MF of X_{pj}, $\beta_a \geqslant 1$ is the annihilation parameter that quantifies how much the FS X_{pi} covers the FS X_{pj}. The larger is the value of β_a, the more severe is the annihilation criterion [11,26]. The value of 1.5 for β_a was found by trial-and-error to produce optimal results in the case study analyzed in the present work. The degree of membership to X_{pi} of the symmetry center $\frac{z_{j,1} + z_{j,2} + z_{j,3} + z_{j,4}}{4}$ of FS X_{pj} is introduced in (10) because it is representative of the level of coverage of the two FSs [11,30,31]. If the two FSs do not intersect themselves, the membership value is 0; on the contrary, if they are identical the membership value is 1.

The FRB is appropriately modified by replacing the canceled FS X_{pj} with the FS X_{pi}.

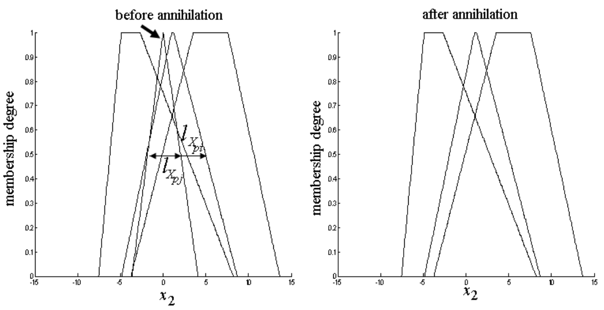

Fig. 9. Annihilation of a narrow FS (arrow)

4.2.4. *Fusion of similar FSs*

If two FSs describing the same variable are sufficiently overlapped, then they should be fused into a single FS because similar [11,26]. Appropriate measures can be used in order to asses the pairwise similarity of the FSs in the FRB.

The similarity measure Ω of the two FSs X_{pi} and X_{pj} here adopted is given by the ratio between the intersection and the union of their two areas [32]:

$$\Omega(X_{pi}, X_{pj}) = \frac{|X_{pi} \cap X_{pj}|}{|X_{pi} \cup X_{pj}|} = \frac{|X_{pi} \cap X_{pj}|}{|X_{pi}| + |X_{pj}| - |X_{pi} \cap X_{pj}|} \qquad (11)$$

If the value of Ω is higher than a pre-established threshold, the two FSs are deemed similar and they are fused. The four parameters $z_{\text{fus},s}$, $s = 1, 2, 3, 4$, of the new, fused trapezoidal MF will be:

$$z_{\text{fus},s} = \frac{z_{i,s} l_{X_{pi}} + z_{j,s} l_{X_{pj}}}{l_{X_{pi}} + l_{X_{pj}}}; s = 1, 2, 3, 4 \qquad (12)$$

where the $z_{\text{fus},s}$ are the values corresponding to the four vertices of the trapezoidal MF [11,30,31] resulting from the fusion, $z_{i,s}$, $z_{j,s}$ are the four vertices of the two fused FSs, and $l_{X_{pi}}$, $l_{X_{pj}}$ are the half-height widths of the FSs X_{pi} and X_{pj}, respectively.

The FRB is modified by replacing the fused FSs with their fusion (Fig. 10).

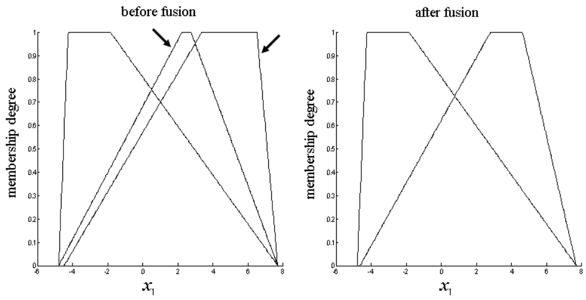

Fig. 10. Fusion of two similar FSs (arrows)

