Abstract—Synthetic aperture radar (synthetic aperture radar, SAR) use a two-dimensional matched filter to improve the signal to noise ratio, but the signal-to-noise ratio (SNR) gain of synthetic aperture radar (SAR) is controversial. The traditional method analyzes the SAR signal-to-noise ratio based on pulse compression and coherent integration, but it will have different calculation results. In this paper, in connection with the characteristics of the SAR digital signal processing, the SNR gain of the two-dimensional matched filter is derived to solve the contentious issue of SAR signal-to-noise ratio, which is based on the assumption of the band-limited white noise in the digital domain, and the impact of matched filtering on SNR is analyzed and the optimized design criteria of the SAR system design.

Keywords—synthetic aperture radar (SAR); SNR; pulse compression; optimized design; matched filter

I. INTRODUCTION

Synthetic aperture radar (synthetic aperture radar, SAR) combine the pulse compression and synthetic aperture radar technology[1,2], which uses pulse compression technology to achieve high resolution in the distance dimension. The azimuth dimension high resolution rely on coherent processing of the long synthetic aperture. In the pulse compression radar, the improvement for SNR ratio is presented with the compressed ratio form of the pre-compression pulse width $\tau_i$ and the compressed pulse width $\tau_0$, it is easy to know that the signal-to-noise gain for SAR distance of the pulse compression processing is the product of time-width and band-width[3]. In using the synthetic aperture forming techniques to achieve the required azimuth resolution of the radar, because of the accumulation of a large number of pulses, so the processing to the azimuth signal is also increased by a improvement factor of SNR.

SAR coherently accumulate $n$ echo signals within a synthetic aperture time $T_a$, from the perspective of coherent integration, the azimuth processing gain of SNR is the accumulated number of pulses in azimuth, which is equal to the product of the pulse repetition frequency and the time required to form a synthetic aperture[4]; however, from the perspective of matched filtering, SAR azimuth processing is also equivalent to the matched filtering process, then the SNR gain is the product of azimuth Doppler signal bandwidth and the accumulation time[5]. From this point, there are two calculation methods about SNR gain currently[6-8], but the calculation results will be different. Therefore, the SAR SNR gains for a two-dimensional signal processing need be analyzed.

In this paper, Aiming at the characteristics of SAR signal processing, the SNR gain formula for digital matched filter is derived from a continuous signal matched filtering process, then the two-dimensional SNR gain after SAR matched filtering process is mathematical analyzed, and this paper verify the theoretical analysis through simulation results finally, solving the disputed issue of SAR SNR gain.

II. THE SNR GAIN OF DIGITAL MATCHED FILTER

In the process of SAR imaging, the method of two-dimensional matched filtering can improve the signal-to-noise ratio[9,10], but there is debating about the SNR gain. For example on the SNR gain of SAR radar equation, Azimuth matched filter SNR gain sometimes use the product of the Doppler bandwidth and the accumulation time, while others are the product of the pulse repetition frequency and accumulation time . According to the literature[12,13], it is easy to know the matched filter is the output signal-to-noise largest ratio of the filter under the white noise condition, but the SNR gain analysis is primarily based on continuous time signal, and on the formation of the SAR echo signal, The form of azimuth signal has been discrete based on the assumption of "stop and go". In addition, in the process of project implementation, it tend to adopt the way of digital signal processing to realize two-dimensional matched filtering, so it is necessary to analyze the SNR gain after discrete matched filtering[14]. So there has discussion about one-
dimensional discrete time signals for the improvement of signal-to-noise ratio.

Suppose the input signal of matched filter is \( s(t) \), and its corresponding frequency spectrum can be represented as:
\[
S(f) = \begin{cases} E_0 & 0 \leq f < f_c + B_s/2 \\ 0 & \text{else} \end{cases}
\]  
(1)
Where \( E_0 \) is the amplitude spectrum of the signal, \( B_s \) is the signal bandwidth, \( f_c \) is the Center frequency.

Suppose the input random noise is the smooth band-limited white noise, and its power spectrum can be represented as:
\[
N(f) = \begin{cases} N_0 & -B_n/2 < f < B_n/2 \\ 0 & \text{else} \end{cases}
\]  
(2)
Where \( N_0 \) is the power spectral density, \( B_n \) is noise bandwidth, so the input noise power is:
\[
P_{in} = \int_{-\infty}^{\infty} N(f) df = N_0 B_n
\]  
(3)
Then, after the matched filter \( H(\omega) = S^*(\omega) \), the matched filter SNR gain can be expressed as:
\[
G_{SNR} = \frac{E_P^2}{\int_{-\infty}^{\infty} N(f) |H(f)|^2 df} = \frac{E_P^2}{\int_{-\infty}^{\infty} N(f) df} N_0 E_P
\]  
(4)
Where \( SNR_0 \) is the output signal-to-noise ratio, \( SNR_1 \) is the input signal-to-noise ratio, the energy of the signal \( E_P \) can be represented as:
\[
E_P = \int_{-\infty}^{\infty} |S(f)|^2 df
\]  
(5)
The SNR gain of the formula (4) can be expressed as following expression for the condition of continuous time signals:
\[
G = \frac{E_P T_P}{\int_{-\infty}^{\infty} N(f) df} = \frac{E_P T_P}{\int_{-\infty}^{\infty} N(f) df} N_0 E_P
\]  
(6)
After the input continuous random noise digital sampling, white noise becomes colored noise, the noise power spectrum will change along with the change of sampling rate, discrete SNR gain can be obtained:
\[
G_m = \frac{E_P T_P}{\int_{-\infty}^{\infty} N(f) df} = \frac{B_s T_P}{\sum_{n=-\infty}^{\infty} \int_{n f_s/2}^{(n+1) f_s/2} N(f - n f_s) df} = \frac{B_s T_P}{\sum_{n=-\infty}^{\infty} \int_{n f_s/2}^{(n+1) f_s/2} N(f - n f_s) df}
\]  
(7)
Where \( f_s \) is the sampling rate of the signal. Molecular summation item of the formula (7) represents the input noise power after sampling, and The denominator summation item represents output noise power after matched filtered, make:
\[
P_m = \sum_{n=-\infty}^{\infty} \int_{n f_s/2}^{(n+1) f_s/2} N(f - n f_s) df = \sum_{n=-\infty}^{\infty} \int_{n f_s/2}^{(n+1) f_s/2} N(f - n f_s) df
\]  
(8)
\[
P_{in} = \sum_{n=-\infty}^{\infty} \int_{n f_s/2}^{(n+1) f_s/2} N(f) df = \sum_{n=-\infty}^{\infty} \int_{n f_s/2}^{(n+1) f_s/2} N(f) df
\]  
(9)
Suppose \( x = (f - nf_s)/f_s \), The input noise power is:
\[
P_{in} = \sum_{n=-\infty}^{\infty} \int_{n f_s/2}^{(n+1) f_s/2} N(s_f) df = \sum_{n=-\infty}^{\infty} \int_{n f_s/2}^{(n+1) f_s/2} N(s_f) df
\]  
(10)
Where:
\[
N(s_f) = \begin{cases} N_0 & -B_n/2 < f < B_n/2 \\ 0 & \text{else} \end{cases}
\]  
(11)
In the same way, the available output noise power is:
\[
P_{on} = \sum_{n=-\infty}^{\infty} \int_{n f_s/2}^{(n+1) f_s/2} \frac{B_s T_p}{N_0 f_s} N(s_f) df = \sum_{n=-\infty}^{\infty} \int_{n f_s/2}^{(n+1) f_s/2} \frac{B_s T_p}{N_0 f_s} N(s_f) df
\]  
(12)
The following is mainly for different noise bandwidth \( B_n \), the sampling rate \( f_s \) and the bandwidth of the signal \( B_s \), analyzing the gain of the signal to noise ratio:\n
\[ (1) \text{ if } B_n < 2 f_s \leq 1, \text{then } f_s \geq B_n \geq B_s \]  
(13)
\[ P_{in} = \sum_{n=-\infty}^{\infty} \int_{n f_s/2}^{(n+1) f_s/2} N(s_f) df = N_0 B_n \]  
(14)
Thus you can get the SNR gain:
\[
G_m = \frac{B_s T_P}{\sum_{n=-\infty}^{\infty} \int_{n f_s/2}^{(n+1) f_s/2} N(f - n f_s) df} = \frac{B_s T_P}{B_n T_P}
\]  
(15)
From the formula can be seen, when meeting the relation of condition (1), the signal-to-noise ratio after the matched filtering gain is the product of noise bandwidth and time-width of signal, filter bandwidth and signal bandwidth is usually approximately equal in radar receiving[14], we can think SNR gain as the product of the time-width and signal bandwidth. However, when the radar system needs to receive a variety of different bandwidth of the signal, if the coverage of signal bandwidth is very large, in order to reduce hardware resources and reduce the complexity of the system, the receiver filter often require the biggest bandwidth according to the signal. When receiving small signal bandwidth, noise bandwidth will be greater than the signal bandwidth, so this paper need to analysis the different conditions SNR gain.