Finally, notice that the implementation of the steps described in this Section 4 modifies the fuzzy KB whose j-th rule takes the form:

$$\begin{aligned} &\textit{if } (x_1 \textit{ is } X_{1j}) \textit{ and } \ldots \textit{ and } (x_n \textit{ is } X_{nj}) \\ &\textit{then } (y_1 \textit{ is } Y_{1j}) \textit{ and } \ldots \textit{ and } (y_c \textit{ is } Y_{cj}) \end{aligned} \qquad (13)$$

The fuzzy rules thereby obtained are used for building the Fuzzy Inference Engine described in Section 2.

5. Case study: classification of transients in the feedwater system of a Boiling Water Reactor

5.1. Problem statement

The identification of a predefined set of faults in a Boiling Water Reactor (BWR) is considered. Transients corresponding to the faults have been simulated by the HAMBO simulator of the Forsmark 3 BWR plant in Sweden [33]. Figure 11 shows a sketch of the system [33].

The considered faults occur in the section of the feedwater system where the feedwater is preheated from 169 °C to 214 °C in two parallel lines of high-pressure preheaters while going from the feedwater tank to the reactor. Process experts have identified a set of 18 faults that are generally hard to detect for an operator and that produce efficiency losses if undetected [34]. The $c = 6$ faults regarding line 1 are here considered as the classes to be distinguished by the classification. These are numbered F1–F5 and F7, coherently with the original numbering [33]. Appendix B provides a brief description of the faults considered.

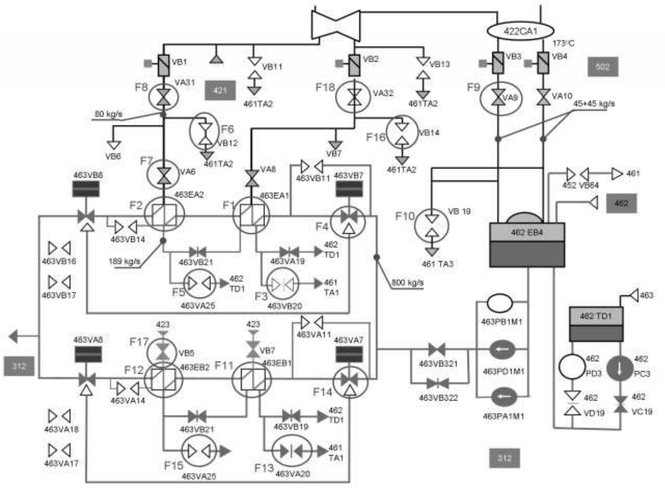

Fig. 11. Sketch of the feedwater system [33]

For each type of fault, the patterns to be used for building the classification model have been constructed by simulating transients with the plant at 80% of full power, taking values every 6 seconds from $t_{\text{in}} = 80\,s$ to $t_{\text{fin}} = 200\,s$.

Among the 363 monitored variables, only $n = 5$ have been chosen for the transient classification using the feature selection algorithm proposed in [35]: position level of control valve for preheater EA1 (PLV), temperature of drain 4 before valve VB3 (T1), water level of tank TD1 (WL), feedwater temperature after preheater EA2 (T2) and feedwater temperature after preheater EB2 (T3). Figure 12 reports an example of the evolution of the five monitored variables in correspondence of the six different simulated faults. The difference of the variables values from the steady state values are reported, because such deviations are those upon which the fault classification is based.

5.2. Application and results

The objective of the application is that of using the available pre-classified patterns of variables deviation for building a classifier based on fuzzy clustering and then extracting from it a set of transparent and accurate diagnostic rules for classifying the feedwater system faults. 80% of the available patterns have been used for building the classifier and the remaining 20% for testing its accuracy.

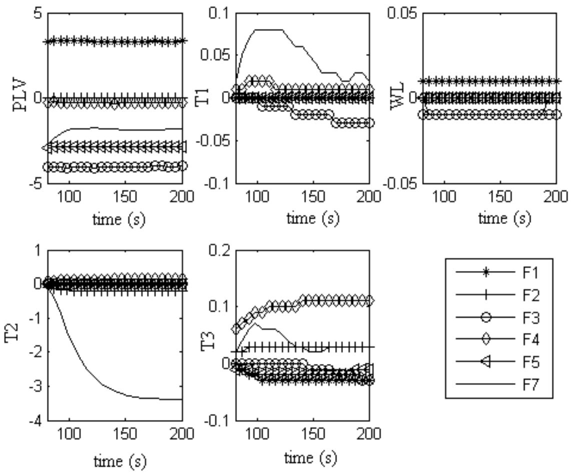

Fig. 12. Time-evolution of the five monitored variables deviations (in abscissa the time in seconds and in ordinate the variable deviation from steady state)

The application of the evolutionary algorithm for optimizing the possibilistic clustering model described in Section 3, supervised by the pre-classified data available from the simulated fault scenarios, leads by construction to 6 clusters, each one corresponding to a different type of fault. These are translated into a possibilistic clustering classifier, based on a KB of the form of Eq. (5) in which the multi-dimensional input FSs correspond to the multi-dimensional fuzzy clusters. The corresponding classification performance on the test patterns is reported in Table 1. Two defuzzification methods have been considered for the final class assignment of an incoming pattern $(x_1, x_2, \ldots, x_n)$ starting from the inferred output FSs $Y'_1, Y'_2, \ldots, Y'_c$ (Fig. 13):

- method I – the input pattern is crisply assigned to the class whose corresponding output y_q has the FS Y'_q with the largest membership grade to the linguistic label $\{\text{YES}\}$, $q = 1, 2, \ldots, c$.
- method II – the pattern is possibilistically assigned to all the classes whose corresponding output y_q, $q = 1, 2, \ldots, c$, has the FS Y'_q with membership value to the linguistic label $\{\text{YES}\}$ larger than a threshold γ (here chosen equal to 0.6). If none of the membership grades to the label $\{\text{YES}\}$ is larger than γ, then the pattern is labeled

'atypical'. If more than one membership grade is larger than γ, then the pattern is labeled 'ambiguous'.