\[ (2) \text{ if } B_n > f_s > 1, \text{ then } B_n \geq f_s \geq B_s \]  
(2)
Suppose \( p = \left[ \frac{f_s}{B_s} + 1 \right] \), where \( [\cdot] \) is presented as rounded part. \( q = \frac{B_n}{f_s} + 1 - p \), the formula (15) the denominator summation item contains \( 2 p + 1 \) integral interval, including \( 2 p - 1 \) complete integral and two non-completed integral interval.
\[
p_{in} = \sum_{n=-\infty}^{\infty} \int_{n f_s/2}^{(n+1) f_s/2} N(s_f) df = f_n N_0 \left( 2 p - 1 + 2 q \right) = N_0 B_n
\]  
(16)
\[
p_{on} = \left( 2 p - 1 \right) N_0 B_n + \left[ \frac{B_n}{f_s} - \frac{B_s}{f_s} \right] N(s_f) df = \left( 2 p - 1 \right) N_0 B_n + \left[ \frac{B_n}{f_s} - \frac{B_s}{f_s} \right] N(s_f) df
\]  
(17)
Where, the output noise power of the formula (17) needs the relations between the noise bandwidth, signal bandwidth, and sampling rate, according to this, the following which can be divided into three conditions are discussed:
\[ (1) \text{ if } q \geq \frac{1 + \frac{B_n}{2 f_s}}{2}, \text{then} \]  
(18)
\[ p_{on} = (2p-1)N_0B_s + 2N_0B_k = (2p+1)N_0B_k \] (18)

\[ P_{on} = (2p-1)N_0B_k \]

\[ \text{if } q \leq 1 - \frac{B_k}{2f_s}, \text{ then} \]

\[ P_{on} = N_0 \left[ B_n - 2p \left( f_s - B_k \right) \right] \]

The prior to azimuth matched filtering process, the analogy with the distance to the matched filter can be considered as input the noise bandwidth \( B_s \), the sampling rate \( F_r \), azimuth signal bandwidth \( B_d \). The same

\[ k B F_s \quad l B F k_s \]

\[ l \geq \frac{1}{2} \quad \frac{B_d}{2 F_r} \]

\[ l \leq \frac{1}{2} \quad \frac{B_d}{2 F_r} \]

\[ N_0 \left( 2kB_d + 2F_r - F_s \right) \]