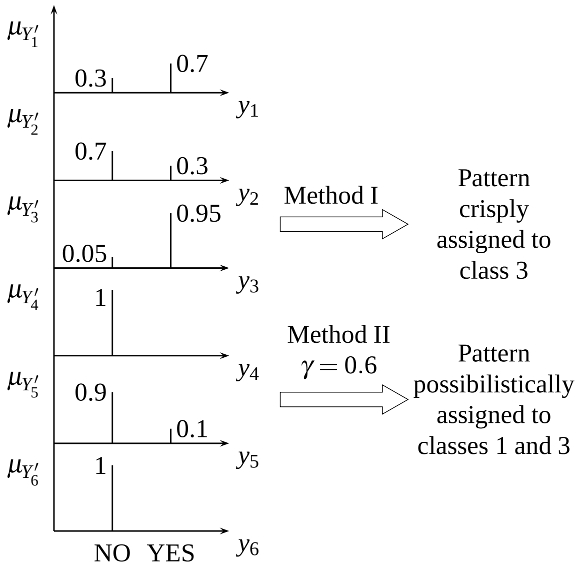

Fig. 13. Defuzzification methods I and II

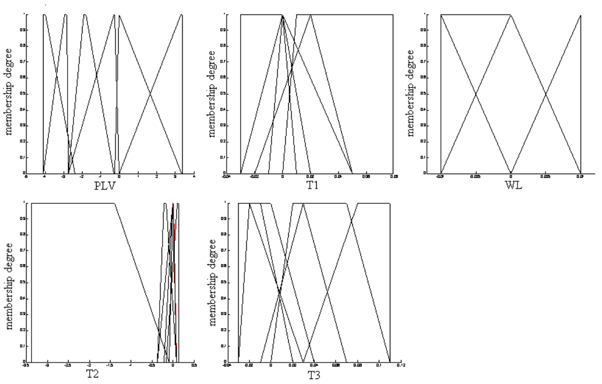

Fig. 14. Projection of the six clusters onto the input variables

Projecting the multi-dimensional clusters onto the UODs of the five antecedents corresponding to the five input variables, the FSs reported in Fig. 14, are obtained. With this partition of the $n = 5$ mono-dimensional antecedents UODs, a new fuzzy rule-based classification model is built, based on a KB with rules of the form of Eq. (13) and equal in number to the fault classes. The classification results of the test patterns are also presented in Table 1. Comparing these results with those obtained directly from the multi-dimensional input FSs representing the clusters, a minor deterioration of the classification performance is observed: one pattern previously correctly classified is now found atypical with both defuzzification methods. This minor decrease in the performance is due to the loss of information following the projection of the multi-dimensional FSs into their mono-dimensional constituents.

Applying the transparency constraints of Section 4 for obtaining an optimal partition of the UODs U_{x_p} of the input variables x_p, $p = 1, \ldots, 5$, the FSs in Fig. 15, are obtained.

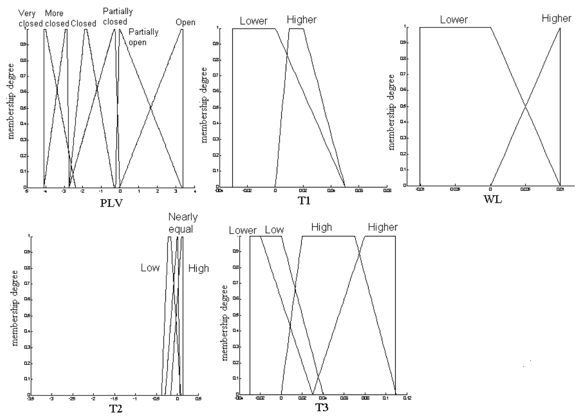

Fig. 15. Final partition of the inputs UODs

Table 1 reports the classification results after the application of each step of the procedure for obtaining a transparent FRB (Section 4). In particular, the step of pruning the FSs covering a large portion of the UOD results in the elimination of the two FSs obtained from the projection of the sixth cluster, representing the sixth class of fault, and in the consequential canceling of the two corresponding variables x_2 and x_4 from the antecedents of the associated sixth rule. After this modification of the FRB, the percentage of atypical patterns decreases from 8% (2 patterns) to 4% (1 pattern) because a pattern with the value of the input variable x_4 out of range, and thus previously labeled as atypical because not activating any rule, is now correctly classified. This is due to the fact that the input variable x_4 is no longer an antecedent of the sixth rule in the new FRB, so that the strength with which this rule is activated depends only on the values of the remaining three input variables that are such to give a non-zero strength of the sixth rule, thus leading to a correct classification of the pattern to the sixth class.

The successive steps of the procedure result in a more transparent FRB without decreasing the classification performance. In particular, all the test patterns are correctly classified except one pattern characterized by the first input variable with a value out of the range of the training patterns. This pattern is correctly labeled as atypical by the FRB of the classification model.

Table 1. Classification performances

Type of KB		Defuzzification method	Correct [%]	Error [%]	Ambiguous [%]	Atypical* [%]
Multi-dimensional input FSs		I	100	0	0	0
		II	96	0	0	4
Mono-dimensional input FSs after:	Projection	I	92	0	0	8
		II	92	0	0	8
	Pruning of FSs covering UOD	I	96	0	0	4
		II	96	0	0	4
	Addition of FS “nearly zero”	I	96	0	0	4
		II	96	0	0	4
	Annihilation of narrow FSs	I	96	0	0	4
		II	96	0	0	4
	Fusion of similar FS	I	96	0	0	4
		II	96	0	0	4

* Including the patterns with an input variable out of range.

To appreciate the transparency of the seven rules obtained after the last step of the proposed approach, Table 2 reports the resulting KB. Note that rule number 7 derives from the need of distinguishing patterns corresponding to the no-fault, stationary state (Section 4.2.2). Again, the original rule forced into the KB to meet the purpose may come out modified by the pruning process because of the elimination or fusion of some of the FSs “Nearly 0”.

Figure 16 presents the fraction of the 25 test data points correctly (top), incorrectly (middle) and not assigned (bottom) using defuzzification method II, as a function of the value of the threshold γ. The classification threshold γ offers an additional flexibility to the modelling and can be interpreted as a measure of confidence in the classification. For high confidence in the classification, one must assign a pattern to a class only if its membership is close to 1, e.g. by imposing a threshold $\gamma = 0.98$. In this case, many patterns may not be assigned to any class: in the present application, with $\gamma = 0.98$, about 40% of the patterns are assigned to the right class, 60% are not assigned and no misclassification or assignment to more than one class occurs; on the other hand, if one wishes to assign all patterns to a class, a low value of the threshold γ must be adopted, e.g. $\gamma = 0.28$. In this case, the number of patterns assigned to more than one class might increase unacceptably: in the present application, with $\gamma = 0.28$ about 58% are assigned to the correct class and 38% are assigned to more than one class. Thus, a compromise is sought. In the present application this is obtained with a value of $\gamma = 0.6$ which leads to satisfactory results since 96% of the test patterns are correctly classified and 4% are not assigned to any class, whereas there are no erroneously classified patterns, nor patterns assigned to more than one class.

6. Conclusions

An innovative procedure for building a transparent fuzzy logic model for pattern classification has been propounded, for use in fault diagnosis tasks. Differently from other classification techniques, the proposed approach aim at mining transparent fuzzy rules from data, emphasizing the linguistic interpretability of the acquired knowledge, which is a fundamental requirement for the application of any diagnostic tool in safety-critical fields like nuclear technology.

Starting from an available set of labeled patterns of monitored variables partitioned in different physical classes, a supervised evolutionary clustering algorithm, based on the Mahalanobis metric, is applied to find optimal geometric clusters, in the space of the monitored variables, which are as close as possible to the physical classes. The clusters found induce classification rules which are translated into a transparent if-then format by a procedure of fuzzy cluster projection, enforcement of appropriate constraints to the mono-dimensional FSs thereby obtained and their combination into transparent and physically interpretable fuzzy rules.

Table 2. The Table of rules of the KB

Rule		PLV	T1	WL	T2	T3		F1	F2	F3	F4	F5	F7
1	IF	Open	Lower	Higher	Nearly equal	Lower	THEN	Yes	No	No	No	No	No
2		Partially open	Lower	Lower	Low	High		No	Yes	No	No	No	No
3		Very closed	Lower	Lower	Nearly equal	Low		No	No	Yes	No	No	No
4		Partially closed	Higher	Lower	High	Higher		No	No	No	Yes	No	No
5		More closed	Lower	Lower	Nearly equal	Low		No	No	No	No	Yes	No
6		Closed	-	Lower	-	High		No	No	No	No	No	Yes
7		Partially open	Lower	Lower	Nearly equal	Low		No	No	No	No	No	No

The obtained fuzzy logic-based fault classification model provides as output the possibilistic membership grades to the different classes, thus explicitly accounting for the ambiguities of the classification problem inherent in its characterizing input features, which may lead to the misclassification or vague classification of certain patterns.

The methodology has been successfully applied to a test case regarding the classification of a predefined set of faults in the feedwater system of a Boiling Water Reactor. The considered faults have been identified by experts as non-critical from a safety point of view but of major concern because leading to significant losses of energy production while quite difficult to detect and classify.