The noise power is kept constant after sampling, then:

\[ \sum_{n=-\infty}^{\infty} \frac{B_k/2}{B_n/2} N \left( f - nF_s \right) df = \sum_{n=-\infty}^{\infty} \frac{F_r/2}{B_d/2} N_L \left( f - nF_r \right) df \]

Thus you can get the SNR gain of the matched filtering after the discrete sampling:

\[ G_m = \begin{cases} \frac{1}{2p+1} \frac{B_k}{B_n} & q > \frac{1}{2} \frac{B_k}{f_s} \\ \frac{1}{2p+1} \frac{B_k}{B_n} & q \leq \frac{1}{2} \frac{B_k}{f_s} \end{cases} \]

The SNR gain of the two-dimensional matched filtering can be expressed as:

\[ G_s = \frac{B_s T_p}{\sum_{n=-\infty}^{\infty} \frac{f_s/2}{B_k/2} N \left( f - nF_s \right) df} \times \]

\[ \sum_{n=-\infty}^{\infty} \frac{B_k/2}{B_d/2} N_L \left( f - nF_r \right) df \]

\[ \frac{B_s T_p}{B_d T_s N_0 B_n} \]

When matching filter in azimuth, the analogy with the distance to the matched filter can be considered as input the noise bandwidth \( B_k \), the sampling rate \( F_r \), azimuth signal bandwidth \( B_d \). The same

\[ k B F_s \quad l B F k_s \]

\[ l \geq \frac{1}{2} \quad \frac{B_d}{2 F_r} \]

\[ l \leq \frac{1}{2} \quad \frac{B_d}{2 F_r} \]

\[ N_0 \left( 2kB_d + 2F_r - F_s \right) \]

Where, \( N_0 \) is the equivalent noise power spectrum before azimuth processing. After the distance processing is completed, the noise bandwidth is limited within the signal bandwidth, the equivalent noise power spectrum is:
From the formula (27), (28) and (29) can be seen, the signal-to-noise ratio of the two-dimensional matched filtering gain are related with signal bandwidth, Doppler-bandwidth, noise bandwidth and $F_r$. In order to meet the requirements of the two dimensional resolution usually in SAR system design, the formula $B_s >> F_r \geq B_d$, $k \geq 1$ is easy to satisfy, and the SNR gain can be approximate to:

$$G_s = \frac{B_s T_p T_r B_n}{2 \pi} = \frac{B_s T_p T_r B_n}{B_d / F_r} = \frac{T_p T_r B_n F_r}{2 \pi}$$  

Therefore, the two-dimensional processing gain of the SAR can approximate thought as the following conditions, the processing gain of the distance is the product of the noise bandwidth and signal time width, Azimuth processing gain is the number of accumulated pulses. and the SAR SNR gain according to the formula (30) need be calculated precise under the condition of other specific parameters.

IV. SIMULATION ANALYSIS

In the system parameter conditions of the following table 1, simulation on the signal-to-noise ratio gain of digital matched filtering processing.

<table>
<thead>
<tr>
<th>TABLE 1</th>
<th>SYSTEM PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters name</td>
<td>value</td>
</tr>
<tr>
<td>Signal bandwidth $B_s$</td>
<td>5MHz</td>
</tr>
<tr>
<td>Sampling rate $f_s$</td>
<td>50MHz</td>
</tr>
<tr>
<td>Pulse width $T_p$</td>
<td>10μs</td>
</tr>
<tr>
<td>Accumulation time $T_r$</td>
<td>0.2s</td>
</tr>
<tr>
<td>The pulse repetition frequency $F_r$</td>
<td>2000Hz</td>
</tr>
</tbody>
</table>

Adopting the method of theoretical calculation, matched filtering SNR gain and noise bandwidth relations can be achieved in Fig 1, from the Fig 1 we can see, when the noise bandwidth $2nf_s + B_s \leq B_n \leq 2(n + 1)f_s - B_s$ is met, SNR gain increase monotonously with the noise bandwidth; When the following formula $2(n + 1)f_s - B_s \leq B_n \leq 2f_s + B_s$ is met, the SNR gain monotonously decrease with noise bandwidth. Therefore, the signal-to-noise ratio of the matched filtering gain is relevant with noise bandwidth under the condition of the discrete band-limited white noise, it can present a cyclical change, and it can obtain the maximum periodically.