The diagnostic results obtained with the proposed approach are satisfactory in terms of both classification accuracy and model interpretability.

On the other hand, other diagnostic problems may be characterized by physical classes that are highly overlapping and little compact in the monitored variables space. In this case, direct application of the procedure currently developed for cluster projection into convex fuzzy sets is most likely to lead to non informative fuzzy sets that would result in a low classification performance. In this respect, work is ongoing for the extension of the procedure by allowing the possibility of projecting a single cluster into more than a single FS, i.e. a non-convex FS, for higher model resolution.

Finally, the sensitivity of the model to the parameters controlling the pruning, annihilation and fusion steps of the procedure is under study, with the objective of providing additional guidelines for their selection.

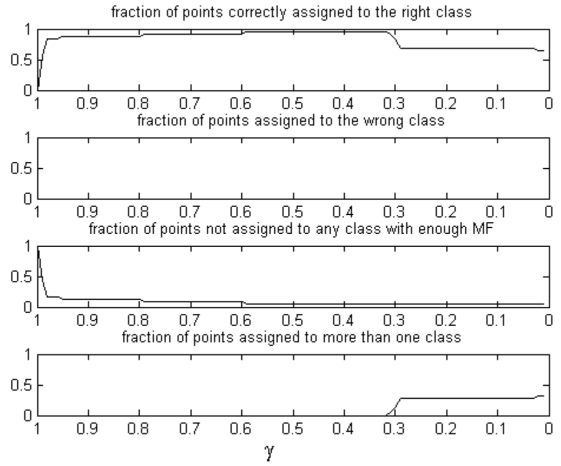

Fig. 16. Classification performance as a function of the confidence threshold γ

Acknowledgements

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Nomenclature

BWR	Boiling Water Reactor
FL	Fuzzy Logic
FRB	Fuzzy Rule Base
FS	Fuzzy Set
KB	Knowledge Base
MF	Membership Function
UOD	Universe of Discourse
PLV	Position Level of control Valve for preheater EA1
T1	Temperature of drain 4 before valve VB3
T2	feedwater Temperature after preheater EA2
T3	feedwater Temperature after preheater EB2
WL	Water Level of tank TD1
R	number of rules in FRB
j	index of the generic rule, $j = 1,\ldots,R$
x_p	antecedent linguistic variable, $p = 1,\ldots,n$
X_{pj}	FS of the p-th antecedent in the j-th rule
U_{x_p}	UOD of x_p
$\mu_{X_{pj}}(x_p)$	MF of x_p to X_{pj}
y_q	consequent linguistic variable, $q = 1,\ldots,m$
Y_{qj}	FS of the q-th consequent in the j-th rule
U_{y_q}	UOD of y_q
$\mu_{Y_{qj}}(y_q)$	MF of y_q to Y_{qj}
t	t-norm intersection operator, in this work taken as the minimum operator
s	s-norm, in this work taken as the maximum operator
X'_p	generic FS of the p-th antecedent in U_{x_p} representing the "fact"
Y'_q	generic FS of the q-th consequent in U_{y_q} representing the "conclusion"
$\overrightarrow{x}_k$	k-th generic n-dimensional pattern
N	total number of available patterns $\overrightarrow{x}_k$
c	a priori known number of classes
$\Gamma^t \equiv (\Gamma^t_1,\ldots,\Gamma^t_c)$	a priori known physical class-membership partition

$\mu_i(\overrightarrow{x}_k)$	possibilistic membership of the pattern $\overrightarrow{x}_k$ in the i-th cluster, i.e. degree of similarity of the pattern $\overrightarrow{x}_k$ with the cluster center $\overrightarrow{v}_i^*$
$\Gamma \equiv (\Gamma_1, \ldots, \Gamma_c)$	obtained geometrical cluster partition
$\mu_i^t(\overrightarrow{x}_k)$	a priori known possibilistic membership of $\overrightarrow{x}_k$ to class i
Γ_i	generic cluster i
X_i	FS associated to the i-th cluster
$Y_q^{\mathrm{NO}}, Y_q^{\mathrm{YES}}$	consequent singleton FSs
w_j	degree of fulfillment of an input pattern to the premise of the j-th rule
ε	minimum coverage level
$l_{X_{pi}}$	width at half height of the FS X_{pi} of the variable x_p
β_o	overlap parameter
β_a	annihilation parameter
$z_{j,s}$	$s = 1,2,3,4$, four vertices of the trapezoidal FS X_{pj}
$\Omega(X_{pi}, X_{pj})$	similarity measure of the two FSs X_{pi} and X_{pj}
γ	confidence threshold used for the classification
ξ	predefined threshold used in the clustering algorithm
$\overrightarrow{v}_i^*$	i-th cluster center
s_{ik}	distance between $\overrightarrow{x}_k$ and $\overrightarrow{v}_i^*$
$\underline{\underline{M}}_i$	matrix defining the Mahalanobis metric of the i-th cluster
T	transpose operator

Appendix A. The supervised evolutionary possibilistic clustering classifier

The overall iterative training scheme can be summarized as follows:

1. At the first iteration ($\tau = 1$), initialize the metrics of all the c clusters to the Euclidean metrics, i.e. $\underline{\underline{M}}_i(1) = \underline{\underline{I}}$, $i = 1,2,\ldots,c$, where $\underline{\underline{I}}$ is the identity matrix.

2. At the generic iteration step τ, run the possibilistic clustering algorithm [17] to partition the N training data into c clusters of memberships $\Gamma(\tau) = \{\Gamma_1(\tau), \ldots, \Gamma_c(\tau)\}$, based on the current metrics $\underline{\underline{M}}_i(\tau)$ and on the "supervising" initial partition Γ^t which sets the initial memberships of the N patterns to c clusters equal to the true memberships to the a priori known classes. Then $\Gamma(\tau)$ is set equal to the obtained optimal partition $\Gamma^* = \{\Gamma_1^*, \ldots, \Gamma_c^*\}$.

3. Compute the distance $D(\Gamma^t, \Gamma(\tau))$ between the a priori known physical classes and the geometric possibilistic clusters by Eq. (2). At the first iteration ($\tau = 1$) initialize the best distance D^+ to $D(\Gamma^t, \Gamma(1))$, D_i^+ to $D(\Gamma_i^t, \Gamma_i(1))$ and the best metrics $\underline{\underline{M}}_i^+$ to $\underline{\underline{M}}_i(1)$ and go to step 5.

4. If $\Gamma(\tau)$ is close to Γ^t, i.e. $D(\Gamma^t, \Gamma(\tau))$ is smaller than a predefined threshold ξ, or if the number of iterations τ is greater than the predefined maximum allowed number of iterations $\tau_{\max}$, stop: $\Gamma(\tau)$ is the optimal cluster partition Γ^*; otherwise, if $D(\Gamma^t, \Gamma(\tau))$ is less than D^+ upgrade D^+ to $D(\Gamma^t, \Gamma(\tau))$, $\underline{\underline{M}}_i^+$ to $\underline{\underline{M}}_i(\tau)$ and $D_i^+ = D(\Gamma_i^t, \Gamma_i(\tau))$.