This paper uses the method of monte carlo simulation results which can verify the theoretical derivation, the signal-to-noise ratio of the matched filtering gain simulation results are shown in Fig 2 under the condition of test times for 5000 times, after many repeated trials, the SNR gain take the average of the many test results, table 2 is presented under the condition of different noise bandwidth, the matched filtering SNR gain simulation results can be contrast with the theoretical calculation. Because of the noise statistical characteristics, the theoretical value and the simulation results are basically consistent, so as to verify the validity of the theoretical derivation. when the noise bandwidth $B_n=95MHz$, SNR gain maximum, the relationship $B_n = 2f_s - B_s$ can be met at this time, the calculation result is consistent with the theory of formula (21). In addition, when the system noise bandwidth and signal bandwidth are fixed, output signal-to-noise ratio of the system can achieve the optimal value by reasonably choosing the sampling rate, the optimal sampling rate can be designed as $f_s^* = (B_n + B_s) / 2$.
Figure 2. The noise bandwidth SNR gain of different conditions

<table>
<thead>
<tr>
<th>Noise-bandwidth Bn (MHz)</th>
<th>The simulation results (dB)</th>
<th>The theoretical calculation values (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>24.08</td>
<td>23.98</td>
</tr>
<tr>
<td>95</td>
<td>29.38</td>
<td>29.78</td>
</tr>
<tr>
<td>105</td>
<td>25.81</td>
<td>25.44</td>
</tr>
</tbody>
</table>

The following is two-dimensional matched filtering SNR gain simulation, as shown below, from Fig. 3 can be seen, SNR gain of the two-dimensional matched filtering is a periodic variation along with the noise bandwidth.

Two-dimensional matched filtering is also cyclical changes, the comparing simulation results with theoretical calculations of the two-dimensional matched filter SNR gain are given in table 2 under different noise bandwidth conditions, from table 3 can be seen, when the noise bandwidth $B_n = 2f_s - B_s = 95\text{MHz}$, the two-dimensional matched filtering SNR gain maximum. This is mainly due to that the Doppler bandwidth and the transmitted signal bandwidth have several orders of magnitude, the impact on SNR is almost negligible.

Simulation on the two-dimensional matched filter SNR gain and PRF at the different Doppler bandwidth conditions, from Fig. 4 (a) can be seen, the SNR gain increases with increasing PRF, which is coincide with analysis conclusions of formula (30). From a partial enlarged view of Fig. 4 (b) can be seen, in a typical SAR system parameters, the SNR gain is positively correlated with the PRF, and the affect of the Doppler bandwidth is small, it can be ignored, which are coincide with analysis conclusions of the formula (30).
Figure 4. Two-dimensional matched filtering of the relationship between SNR gain and PRF.

(b) After the partial enlarged

V. CONCLUSIONS

According to the theoretical analysis and simulation results, discrete sampling will lead white noise to colored noise, SNR gain of the two-dimensional matched filters has relationship with noise bandwidth pulse repetition frequency, signal bandwidth, sampling rate in the colored noise conditions, when certain conditions are met, it is equivalent to the traditional matching filter SNR gain. Secondly, SNR gain maxima occur periodically by analyzing the impact of noise bandwidth of SNR under the discrete band-limited white noise conditions, when the sampling rate, the noise bandwidth, signal bandwidth meet the \( f_s^* = \frac{B_s + B_n}{2} \), matched filtering SNR gain reaches maximum, signal to noise ratio can be optimized design based on the maximum sampling rate of SAR guidelines.

REFERENCES