5. Increment τ by 1. Update each matrix $\underline{\underline{M}}_i^+$ by exploiting its unique decomposition into Cholesky factors [5], $\underline{\underline{M}}_i^+ = \left\{\underline{\underline{G}}_i^+\right\}^{\mathrm{T}} \underline{\underline{G}}_i^+$, where $\underline{\underline{G}}_i^+$ is a lower triangular matrix with positive entries on the main diagonal. More precisely, at iteration τ, the entries $g_{l_1,l_2}^i(\tau)$ of the Cholesky factor $\underline{\underline{G}}_i(\tau)$ are updated as follows:

$$g_{l_1,l_2}^i(\tau) = g_{l_1,l_2}^{i+} + N_{l_1,l_2}^i(0, \delta_i^+) \quad \text{if } l_1 < l_2 \tag{A.1}$$

$$g_{l_1,l_2}^i(\tau) = \max\left(10^{-5}, g_{l_1,l_2}^{i+} + N_{l_1,l_2}^i(0, \delta_i^+)\right) \quad \text{if } l_1 = l_2 \tag{A.2}$$

where $\delta_i^+ = \alpha D_i^+$, α is a parameter that controls the size of the random step of modification of the Cholesky factor entries g_{l_1,l_2}^{i+}, N_{l_1,l_2}^i denotes a Gaussian noise with mean 0 and standard deviation δ, and

Eq. (A.2) ensures that all entries in the main diagonal of the matrices $\underline{\underline{G}}_i(\tau)$ are positive numbers and so $\underline{\underline{M}}_i(\tau)$ are definite positive distance matrices. Notice that the elements of the *i*-th Mahalanobis matrix are updated proportionally to the distance D_i^+ between the *i*-th a priori known class and the *i*-th cluster found. In this way, only the matrices of those clusters which are not satisfactory for the classification purpose are modified.

6. Return to step 2.

Appendix B. Details on the fault types considered in the case study

F1	leakage through the second high-pressure preheater (line 1). A leakage from the primary incoming side to the primary outgoing side means that part of the feedwater will pass the heater without being heated. If it is a big leak it can be detected by looking at the temperature after the preheater, but it's hard to detect if you do not know it. The consequences are loss of efficiency. Equipment involved: 463EA1 - preheater
F2	leakage in the first high-pressure preheater (line 1) to the drain tank. A leakage from primary side to secondary side means that part of the feedwater will go to the drain side of the preheater instead of continuing to the reactor. If it is a bigleak it can be detected by the position of the drain valve which is more open than it should be, but it is hard to detect if you do not know it. The consequences are loss of efficiency. Equipment involved: 463EA2 - second preheater
F3	Leakage through the first high-pressure preheater drain back-up valve (line 1) to the condenser. A leakage here means that part of the drain water will go to the condenser instead of going to the feed water tank. If it is a big leak it can be detected by the position of the ordinary drain valve, it is less open than itshould be, but it's hard to detect if you do not know it. The consequences are loss of efficiency. Equipment involved: 463VB20 -drain back-up valve. Fault simulated by: Introduce a leak through the closed valve
F4	Leakage through line 1 high-pressure preheaters bypass valve. A leakage here means that part of the feedwater will not be heated in the preheater line. If it is a big leak it can be detected by seeing that the temperature after that line is lower than the temperature after the other line. But it's hard to detect if you don't know it. The consequences are loss of efficiency. Equipment involved: 463VB7.2 - bypass valve. Fault simulated by introducing a leak through the closed valve
F5	Leakage through the second high-pressure preheater drain back-up valve(line 1) to the feedwater tank. A leakage here means that part of the drain water will go to the feedwater tank directly instead of going through the first preheater. If it is a big leak it can be detected by the position of the drain valve, it's less open than it should be but it's hard to detect if you don't know it. The consequences are loss of efficiency. Equipment involved: 463VA25 - drain back-up valve. Fault simulated by introducing a leak through the closed valve
F7	Steam line valve to the second high-pressure preheater (line 1) closing. Closing of this valve means that less steam will go through the preheater. If it is closing much it can be detected by the position of the drain valve, it's less open than it should be, but it's hard to detect if you don't know it. The consequences are loss of efficiency. Equipment involved: 423VA6 - Steam line valve. Fault simulated by closing a valve that is monitored to be open and re-defining the state "valve open" as "valve 40% open". Closing the valve, up to 60% (i.e. 40% open)

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